

MIDTERM EXAM 1

DATE : SEP. 13 (WED)

TIME : 6:30 PM - 7:45 PM

PLACE : SECTION A - BOGGS B5
(8:25)

SECTION E - HOWEY-PHYSICS
(11:00) L3

COVERAGE : UP TO MON CLASS

REVIEW : SEP. 13 in CLASS

(SAMPLE EXAMS : S22, F22,
 ↑ S23)
 MASTER WEBPAGE .

Midterm 1

PLEASE PRINT YOUR NAME CLEARLY IN ALL CAPITAL LETTERS

Name: _____ GTID Number: _____

Student GT Email Address: _____@gatech.edu

Section Number (e.g. A3, G2, etc.) _____ TA Name _____

Circle your instructor:

Prof Kim Prof Barone Prof David/Schroeder Prof Kumar

Student Instructions

- **Show your work** and justify your answers for all questions unless stated otherwise.
- **Organize your work** in a reasonably neat and coherent way.
- **Simplify your answers** unless explicitly stated otherwise.
- **Fill in circles** completely. Do not use check marks, X's, or any other marks.
- Calculators, notes, cell phones, books are not allowed.
- Use dark and clear writing: your exam will be scanned into a digital system.
- Exam pages are double sided. Be sure to complete both sides.
- Leave a 1 inch border around the edges of exams.
- The last page is for scratch work. Please use it if you need extra space.
- This exam has 8 pages of questions.

Midterm 1. Your initials: _____

You do not need to justify your reasoning for questions on this page.

1. (a) (8 points) Suppose A is an $m \times n$ matrix and $\vec{b} \in \mathbb{R}^m$ unless otherwise stated. Select **true** if the statement is true for all choices of A and \vec{b} . Otherwise, select **false**.

true false

- | | | |
|-----------------------|-----------------------|--|
| <input type="radio"/> | <input type="radio"/> | If a vector \vec{b} can be written uniquely as a linear combination of vectors $\vec{a}_1, \vec{a}_2, \vec{a}_3$ then there is a pivot in the first three columns of the matrix $(\vec{a}_1 \ \vec{a}_2 \ \vec{a}_3 \ \vec{b})$. |
| <input type="radio"/> | <input type="radio"/> | If $A\vec{x} = \vec{b}$ is consistent, then \vec{b} is in the span of the columns of A . |
| <input type="radio"/> | <input type="radio"/> | If \vec{v} and \vec{w} are solutions to an inhomogeneous system $A\vec{x} = \vec{b}$, then $\vec{v} - \vec{w}$ is a solution to $A\vec{x} = \vec{0}$. |
| <input type="radio"/> | <input type="radio"/> | If $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$ is linearly dependent, then $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly dependent. |
| <input type="radio"/> | <input type="radio"/> | If A is size 3×4 and none of the rows of A consist entirely of zeros, then A has 3 pivots. |
| <input type="radio"/> | <input type="radio"/> | If A and B are square $n \times n$ matrices, then $A^2 - B^2 = (A - B)(A + B)$. |
| <input type="radio"/> | <input type="radio"/> | If A is size $m \times n$ with $m \neq n$ and the columns of A are linearly independent, then the transformation $T(\vec{x}) = A\vec{x}$ is onto. |
| <input type="radio"/> | <input type="radio"/> | If the coefficient matrix A for a system of linear equations has a pivot in every row, then the system $A\vec{x} = \vec{b}$ has a solution for any \vec{b} in \mathbb{R}^m . |
-

Midterm 1. Your initials: _____

You do not need to justify your reasoning for questions on this page.

