

In-Class Final Exam Review Set A, Math 1554, Fall 2019

1. Indicate whether the statements are true or false.

| true | false | |
|-------------------------------------|-------------------------------------|--|
| <input checked="" type="checkbox"/> | <input type="checkbox"/> | If a linear system has <u>more unknowns</u> than <u>equations</u> , then the system has either no solutions or infinitely many solutions. $n > m$ $A : m \times n$ |
| <input type="checkbox"/> | <input checked="" type="checkbox"/> | A $n \times n$ matrix A and its echelon form E will always have the same eigenvalues. $\lambda^2 - \lambda - 1 = 0 \rightarrow \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} \lambda = 1, -1$ |
| <input type="checkbox"/> | <input type="checkbox"/> | $x^2 - 2xy + 4y^2 \geq 0$ for all real values of x and y . |
| <input type="checkbox"/> | <input type="checkbox"/> | If matrix A has linearly dependent columns, then $\dim((\text{Row } A)^\perp) > 0$. |
| <input type="checkbox"/> | <input type="checkbox"/> | If λ is an eigenvalue of A , then $\dim(\text{Null}(A - \lambda I)) > 0$. |
| <input type="checkbox"/> | <input type="checkbox"/> | If A has QR decomposition $A = QR$, then $\text{Col } A = \text{Col } Q$. |
| <input type="checkbox"/> | <input type="checkbox"/> | If A has LU decomposition $A = LU$, then $\text{rank}(A) = \text{rank}(U)$. |
| <input type="checkbox"/> | <input type="checkbox"/> | If A has LU decomposition $A = LU$, then $\dim(\text{Null } A) = \dim(\text{Null } U)$. |

2. Give an example of the following.

i) A 4×3 lower triangular matrix, A , such that $\text{Col}(A)^\perp$ is spanned by

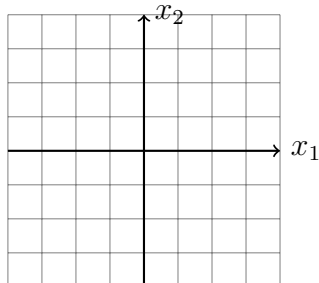
the vector $\vec{v} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 1 \end{pmatrix}$. $A = \begin{pmatrix} & & & \\ & & & \\ & & & \\ & & & \end{pmatrix}$

ii) A 3×4 matrix A , that is in RREF, and satisfies $\dim((\text{Row } A)^\perp) = 2$ and $\dim((\text{Col } A)^\perp) =$

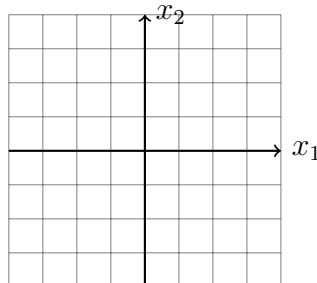
2. $A = \begin{pmatrix} & & & \\ & & & \\ & & & \end{pmatrix}$

3. (3 points) Suppose $A = \begin{pmatrix} 3 & 1 \\ 6 & 2 \end{pmatrix}$. On the grid below, sketch a) $\text{Col}(A)$, and b) the eigenspace corresponding to eigenvalue $\lambda = 5$.

(a) $\text{Col}(A)$



(b) $\lambda = 5$ eigenspace



4. Fill in the blanks.

(a) If $A \in \mathbb{R}^{M \times N}$, $M < N$, and $A\vec{x} = 0$ does not have a non-trivial solution, how many pivot columns does A have?

(b) Consider the following linear transformation.

$$T(x_1, x_2) = (2x_1 - x_2, 4x_1 - 2x_2, x_2 - 2x_1).$$

The domain of T is . The image of $\vec{x} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ under $T(\vec{x})$ is $\begin{pmatrix} \\ \\ \end{pmatrix}$. The co-domain of T is . The range of T is:

5. Four points in \mathbb{R}^2 with coordinates (t, y) are $(0, 1)$, $(\frac{1}{4}, \frac{1}{2})$, $(\frac{1}{2}, -\frac{1}{2})$, and $(\frac{3}{4}, -\frac{1}{2})$. Determine the values of c_1 and c_2 for the curve $y = c_1 \cos(2\pi t) + c_2 \sin(2\pi t)$ that best fits the points. Write the values you obtain for c_1 and c_2 in the boxes below.

