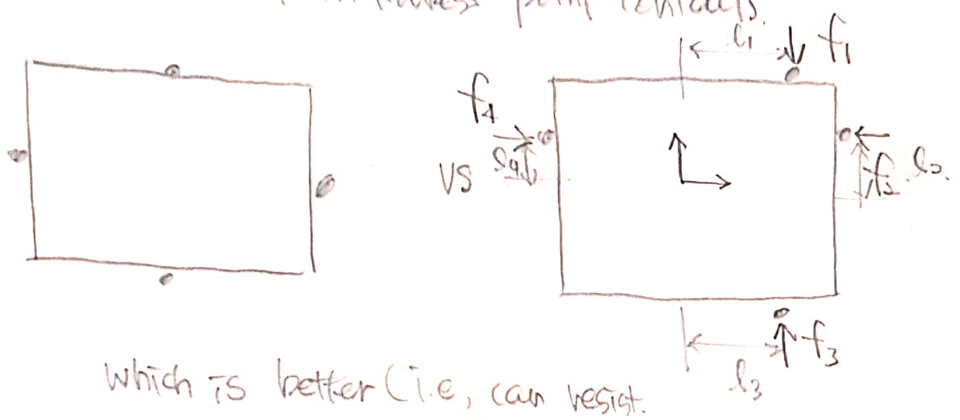
(Week 2) Grasp Statics (2): Planar Grasps with Friction

<VOD 1> Review of planar frictionless grasps

· Last Time: 4 frictionless point contacts.



which is better (i.e, can resist external forces/moments)?

$$f_1=\begin{bmatrix}0\\-x_1\end{bmatrix},\quad f_2=\begin{bmatrix}-x_2\\0\end{bmatrix},\quad f_3=\begin{bmatrix}0\\x_3\end{bmatrix},\quad f_4=\begin{bmatrix}x_4\\0\end{bmatrix}$$

$x_1, x_2, x_3, x_4 \geq 0$
(only pushing allowed).

· Static equilibrium:

Σ forces $=0$

Σ moments $=0$

$$\Rightarrow \underbrace{\begin{bmatrix}0 & -1 & 0 & 1\\ -1 & 0 & 1 & 0\\ -l_1 & l_2 & l_3 & -l_4\end{bmatrix}}_{A}\underbrace{\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}}_{x}=\begin{bmatrix}-f_{ext,x}\\-f_{ext,y}\\-m_{ext,z}\end{bmatrix}=\begin{bmatrix}b_1\\b_2\\b_3\end{bmatrix}$$

Equation of form $Ax=b$

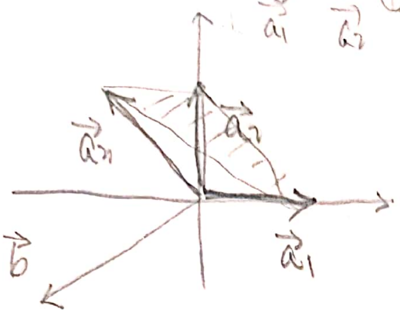
Q: Does there exist, for any arbitrary b, a solution $x \geq 0$?

To answer this.

consider

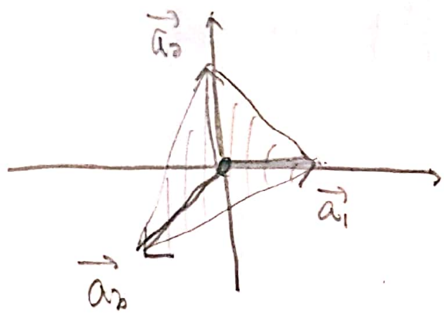
$$A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$\vec{a}_1 \quad \vec{a}_2 \quad \vec{a}_3$



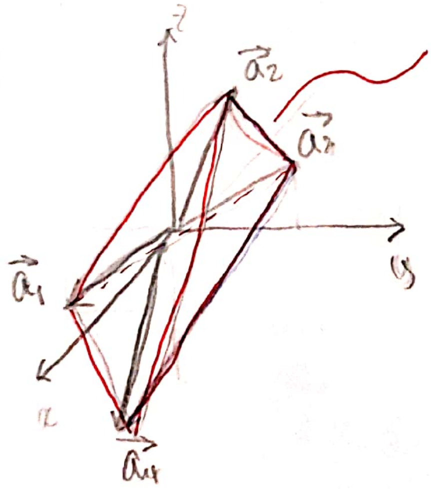
$\rightsquigarrow$ No sol'N.