(b) (4 points) Indicate whether the following situations are possible or impossible.

possible impossible

- | | | |
|-----------------------|-----------------------|---|
| <input type="radio"/> | <input type="radio"/> | An $m \times n$ matrix A with a pivot in its last column such that $A\vec{x} = \vec{0}$ is inconsistent. |
| <input type="radio"/> | <input type="radio"/> | Two nonzero vectors \vec{v}_1, \vec{v}_2 such that $\{\vec{v}_1, \vec{v}_2\}$ is linearly independent and $\{\vec{v}_1 - \vec{v}_2, \vec{v}_1 + \vec{v}_2\}$ is linearly dependent. |
| <input type="radio"/> | <input type="radio"/> | A matrix A of size 4×3 with linearly dependent columns. |
| <input type="radio"/> | <input type="radio"/> | A transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^4$ that is onto. |
-

(c) (2 points) If A is an $m \times 5$ matrix and $A\vec{x} = 0$ has a unique solution, then which of the following is true. *Select only one.*

- ☐ $m \geq 5$
- ☐ $m = 5$
- ☐ $m \leq 5$
- ☐ m can be any natural number

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You do not need to justify your reasoning for questions on this page.

(d) (3 points) For the vectors

$$\vec{v}_1 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \vec{v}_2 = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \vec{v}_3 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}.$$

Which of the following sets are linearly independent? *Select all that apply.*

- ☐ $\{\vec{v}_1, \vec{v}_2\}$
- ☐ $\{\vec{v}_2, \vec{v}_3\}$
- ☐ $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$

(e) (2 points) Consider the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}.$$

Which of following accurately describes the transformation $T(\vec{x}) = A\vec{x}$?

Select only one.

- ☐ Rotation by $\frac{\pi}{2}$ radians around the x axis.
- ☐ Rotation by $\frac{\pi}{2}$ radians around the z axis.
- ☐ Reflection across the $x = 0$ plane.
- ☐ Reflection across the $y = z$ plane.

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2. (2 points) If possible, fill in the box with the missing element of the vector \vec{w} with a number so that $\{\vec{u}, \vec{v}, \vec{w}\}$ is linearly dependent. If it is not possible write NP in the space.

$$\vec{u} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad \vec{v} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}, \quad \vec{w} = \begin{pmatrix} 5 \\ \boxed{} \\ 3 \end{pmatrix}$$

3. (4 points) Find b and c such that $AB = BA$.

$$A = \begin{pmatrix} 2 & b \\ -3 & c \end{pmatrix} \quad B = \begin{pmatrix} 4 & -5 \\ 3 & 5 \end{pmatrix}$$

$$b = \boxed{} \quad c = \boxed{}$$

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You do not need to justify your reasoning for questions on this page.

4. (8 points) Let T be the linear transformation defined by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ 3x_2 \\ 2x_1 - 4x_2 \end{bmatrix}.$$

(i) What is domain and codomain of T ?

domain is

codomain is

(ii) What is the standard matrix of T ?

(iii) Find a vector \vec{x} such that $T(\vec{x}) = \vec{b}$, where $\vec{b} = \begin{bmatrix} -2 \\ 9 \\ -4 \end{bmatrix}$.

$\vec{x} =$

(iv) Is T one-to-one?

☐ yes

☐ no

(v) Is T onto?

☐ yes

☐ no

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5. (5 points) **Show all work for problems on this page.**

For what value(s) of h is the following set of vectors linearly dependent?

$$\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ h \\ h^2 \end{pmatrix} \right\}$$

$$h = \boxed{}$$

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6. (5 points) **Show your work in the space below and put your answer in the box.**

Provide a *dependence relation* on $\vec{v}_1, \vec{v}_2, \vec{v}_3$, i.e., a nontrivial linear combination of $\vec{v}_1, \vec{v}_2, \vec{v}_3$ equaling the zero vector which demonstrates that the vectors $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ are linearly dependent.

For full credit, show how the dependence relation is obtained by row reducing the appropriate coefficient matrix.

