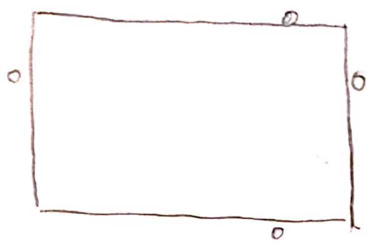
* Grasp Statics (3) : Spatial Grasps

(VOD 1.) Review of planar grasp statics

Last Time : Force / Form Closure



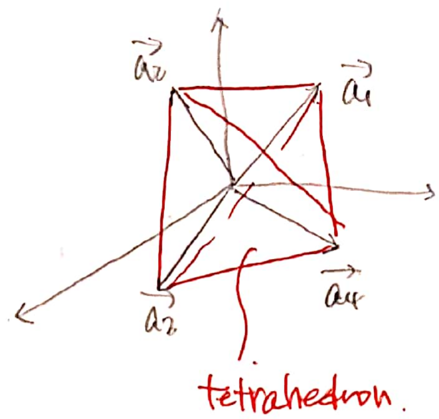
4 frictionless point contact

- Force (form) closure $\leftrightarrow$ There exists $x \geq 0$ that satisfies $Ax = b$ for all arbitrary b

  ($A = \left[\vec{a_1} \ \vec{a_2} \ \vec{a_3} \ \vec{a_4} \right] \in \mathbb{R}^{3 \times 4}$

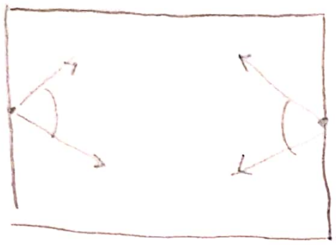
  $b \in \mathbb{R}^3$

$\Leftrightarrow$



- There exists some 3-dim open ball, centered at 0, that lies in interior of tetrahedron.

2 Point contacts with friction



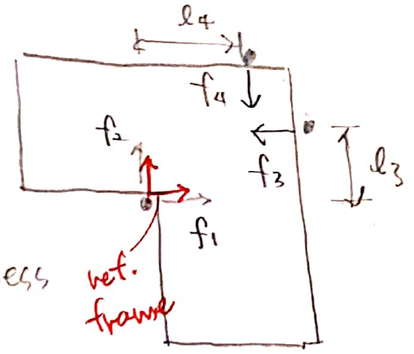
Force closure $\leftrightarrow$ same condition as before

(for example in class.

$$A = \begin{bmatrix} 1 & 1 & -1 & -1 \\ \mu & -\mu & \mu & -\mu \\ -\mu r & \mu r & \mu r & -\mu r \end{bmatrix}$$
)

A computational method for force closure would be nice.

(Ex)



3 frictionless PCs

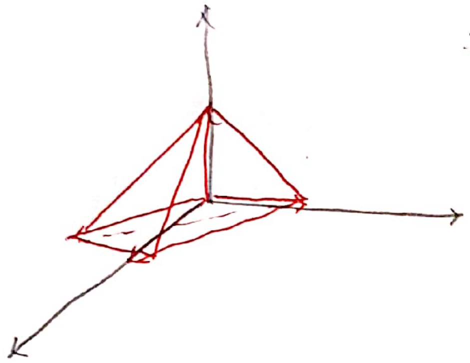
$$A = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & l_3 & -l_4 \end{bmatrix}$$

Gaussian elimination $Ax=b$:

$$\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & l_3 & -l_4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \rightarrow$$

$$\rightarrow \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -\frac{l_4}{l_3} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_4 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3/l_3 \end{bmatrix}$$

$$\rightarrow \begin{array}{c} \begin{bmatrix} 1 & 0 & 0 & -l_4/l_3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -l_4/l_3 \end{bmatrix} \\ \vec{a}_1 \quad \vec{a}_2 \quad \vec{a}_3 \quad\quad \vec{a}_4 \end{array} \begin{bmatrix} x_1 \\ \vdots \\ x_4 \end{bmatrix} = \begin{bmatrix} b_1 + b_3/l_3 \\ b_2 \\ b_3/l_3 \end{bmatrix}$$

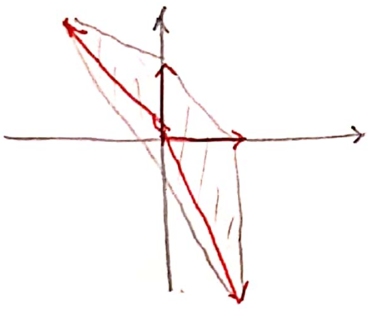


※ Because last column hass all entries < 0

→ origin lies in interior of tetrahedron.

· What if there are more than 4 columns?

(ex) $A = \begin{bmatrix} 1 & 0 & -2 & 1 \\ 0 & 1 & 2 & -2 \end{bmatrix}$



cf) $A = \begin{bmatrix} 1 & 0 & -2 & 10 \\ 0 & 1 & 10 & -2 \end{bmatrix}$

→ not force closure!

For general case $Ax = b$:

(1) Gauss Elimination to

$$\begin{bmatrix} I & | & S \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}$$

* Does there exist $w \geq 0$

  such that $Sw < 0$?

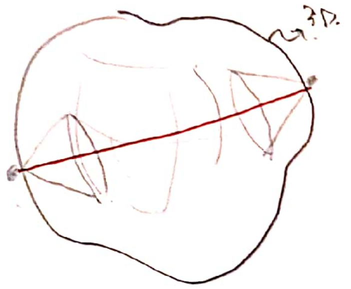
  - If yes → force closure

(VOD2) <u>Force close for spatial grasps.</u>

* <u>Spatial force closure:</u>

  <u>frictional point contacts.</u>

(1)

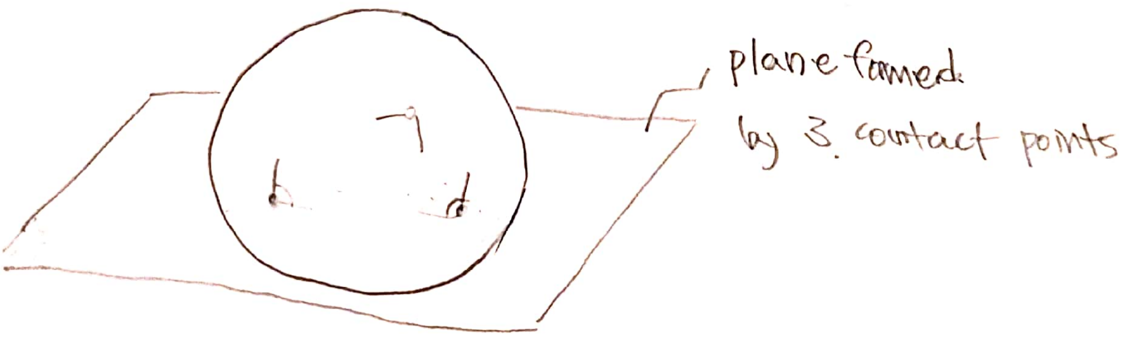


2 contacts are not enough.

→ 회전할 수 있음!

(2) 3 contacts is enough? YES!

A result states the following:



If the plane is in planar force closure

↔ spatial force closure.