<Week2> 1

* Grasp Statics(1): Planar Frictionless Grasps

- VOD1. Degrees of freedom, force closure, form closure

Last time: Mobility

Grubler's formula:

- Planar version:

  - $DOF = 3(N-1-J) + \sum_{i=1}^{J} f_i$

  N = # of links (including ground)
  J = # of joints
  f_i = dof of joint i

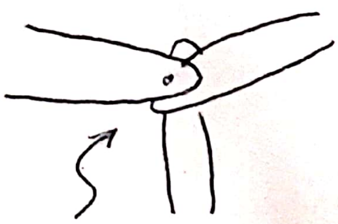
- Spatial version:

  - $DOF = 6(N-1-J) + \sum_{i=1}^{J} f_i$

- Points of Caution:

(i) Use the right version!

(ii)

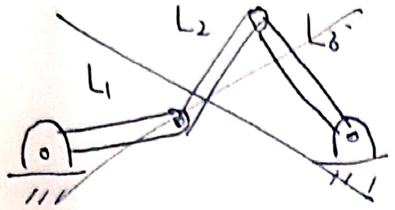


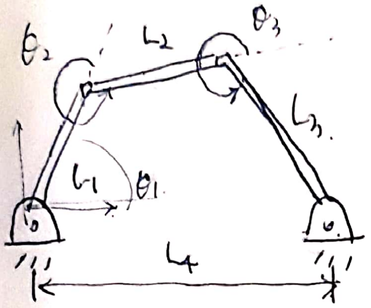
several overlapping joints

(iii) Singular mechanisms (Grubler fails)

· Configuration space (will discuss in closed chapter):

Ex)

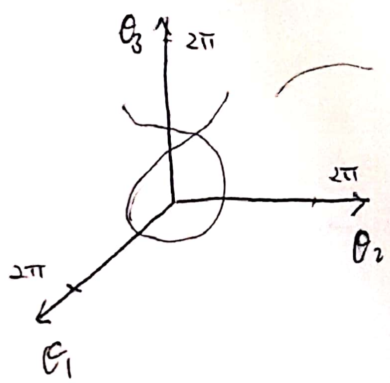




- Regard as 3-link planar open chain with tip fixed at $(L_4, 0)$:

$$L_1 \cos\theta_1 + L_2 \cos(\theta_1+\theta_2) + L_3 \cos(\theta_1+\theta_2+\theta_3) = L_4$$
$$L_1 \sin\theta_1 + L_2 \sin(\theta_1+\theta_2) + L_3 \sin(\theta_1+\theta_2+\theta_3) = 0$$

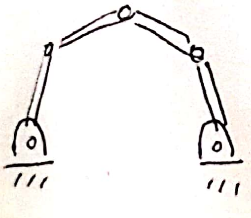
2 equations
3. unknowns $(\theta_1, \theta_2, \theta_3)$
→ 1 dof

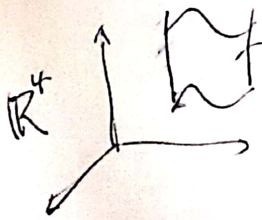


curve in $\mathbb{R}^3$ is configuration space

↳ The set of all possible configuration of the robot.

Ex)





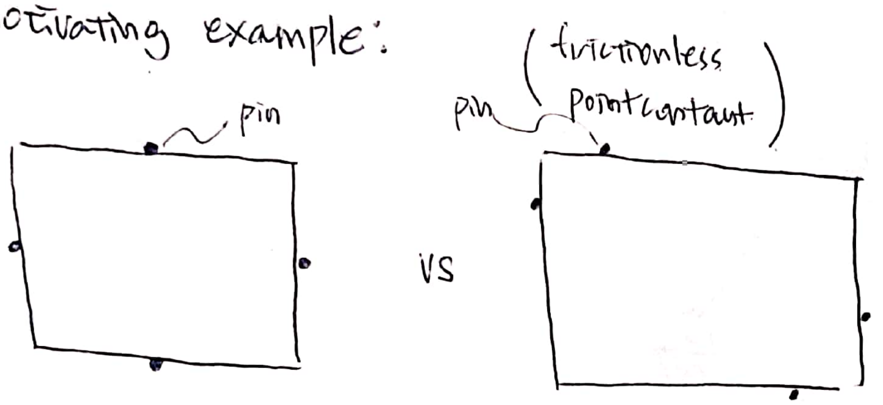
(고차원) 2-dim surface in $\mathbb{R}^4$

2 equations
4. unknowns
→ 2 dof.