You may leave your answer in terms of $\vec{v}_1, \vec{v}_2, \vec{v}_3$.

$$\vec{v}_1 = \begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix}, \quad \vec{v}_2 = \begin{pmatrix} -1 \\ -2 \\ -1 \end{pmatrix}, \quad \vec{v}_3 = \begin{pmatrix} 5 \\ 4 \\ 3 \end{pmatrix}$$

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7. Show your work in the space below the first box and put your answers in the boxes.

- (a) (5 points) Write the parametric vector form for the general solution to the inhomogeneous equation $A\vec{x} = \vec{b}$.

$$A = \begin{pmatrix} 2 & 1 & 0 & -4 & 1 \\ 5 & 3 & 0 & -10 & 4 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} 7 \\ 18 \end{pmatrix}$$

- (b) (2 points) For the homogeneous system with the same coefficient matrix A as part (a) above, write down the general solution to $A\vec{x} = \vec{0}$. *Hint: use your answer from part (a).*

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Section Number (e.g. A3, G2, etc.) _____ TA Name _____

Circle your instructor:

Prof Vilaca Da Rocha Prof Kafer Prof Barone Prof Wheeler
Prof Blumenthal Prof Sun Prof Shirani

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1. (a) (8 points) Suppose A is an $m \times n$ matrix and $\vec{b} \in \mathbb{R}^m$ unless otherwise stated. Select **true** if the statement is true for all choices of A and \vec{b} . Otherwise, select **false**.

true false

-
- | | | |
|-----------------------|-----------------------|--|
| <input type="radio"/> | <input type="radio"/> | If A has a pivot in every column then the system $A\vec{x} = \vec{b}$ has a unique solution. |
| <input type="radio"/> | <input type="radio"/> | Suppose A is a 6×4 matrix with 4 pivots, then there is \vec{b} such that $A\vec{x} = \vec{b}$ has no solution. |
| <input type="radio"/> | <input type="radio"/> | The sets $\{\vec{v}_1, \vec{v}_2\}$ and $\{\vec{v}_1 + \vec{v}_2, -\vec{v}_1 - \vec{v}_2\}$ have the same span. |
| <input type="radio"/> | <input type="radio"/> | If A and B are square $n \times n$ matrices, then $A^2 - B^2 = (A - B)(A + B)$. |
| <input type="radio"/> | <input type="radio"/> | The matrix equation $A\vec{x} = \vec{0}$ is always consistent. |
| <input type="radio"/> | <input type="radio"/> | Suppose $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ are nonzero vectors in \mathbb{R}^n and the sets $\{\vec{v}_1, \vec{v}_2\}$, $\{\vec{v}_1, \vec{v}_3\}$, and $\{\vec{v}_2, \vec{v}_3\}$ are all linearly independent. Then, $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly independent. |
| <input type="radio"/> | <input type="radio"/> | If $A\vec{v} = 0$, $A\vec{u} = 0$ and $\vec{w} = 3\vec{v} - 2\vec{u}$, then $A\vec{w} = 0$. |
| <input type="radio"/> | <input type="radio"/> | Let $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ be a linear transformation such that $T(\vec{x}) = \vec{b}$ has a solution for every $\vec{b} \in \mathbb{R}^m$. Then T is one-to-one. |
-

- (b) (4 points) Indicate whether the following situations are possible or impossible.

possible impossible

-
- | | | |
|-----------------------|-----------------------|--|
| <input type="radio"/> | <input type="radio"/> | A 7×5 matrix A with linearly independent columns. |
| <input type="radio"/> | <input type="radio"/> | A linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ that is not onto and its standard matrix has linearly independent columns. |
| <input type="radio"/> | <input type="radio"/> | $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ that is onto and its standard matrix has exactly one non-pivotal column. |
| <input type="radio"/> | <input type="radio"/> | Two non-zero matrices A, B of size 2×2 with $AB = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$. |
-

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You do not need to justify your reasoning for questions on this page.