However, if $\vec{a}_3 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$



Sol'n x exists for all b.

· $\vec{a}_1 \ \vec{a}_2 \ \vec{a}_3 \ \vec{a}_4$



tetrahedron.

※ tetrahedron should contain origin in its interior for sol'n $x \geq 0$ exist for arbitrary b.
(in fact it does)

(vcp2) <u>Computational tests for force/form closure</u>

- Positive linear combination of columns of A should span all of $\mathbb{R}^3$
- State the above more preasely:

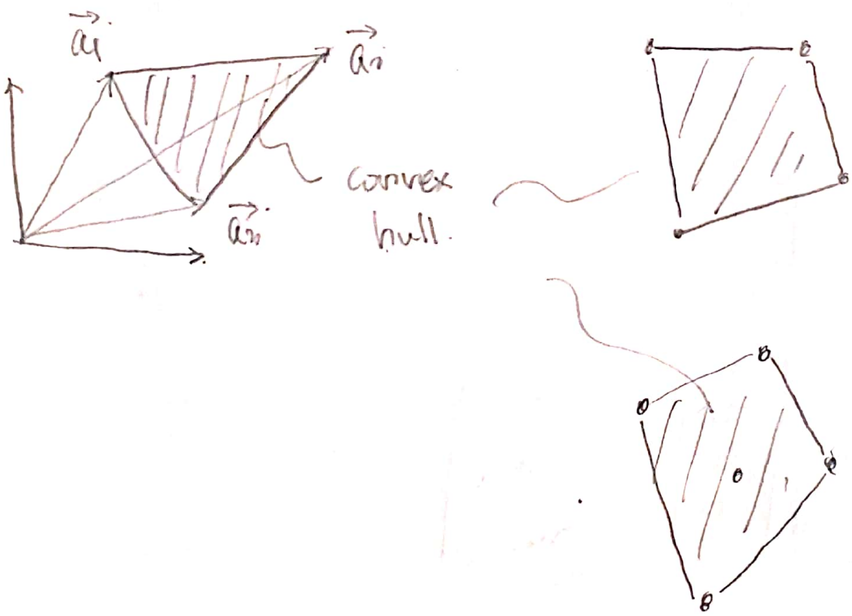
(1) Given $\vec{a_1}, \vec{a_2}, \cdots \vec{a_n} \in \mathbb{R}^m$ (m차원)

the "<u>convex hull</u>" of thesevectors.

is the set.

$$\left\{ \vec{z} = \sum_{i=1}^{n} w_i \vec{a_i} \;\middle|\; \begin{array}{l} w_i \geq 0, \\ w_1 + w_2 + \cdots + w_n = 1 \end{array} \right\}.$$

(ex) In $\mathbb{R}^2$

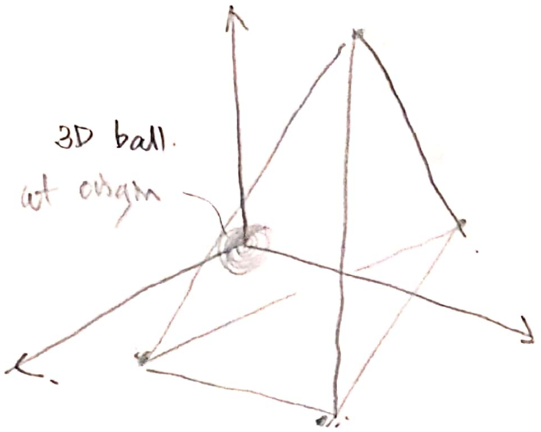


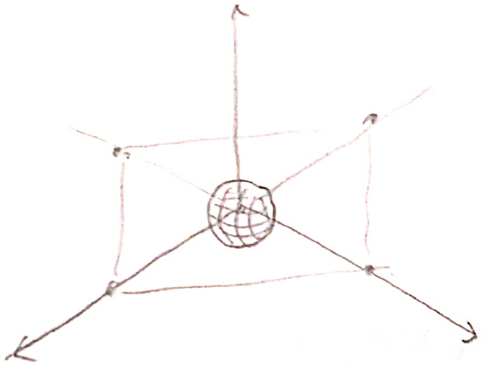
- Consider $Ax = b$, $A \in \mathbb{R}^{m \times n}$.