• VOD2. Grasp contact models

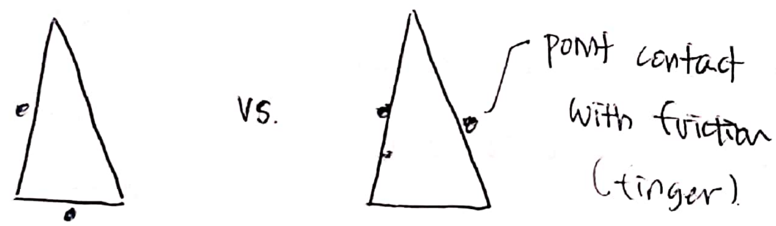
• Grasping: Form and Force closure

· motivating example:



Which is better? Right one

· Another example



Which is better? Right one

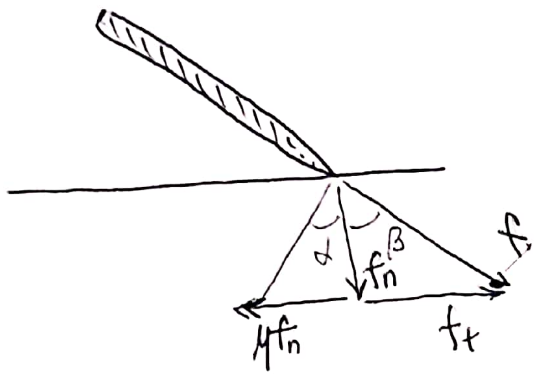
• Form closure -

- an object is completely immobilized by
  a set of point contact

• Force closure

- fingers can apply forces
  to resist any external forces on object.

Contact Models



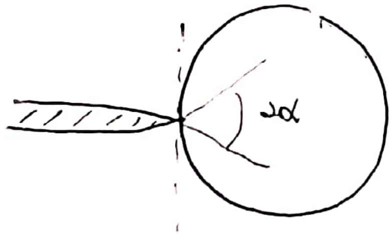
- Slipping occurs when $|f_t| > \mu|f_n|$

  $\mu \sim$ Coulomb friction coefficient

  - Equivalently, slip $\leftrightarrow$ $\alpha < \beta$

    $\alpha = \tan^{-1}\mu$

- Friction cone:



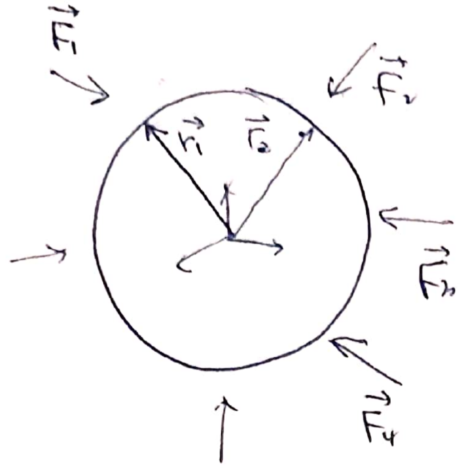
  - If force is outside cone

    $\rightarrow$ slip

- 2 types of contact:
  - frictionless point contact:
    ($\alpha=0$ or $\mu=0$)
    only normal forces can be applied.
  - point contact with friction.

    (Consider planar case
    only today)

VOD3. Static equilibrium and force/form closure

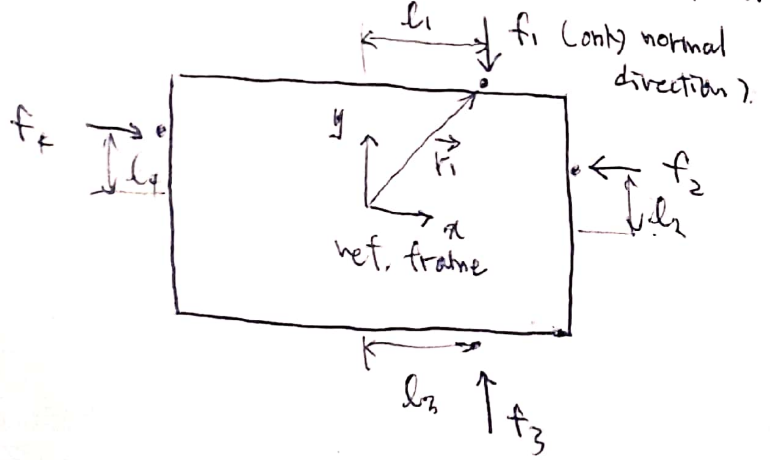


When is the object stationary?

$$\left.\begin{array}{l}\sum \vec{f}_i = 0 \\ \sum \underbrace{\vec{r}_i \times \vec{f}_i}_{\text{moments}} = 0\end{array}\right\} \text{ static equilibrium.}$$

Rigid Body.

- Back to the first example: 4 frictionless Point Contacts.