(c) (2 points) Let

$$\left(\begin{array}{ccc|c} 1 & 3 & 0 & 1 \\ 0 & 3h & 3 & 6 \\ 0 & 0 & 1 & 2 \end{array} \right)$$

be an augmented matrix of a system of linear equations. For which values of h does the system have a free variable? *Choose the best option.*

- ☐ 0 only
- ☐ $\frac{1}{3}$ only
- ☐ 1 only
- ☐ for all values of h
- ☐ for no values of h

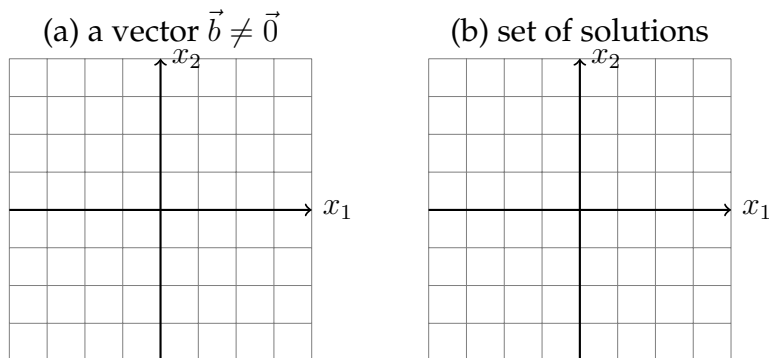
(d) (2 points) A linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^1$ maps each of the standard unit vectors \vec{e}_1 , \vec{e}_2 and \vec{e}_3 to 1. Which of the following statements is TRUE? *Select only one.*

- ☐ T is one-to-one.
- ☐ T is not onto.
- ☐ The solution set of $T(\vec{x}) = \vec{0}$ spans a plane in \mathbb{R}^3 .
- ☐ The range of T is $\{1\}$.

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You do not need to justify your reasoning for questions on this page.

2. (4 points) Suppose $A = \begin{pmatrix} 3 & 1 \\ 6 & 2 \end{pmatrix}$ and sketch (a) a non-zero vector \vec{b} such that $A\vec{x} = \vec{b}$ is consistent, and (b) the set of solutions to $A\vec{x} = \vec{0}$.



3. (2 points) Consider the linear system in variables x_1, x_2, x_3 with unknown constants below.

$$a_1x_1 + a_2x_2 + a_3x_3 = b_1$$

$$c_1x_1 + c_2x_2 + c_3x_3 = b_2$$

Which of the following statements about the solution set of this system are possible?
Select all that apply.

- ☐ The solution set is empty.
- ☐ The solution set is a single point.
- ☐ The solution set is a line.
- ☐ The solution set is a plane.

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You do not need to justify your reasoning for questions on this page.

4. Fill in the blanks.

(a) (3 points) Let A be a coefficient matrix of size 2×2 and B be a coefficient matrix of size 3×2 . Construct an example of two augmented matrices $[A|\vec{b}]$ and $[B|\vec{d}]$ which are both in RREF and such that the systems $A\vec{x} = \vec{b}$ and $B\vec{x} = \vec{d}$ each have the exact same unique solution $x_1 = 3$ and $x_2 = 6$. If this is not possible write NP in each box.

$$\left[A|\vec{b}\right]=\left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right] \qquad \left[B|\vec{d}\right]=\left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right]$$

(b) (2 points) Let $\vec{u}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $\vec{u}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, and $\vec{b} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$. Find c_1, c_2 such that $\vec{b} = c_1\vec{u}_1 + c_2\vec{u}_2$.

$$c_1 = \boxed{} \qquad c_2 = \boxed{}$$

Midterm 1. Your initials: _____

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5. (8 points) Let T be a linear transformation that maps \vec{v}_1 to $T(\vec{v}_1)$ and \vec{v}_2 to $T(\vec{v}_2)$, where

$$\vec{v}_1 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \quad \vec{v}_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \quad T(\vec{v}_1) = \begin{pmatrix} 1 \\ 3 \\ 0 \\ 1 \end{pmatrix}, \quad T(\vec{v}_2) = \begin{pmatrix} 3 \\ -1 \\ -2 \\ 1 \end{pmatrix}.$$

- (i) What is domain and codomain of T ?

domain is

codomain is

- (ii) Is it true that $\mathbb{R}^2 = \text{span}\{\vec{v}_1, \vec{v}_2\}$?

