

Week 1. Robot Configuration Space.

*VoD2. Gruebler's formula for planar robots

Gruebler's formula

(planar version):

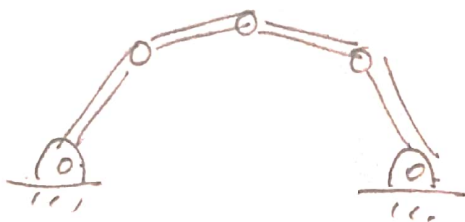
N = # of links (including ground)

J = # of joints.

f_i = dof of joint i , $i=1, \dots, J$

$$\begin{aligned} \text{DOF} &= 3(N-1) - \sum_{i=1}^J (3-f_i) \\ &= 3(N-1-J) + \sum_{i=1}^J f_i \end{aligned}$$

(ex.)



$$N=5, J=5, f_i=1 \quad \forall i \quad (\text{all } i).$$

$$\text{DOF} = 3(5-1-5) + 5 = 2$$

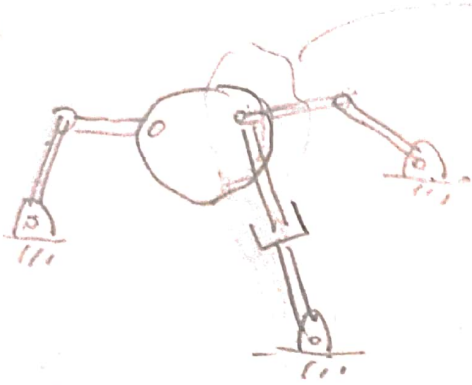
(ex.)



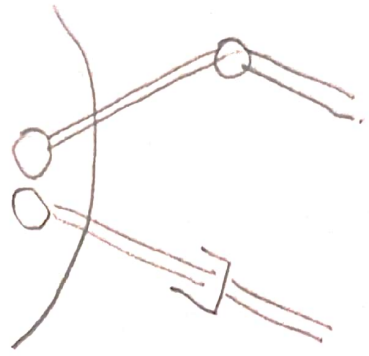
$$N=8, J=9, f_i=1.$$

$$\begin{aligned} \text{DOF} &= 3(8-1-9) + 9 \\ &= 3 \end{aligned}$$

(ex)



이런 식으로!



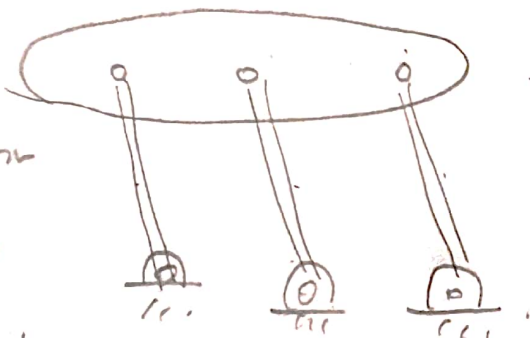
$$N=8, J=8, f_1=1$$

$$\text{DOF} = 3(8-1-8) + 8 = 5$$

⇒ Every joint connects exactly two links!

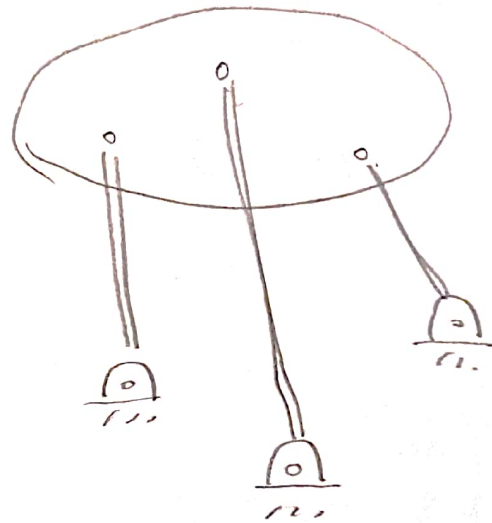
(ex)

link의 motion
은 같음
이다



$$N=5, J=6, f_1=1$$

$$\text{DOF} = 3(5-1-6) + 6 = 0 (?)$$



$$\text{DOF} = 0$$

* There are some exceptions, *
to Gruebler's formula.

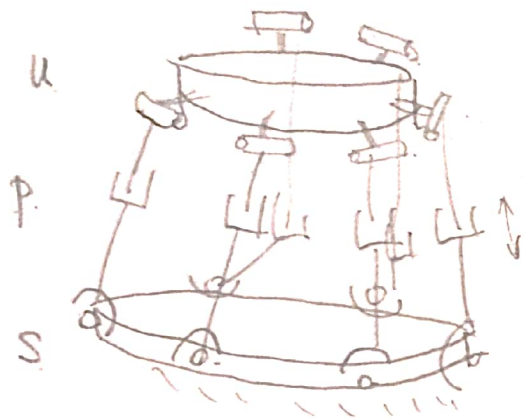
* Spatial Mechanism's (3D)

$$DOF = 6(N-1) - \sum_{i=1}^J (6-f_i)$$

↳ 3 position
3 orientation

$$= 6(N-1-J) + \sum_{i=1}^J f_i$$

(ex) 6 x. UPS platform:

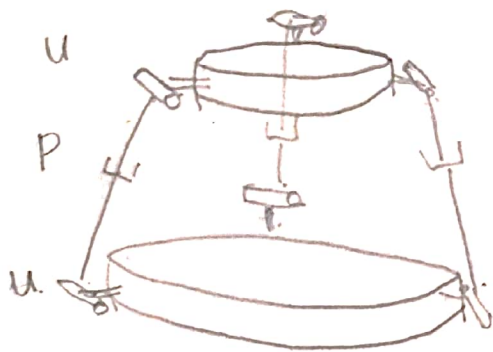


$$N=14, J=18$$

$$f_i = \begin{cases} 6 \text{ S-joint } (3 \times 6) \\ 6 \text{ U-joint } (2 \times 6) \\ 6 \text{ P-joint } (1 \times 6) \end{cases}$$

$$DOF = 6(14-1-18) + (18+12+6) = 6 \rightarrow \text{P-joint를 움직일 수 있다}$$

(ex) 3 x UPU mechanism:



$$N=8, J=9$$

$$f_i = \begin{cases} 6 \text{ U-joint } (2 \times 6) \\ 3 \text{ P-joint } (1 \times 3) \end{cases}$$

$$DOF = 6(8-1-9) + (15)$$

$$= 3 \rightarrow \text{P-joint를 움직일 수 있다}$$

Actually, move like "5."

(Singular Mechanism).

↳ 평행하게 움직일 수 있다

이 경우 dof가 3이지만 5이다!