$b \in \mathbb{R}^m$, $x \in \mathbb{R}^n$. A sol'n $x \geq 0$.

exists for all arbitrary b.

$\Leftrightarrow$ There exists ~~An~~ open ball in $\mathbb{R}^n$ centered at the origin that lies in the interior of the convex hull. of columns of A





$$l_1 = l_2 = l_3 = l_4 = 0.$$

$$\Updownarrow$$

$$\begin{bmatrix} 0 & -1 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -f_{ext,x} \\ -f_{ext,y} \\ -m_{ext,z} \end{bmatrix}$$

→ "unable to resist external moments!"

* Remarks

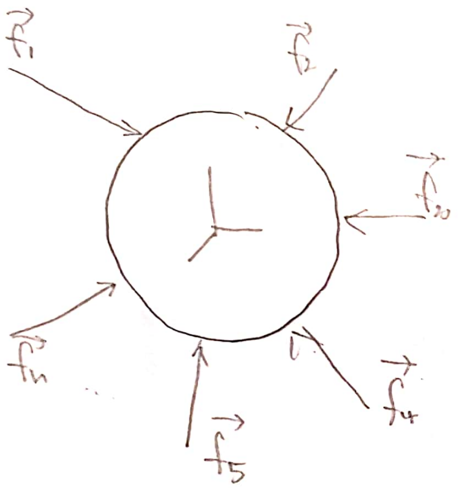
(1). For planar grasps,

a minimum of "four"

frictionless point contacts are required

(needs 2D convex hull).

(2) For spatial grasps (of 3-D objects)



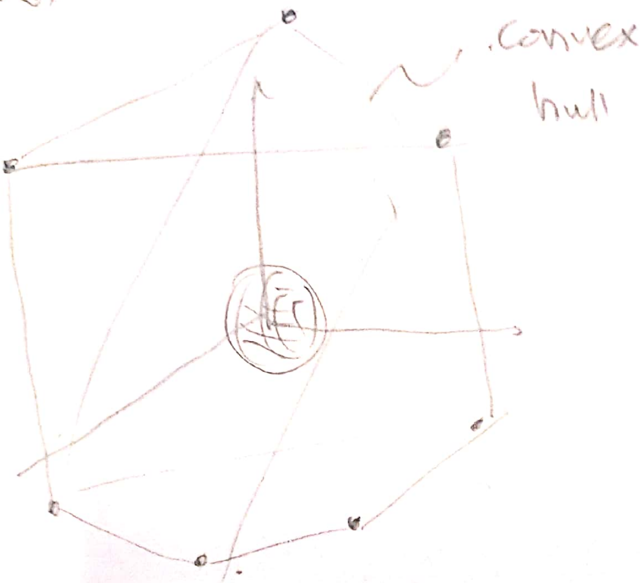
- Static equilibrium:

$$\begin{cases} \sum_{i=1}^{n} \vec{f}_i + \vec{f}_{ext} = 0 & (3 \text{ eqns}) \\ \sum_{i=1}^{n} \vec{r}_i \times \vec{f}_i + \vec{m}_{ext} = 0 & (3 \text{ eqns}) \end{cases}$$

$\vec{f}_i = \hat{n}_i x_i \quad , i=1, \dots, n.$

$$\Rightarrow \underbrace{\begin{bmatrix} \hat{n}_1 & \cdots & \hat{n}_n \\ \\ \vec{r}_1 \times \hat{n}_1 & \cdots & \vec{r}_n \times \hat{n}_n \end{bmatrix}}_{A \in \mathbb{R}^{6 \times n}} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} -\vec{f}_{ext} \\ -\vec{m}_{ext} \end{bmatrix}$$

$\mathbb{R}^6$. (6차원)



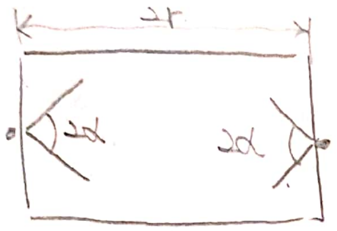
- for spatial grasps.
  need a minimum of "7" frictionless point contacts

(VOD3) Force closure for grasps with friction:

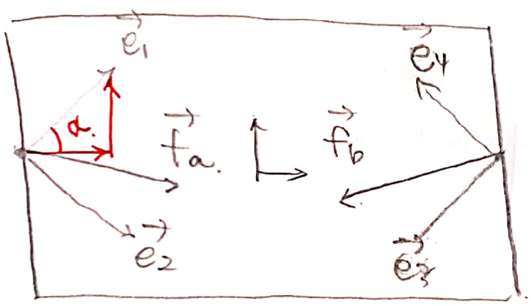
Nguyen's Theorem

- Grasps with friction:

  - let's start with an example:



Two point contacts with friction.



$$\vec{f}_a = x_1\vec{e}_1 + x_2\vec{e}_2$$
$$\vec{f}_b = x_3\vec{e}_3 + x_4\vec{e}_4 \qquad (x_1, x_2, x_3, x_4 \geq 0)$$

$$\vec{e}_1 = \begin{bmatrix} 1 \\ \mu \end{bmatrix}, \quad \vec{e}_2 = \begin{bmatrix} 1 \\ -\mu \end{bmatrix} \quad \vec{e}_3 = \begin{bmatrix} -1 \\ -\mu \end{bmatrix}, \quad \vec{e}_4 = \begin{bmatrix} -1 \\ \mu \end{bmatrix}.$$

(Recall $\mu = \tan\alpha$)

- Static equilibrium:

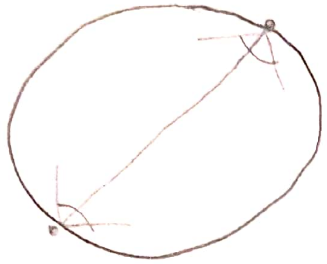
$$\vec{f}_a + \vec{f}_b + \vec{f}_{ext} = 0$$

$$m_a + m_b + m_{ext} = 0$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & -1 & -1 \\ \mu & -\mu & -\mu & \mu \\ -\mu r & \mu r & -\mu r & \mu r \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -f_{ext,x} \\ -f_{ext,y} \\ -m_{ext,z} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Use the same test!

Nguyen's Theorem:

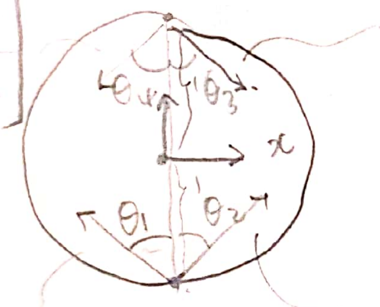


For a planar object constrained by 2 point contact with friction.

force closure $\Leftrightarrow$ line connecting 2 contacts lies inside both friction cones.

An informal proof: (case 1)

$\begin{bmatrix} -\sin\theta_4 \\ -\cos\theta_4 \end{bmatrix}$ $\begin{bmatrix} \sin\theta_3 \\ -\cos\theta_3 \end{bmatrix}$



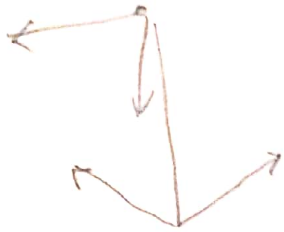
$\begin{bmatrix} -\sin\theta_1 \\ \cos\theta_1 \end{bmatrix}$ $\begin{bmatrix} \sin\theta_2 \\ -\cos\theta_2 \end{bmatrix}$

· $\theta_1, \theta_2, \theta_3, \theta_4$ 방향으로 힘을 준다고 생각!

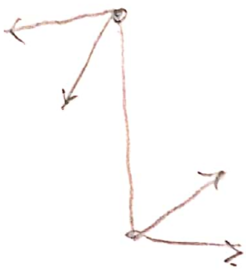
$$\Rightarrow \begin{bmatrix} -\sin\theta_1 & \sin\theta_2 & \sin\theta_3 & -\sin\theta_4 \\ \cos\theta_1 & \cos\theta_2 & -\cos\theta_3 & -\cos\theta_4 \\ -\sin\theta_1 & \sin\theta_2 & -\sin\theta_3 & -\sin\theta_4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

"This case works!"

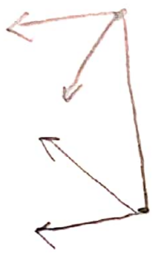
(Case2):



(case3):



(case4):



case 2,3,4 don't work!