☐ yes

☐ no

- (iii) Write $\vec{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\vec{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ as linear combinations of \vec{v}_1 and \vec{v}_2 .

$\vec{e}_1 =$

$\vec{e}_2 =$

- (iv) What is the standard matrix of T ?

- (v) Is T one-to-one?

☐ yes

☐ no

Midterm 1. Your initials: _____

6. Show all work for problems on this page.

(a) (3 points) For what value of k will matrix A have exactly two pivots?

$$A = \begin{pmatrix} 1 & 1 & 1 \\ -1 & 0 & 2 \\ 0 & 1 & k \end{pmatrix}$$

$$k = \boxed{}$$

(b) (4 points) Find b and c such that $AB = BA$.

$$A = \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix} \quad B = \begin{pmatrix} 1 & b \\ c & 0 \end{pmatrix}$$

$$b = \boxed{} \quad c = \boxed{}$$

Midterm 1. Your initials: _____

7. (4 points) **Show your work for problems on this page.**

Write down the parametric vector form for solutions to the homogeneous equation $A\vec{x} = \vec{0}$.

$$A = \begin{bmatrix} 1 & -1 & -2 & -3 & -1 \\ 0 & 1 & 0 & 3 & 1 \\ -1 & 1 & 2 & 3 & 2 \end{bmatrix}$$

8. (4 points) Determine whether the set of vectors $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly independent. Justify your answer in the space below.

$$\vec{v}_1 = \begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 2 \\ -1 \\ 8 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} -2 \\ 2 \\ -9 \end{bmatrix}$$

☐ linearly independent ☐ linearly dependent

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Circle your instructor:

Prof Barone

Prof Shirani

Prof Simone

Prof Timko

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1. (a) (8 points) Suppose A is an $m \times n$ matrix and $\vec{b} \in \mathbb{R}^m$ unless otherwise stated. Select **true** if the statement is true for all choices of A and \vec{b} . Otherwise, select **false**.

true false

- | | | |
|-----------------------|-----------------------|--|
| <input type="radio"/> | <input type="radio"/> | The span of two non-zero vectors in \mathbb{R}^3 is necessarily a plane. |
| <input type="radio"/> | <input type="radio"/> | If an echelon form of A has a row of zeros, then the system $A\vec{x} = \vec{b}$ has a free variable. |
| <input type="radio"/> | <input type="radio"/> | If $A\vec{v} = \vec{b}$, and $[A \mid \vec{b}]$ is row equivalent to $[C \mid \vec{d}]$, then $C\vec{v} = \vec{d}$. |
| <input type="radio"/> | <input type="radio"/> | If the columns of A span \mathbb{R}^m , then $A\vec{x} = \vec{b}$ is consistent for any $\vec{b} \in \mathbb{R}^m$. |
| <input type="radio"/> | <input type="radio"/> | If $A\vec{x} = \vec{0}$ has a non-trivial solution, then the columns of A are linearly dependent. |
| <input type="radio"/> | <input type="radio"/> | If \vec{v} and \vec{u} are solutions of a homogeneous system of linear equations, then $\vec{v} + \vec{u}$ is also a solution of that system. |
| <input type="radio"/> | <input type="radio"/> | If the columns of a matrix A are linearly dependent, then the system $A\vec{x} = \vec{b}$ can not have a unique solution. |
| <input type="radio"/> | <input type="radio"/> | If $T : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is a linear transformation such that $T(\vec{x}) = \vec{b}$ has no solution for some $\vec{b} \in \mathbb{R}^n$, then T is not one-to-one. |
-