- Suppose external forces are applied..,

  object is stationary if

  (1) $\vec{f}_1 + \cdots + \vec{f}_4 + \vec{f}_{ext} = 0$

  (2) $(\vec{r}_1 \times \vec{f}_1) + \cdots + (\vec{r}_4 \times \vec{f}_4) + (\vec{r}_{ext} \times \vec{f}_{ext}) = 0.$

- $f_1 = \begin{bmatrix} 0 \\ -\lambda_1 \end{bmatrix}$, $f_2 = \begin{bmatrix} -\lambda_2 \\ 0 \end{bmatrix}$, $f_3 = \begin{bmatrix} 0 \\ \lambda_3 \end{bmatrix}$, $f_4 = \begin{bmatrix} \lambda_4 \\ 0 \end{bmatrix}$

  $(\lambda_1, \lambda_2, \lambda_3, \lambda_4 \geq 0)$

.. (1) $\begin{bmatrix} -\lambda_2 + \lambda_4 + f_{ext,x} \\ -\lambda_1 + \lambda_3 + f_{ext,y} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

...(2) $\quad \vec{r_1}\times\vec{f_1} = \begin{bmatrix} 0 \\ 0 \\ -l_1 x_1 \end{bmatrix}, \quad \cdots$

$$\Rightarrow \begin{bmatrix} 0 \\ 0 \\ -l_1x_1 + l_2x_2 + l_3x_3 - l_4x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -m_{ext} \end{bmatrix}$$

$\begin{pmatrix} ccw: + \\ cw: - \end{pmatrix}$

$$\Rightarrow \begin{bmatrix} 0 & -1 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ -l_1 & l_2 & l_3 & -l_4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

(↝ linear equation $Ax = B$)

$$\begin{cases} b_1 = -f_{ext,x} \\ b_2 = -f_{ext,y} \\ b_3 = -m_{ext,z} \end{cases}$$

- If a <u>feasible</u> solution $x \in \mathbb{R}^4$ exists for any arbitrary $b \in \mathbb{R}^3$, then object is stationary (form closure)

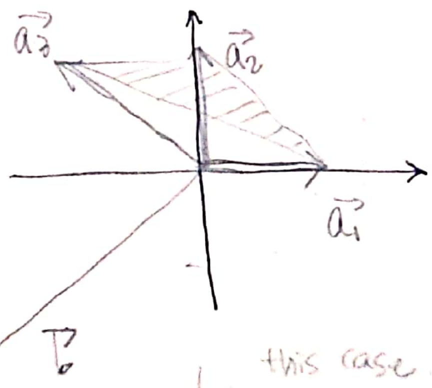
  (For our problem, $x_i \geq 0$.)

- Consider easier version:

$$\begin{array}{c} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \\ \vec{a_1} \quad \vec{a_2} \quad \vec{a_3} \qquad\qquad\quad \vec{b} \end{array}$$

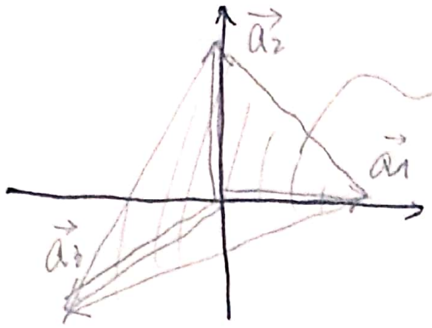
$(x_1, x_2, x_3 \geq 0)$.

$x_1\vec{a_1} + x_2\vec{a_2} + x_3\vec{a_3} = \vec{b}$.



this case

↳ No solution!

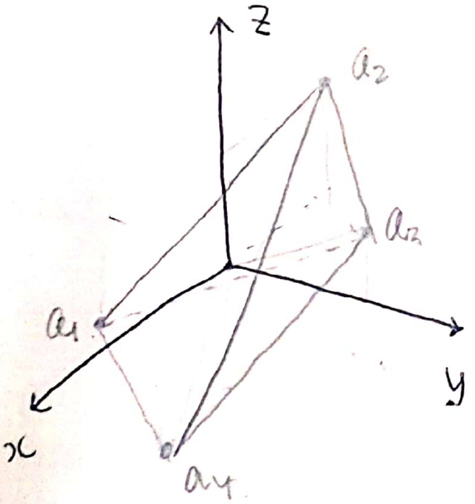


origin is inside △

↳ Solution exists for All $\vec{b}$

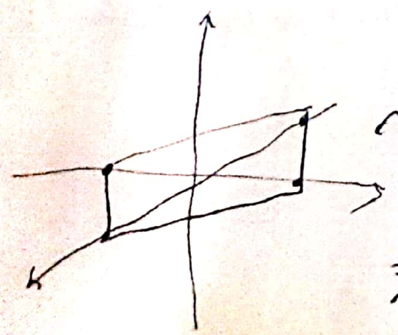
•

$$\begin{array}{cccc} \left[\begin{matrix} 0 & -1 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ -l_1 & l_2 & l_3 & -l_4 \end{matrix}\right] \\ \begin{matrix} a_1 & a_2 & \cdots & \end{matrix} \end{array}$$



tetrahedron (사면체)

- origin lies inside tetrahedron → form closure
- Suppose $l_1 = l_2 = l_3 = l_4 = 0$:



~ no longer tetrahedron

⇒ "Not" form closure.