Fact: W : a subspace in $\mathbb{R}^n$
$\dim(W) + \dim(W^\perp) = n$

Midterm 3 Lecture Review Activity, Math 1554

1. Indicate **true** if the statement is true, otherwise, indicate **false**.

	true	false
a) If S is a two-dimensional subspace of $\mathbb{R}^{50}$, then the dimension of $S^\perp$ is 48.	⊗	○
b) An eigenspace $= \text{Null}(A-\lambda I)$ is a subspace spanned by a ~~single eigenvector~~. eigenvectors corr. to a single eigenvalue λ.	○	⊗
c) The $n \times n$ zero matrix can be diagonalized.	⊗	○
d) A least-squares line that best fits the data points $(0, y_1), (1, y_2), (2, y_3)$ is unique for any values y_1, y_2, y_3.	⊗	○

2. If possible, give an example of the following.

2.1) A matrix, A, that is in echelon form, and $\dim\left((\text{Row}\,A)^\perp\right) = 2$, $\dim\left((\text{Col}\,A)^\perp\right) = 1$

2.2) A singular 2×2 matrix whose eigenspace corresponding to eigenvalue $\lambda = 2$ is the line $x_1 = 2x_2$. The other eigenspace of the matrix is the x_2 axis.

2.3) A subspace S, of $\mathbb{R}^4$, that satisfies $\dim(S) = \dim(S^\perp) = 3$.

2.4) A 2×3 matrix, A, that is in RREF. $(\text{Row}\,A)^\perp$ is spanned by $\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$.

Least square line

Model: $\underline{y = \beta_0 + \beta_1 x}$

Data: $(0, y_1)$ $(1, y_2)$ $(2, y_3)$

$$\begin{cases} y_1 = \beta_0 + \beta_1 \cdot 0 \\ y_2 = \beta_0 + \beta_1 \cdot 1 \\ y_3 = \beta_0 + \beta_1 \cdot 2 \end{cases}$$

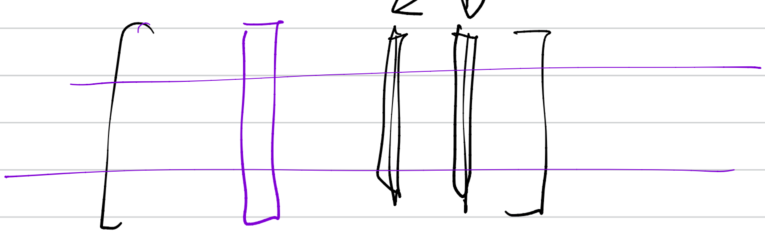
$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \boxed{\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}} \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix}$$

- $\text{Row}(A)^{\perp} = \text{Col}(A^T)^{\perp} = \text{Nul}((A^T)^T) = \text{Nul}(A)$.

$\dim(\text{Row}(A)^{\perp}) = 2 = \underline{\dim(\text{Nul}(A))}$

pivot

Nonpivot = # of Non-pivot



2

$$\begin{bmatrix} 1 & * & * \\ 0 & 0 & 0 \end{bmatrix}$$

- $\dim(\text{Col}(A)^{\perp}) = 1$

$+$

Assume $\dim(\text{Col}(A)) = 1 \Rightarrow$ $\text{Col}(A)$ in $\mathbb{R}^2$

2

3. Circle **possible** if the set of conditions are create a situation that is possible, otherwise, circle **impossible**. For the situations that are possible give an example.

   3.1) A is $n \times n$, $A\vec{x} = A\vec{y}$ for a particular $\vec{x} \neq \vec{y}$, $\vec{x}$ and $\vec{y}$ are in $\mathbb{R}^n$, and $\dim((\text{Row}\, A)^{\perp}) \neq 0$.

   possible **impossible**

   3.2) A is $n \times n$, $\lambda \in \mathbb{R}$ is an eigenvalue of A, and $\dim((\text{Col}(A - \lambda I))^{\perp}) = 0$.

   possible **impossible**

   3.3) $\text{proj}_{\vec{v}}\vec{u} = \text{proj}_{\vec{u}}\vec{v}$, $\vec{v} \neq \vec{u}$, and $\vec{u} \neq \vec{0}$, $\vec{v} \neq \vec{0}$.

   possible **impossible**

4. Consider the matrix A.

$$A = \begin{pmatrix} 1 & -3 & 0 & 2 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

   Construct a basis for the following subspaces and state the dimension of each space.

   4.1) $(\text{Row}A)^{\perp}$

   4.2) $\text{Col}\, A$

   4.3) $(\text{Col}A)^{\perp}$

$A \in \mathbb{R}^{m \times n}$

$$\begin{cases} \mathrm{Col}(A)^{\perp} = \mathrm{Nul}(A^T) \\ \mathrm{Nul}(A)^{\perp} = \mathrm{Col}(A^T) \\ \mathrm{Row}(A) = \mathrm{Col}(A^T) \\ \dim(\underline{\mathrm{Row}(A)}) = \dim(\mathrm{Col}(A)) \\ \dim(\mathrm{Nul}(A)) + \dim(\mathrm{Col}(A)) = n \\ \dim(W) + \dim(W^{\perp}) = n \end{cases}$$

W in $\mathbb{R}^n$,