- (b) (4 points) Indicate whether the following situations are possible or impossible.

possible impossible

- | | | |
|-----------------------|-----------------------|--|
| <input type="radio"/> | <input type="radio"/> | A linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ that is not onto. |
| <input type="radio"/> | <input type="radio"/> | A linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ that is onto and its standard matrix has two non-pivotal columns. |
| <input type="radio"/> | <input type="radio"/> | A linear transformation $T : \mathbb{R}^4 \rightarrow \mathbb{R}^2$ that is onto and its standard matrix has linearly dependent columns. |
| <input type="radio"/> | <input type="radio"/> | Three non-zero matrices A, B, C of size 2×2 with $AC = BC$ and $A \neq B$. |
-

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You do not need to justify your reasoning for questions on this page.

(c) (2 points) Let

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & h^2 & 0 \end{array} \right]$$

be a row echelon form of an augmented matrix of a system of linear equations. For which values of h is the system consistent? Choose the best option.

- ☐ for all values of h
- ☐ 0 only
- ☐ for no values of h
- ☐ 1 and -1 only

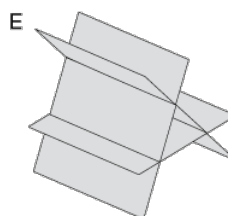
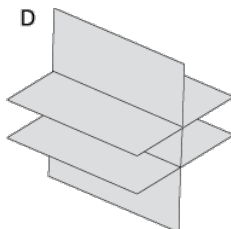
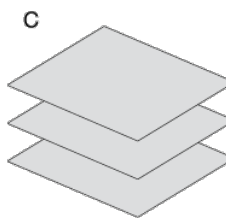
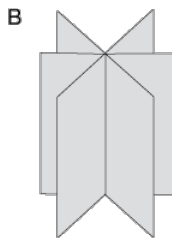
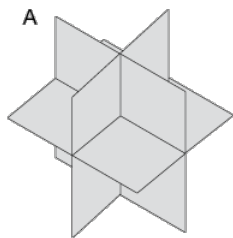
(d) (2 points) Let

$$\left[\begin{array}{ccccc|c} 1 & -1 & 0 & \pi & 2 & -1 \\ 0 & 0 & 1 & -2 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array} \right]$$

be a row echelon form of an augmented matrix of a system of equations. How many solutions does the system have? Choose the best option.

- ☐ 0
- ☐ 1
- ☐ infinitely many

(e) (2 points) Suppose $A\vec{x} = \vec{b}$ is a system of three linear equations in three variables. If the system $A\vec{x} = \vec{b}$ is consistent, which of the following could be the graphs in \mathbb{R}^3 of the three equations represented by the rows of $[A \mid b]$? Circle all pictures that apply.

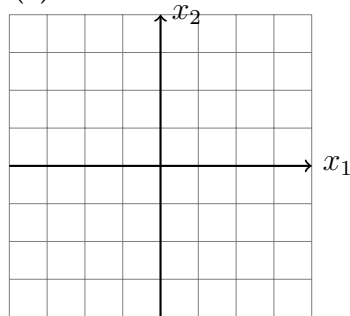


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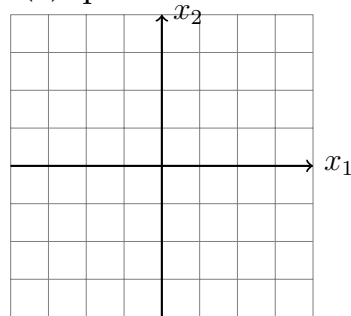
You do not need to justify your reasoning for questions on this page.

2. (4 points) Suppose $A = \begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix}$ and sketch a) a non-zero solution to $A\vec{x} = \vec{0}$, and b) the span of the columns of A .

(a) non-zero solution



(b) span of columns



3. (5 points) Let $A \in \mathbb{R}^{3 \times 5}$, $B \in \mathbb{R}^{4 \times 3}$ and $\vec{a} \in \mathbb{R}^3$, $\vec{b} \in \mathbb{R}^4$, $\vec{c} \in \mathbb{R}^5$. Which of the following are defined? Choose all the expressions which are defined.

- ☐ $B\vec{b}$
☐ $A\vec{c}$
☐ $A(B\vec{a})$
☐ $B(A\vec{c})$
☐ $B(\vec{a} + \vec{b})$

4. (3 points) In each of the following cases, indicate whether $A\vec{x} = \vec{b}$ has no solutions, a unique solution, infinitely many solutions, or if this can not be determined with the given information.

no solution	unique solution	infinitely many solutions	can't be deter- mined	
<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	$A \in \mathbb{R}^{3 \times 4}$, $\vec{b} = \vec{0}$, and A has 2 pivots
<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	$A \in \mathbb{R}^{5 \times 2}$, $\vec{b} = \vec{0}$, and A has 2 pivots
<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	$A \in \mathbb{R}^{3 \times 5}$ and A has 3 pivots

Midterm 1. Your initials: _____

You do not need to justify your reasoning for questions on this page.

5. Fill in the blanks.

- (a) (2 points) If the augmented matrix $[A \mid \vec{b}]$ of a system of equations is 3×6 and the system has two pivot (basic) variables, then how many free variables does it have?

- (b) (2 points) For what value(s) of h is the following set of vectors linearly dependent?

$$\left\{ \begin{pmatrix} 1 \\ 1 \\ h \end{pmatrix}, \begin{pmatrix} 1 \\ h \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ h \end{pmatrix} \right\}$$

$$h = \boxed{}$$

Midterm 1. Your initials: _____

6. Show all work for problems on this page.

(a) (1 point) Let $\vec{b} = \begin{bmatrix} 3 \\ -4 \\ -6 \\ 1 \end{bmatrix}$, $\vec{a}_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$, $\vec{a}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$, and $\vec{a}_3 = \begin{bmatrix} 3 \\ -3 \\ -5 \\ 2 \end{bmatrix}$. Is \vec{b} in the span of \vec{a}_1, \vec{a}_2 , and \vec{a}_3 ?

☐ Yes

☐ No

(b) (2 points) If you answered yes to part (a), write \vec{b} as a linear combination of $\vec{a}_1, \vec{a}_2, \vec{a}_3$.
If you answered no, give an echelon form of the augmented matrix $[\vec{a}_1 \ \vec{a}_2 \ \vec{a}_3 \mid \vec{b}]$.

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Midterm 1. Your initials: _____

7. (3 points) **Show your work for problems on this page.**

Suppose that we have

$$\left[A \mid \vec{b} \right] \sim \left[\begin{array}{cccc|c} 1 & 4 & 0 & -1 & 3 \\ 0 & 0 & 1 & 5 & 2 \end{array} \right]$$

Find the parametric vector form for the solutions of $A\vec{x} = \vec{b}$.

8. (2 points) Suppose $A\vec{v} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ and $A\vec{u} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$. Compute $A(2\vec{v} - \vec{u})$.

Midterm 1. Your initials: _____

9. (8 points) **Show all work for problems on this page.** Consider the linear transformation defined by $T(x_1, x_2) = (x_1 + x_2, x_1, x_1 - x_2)$ with domain \mathbb{R}^2 .

(i) What is the codomain of T ?

(ii) What is the standard matrix of T ?

(iii) Is T onto?

☐ yes

☐ no

(iv) Write an equation using the variables b_1 , b_2 , and b_3 which is satisfied exactly when $T(x_1, x_2) = (b_1, b_2, b_3)$ has a solution for x_1, x_2 .

(v) What is the range of T ?