$$c_1 = \text{} \qquad c_2 = \text{$$

In-Class Final Exam Review Set B, Math 1554, Fall 2019

1. Indicate whether the statements are true or false.

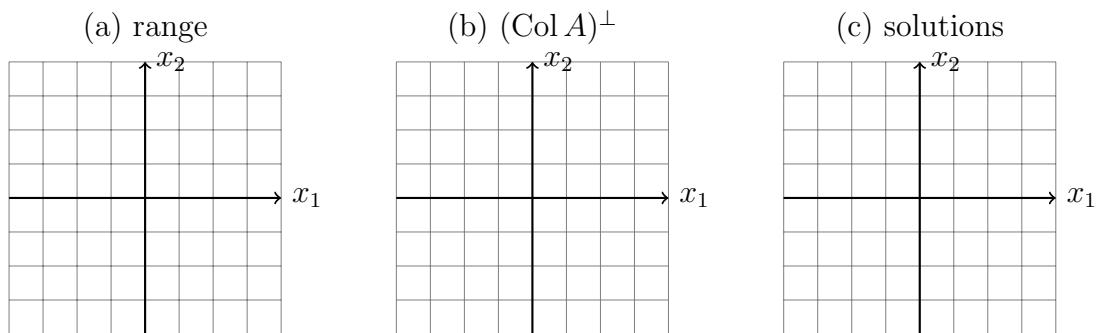
true false

- | | | |
|-----------------------|-----------------------|---|
| <input type="radio"/> | <input type="radio"/> | For any vector $\vec{y} \in \mathbb{R}^2$ and subspace W , the vector $\vec{v} = \vec{y} - \text{proj}_W \vec{y}$ is orthogonal to W . |
| <input type="radio"/> | <input type="radio"/> | If A is $m \times n$ and has linearly dependent columns, then the columns of A cannot span \mathbb{R}^m . |
| <input type="radio"/> | <input type="radio"/> | If a matrix is invertible it is also diagonalizable. |
| <input type="radio"/> | <input type="radio"/> | If E is an echelon form of A , then $\text{Null } A = \text{Null } E$. |
| <input type="radio"/> | <input type="radio"/> | If the SVD of $n \times n$ singular matrix A is $A = U\Sigma V^T$, then $\text{Col} A = \text{Col} U$. |
| <input type="radio"/> | <input type="radio"/> | If the SVD of $n \times n$ matrix A is $A = U\Sigma V^T$, $r = \text{rank} A$, then the first r columns of V give a basis for $\text{Null} A$. |

2. Give an example of:

- a) a vector $\vec{u} \in \mathbb{R}^3$ such that $\text{proj}_{\vec{p}} \vec{u} = \vec{p}$, where $\vec{u} \neq \vec{p}$, and $\vec{p} = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$: $\vec{u} = \begin{pmatrix} \\ \\ \end{pmatrix}$
- b) an upper triangular 4×4 matrix A that is in RREF, 0 is its only eigenvalue, and its corresponding eigenspace is 1-dimensional. $A = \begin{pmatrix} & & & \\ & & & \\ & & & \\ & & & \end{pmatrix}$
- c) A 3×4 matrix, A , and $\text{Col}(A)^\perp$ is spanned by $\begin{pmatrix} 1 \\ -3 \\ -4 \end{pmatrix}$.
- d) A 2×2 matrix in RREF that is diagonalizable and not invertible.

3. Suppose $A = \begin{pmatrix} 2 & -1 \\ 4 & -2 \end{pmatrix}$. On the grid below, sketch a) the range of $x \rightarrow Ax$, b) $(\text{Col } A)^\perp$, (c) set of solutions to $A\vec{x} = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$.



4. Matrix A is a 2×2 matrix whose eigenvalues are $\lambda_1 = \frac{1}{2}$ and $\lambda_2 = 1$, and whose corresponding eigenvectors are $\vec{v}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\vec{v}_2 = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$. Calculate
1. $A(\vec{v}_1 + 4\vec{v}_2)$
 2. A^{10}
 3. $\lim_{k \rightarrow \infty} A^k(\vec{v}_1 + 4\vec{v}_2)$

In-Class Final Exam Review Set C, Math 1554, Fall 2019

1. Indicate whether the statements are possible or impossible.

| possible | impossible | |
|-----------------------|-----------------------|---|
| <input type="radio"/> | <input type="radio"/> | $Q(\vec{x}) = \vec{x}^T A \vec{x}$ is a positive definite quadratic form, and $Q(\vec{v}) = 0$, where \vec{v} is an eigenvector of A . |
| <input type="radio"/> | <input type="radio"/> | The maximum value of $Q(\vec{x}) = ax_1^2 + bx_2^2 + cx_3^2$, where $a > b > c$, for $\vec{x} \in \mathbb{R}^3$, subject to $\ \vec{x}\ = 1$, is not unique. |
| <input type="radio"/> | <input type="radio"/> | The location of the maximum value of $Q(\vec{x}) = ax_1^2 + bx_2^2 + cx_3^2$, where $a > b > c$, for $\vec{x} \in \mathbb{R}^3$, subject to $\ \vec{x}\ = 1$, is not unique. |
| <input type="radio"/> | <input type="radio"/> | A is 2×2 , the algebraic multiplicity of eigenvalue $\lambda = 0$ is 1, and $\dim(\text{Col}(A)^\perp)$ is equal to 0. |
| <input type="radio"/> | <input type="radio"/> | Stochastic matrix P has zero entries and is regular. |
| <input type="radio"/> | <input type="radio"/> | A is a square matrix that is not diagonalizable, but A^2 is diagonalizable. |
| <input type="radio"/> | <input type="radio"/> | The map $T_A(\vec{x}) = A\vec{x}$ is one-to-one but not onto, A is $m \times n$, and $m < n$. |

2. Transform $T_A = A\vec{x}$ reflects points in \mathbb{R}^2 through the line $y = 2 + x$. Construct a standard matrix for the transform using homogeneous coordinates. Leave your answer as a product of three matrices.

3. Fill in the blanks.

- (a) $T_A = A\vec{x}$, where $A \in \mathbb{R}^{2 \times 2}$, is a linear transform that first rotates vectors in \mathbb{R}^2 clockwise by $\pi/2$ radians about the origin, then reflects them through the line $x_1 = x_2$. What is the value of $\det(A)$?
- (b) B and C are square matrices with $\det(BC) = -5$ and $\det(C) = 2$. What is the value of $\det(B)\det(C^4)$?
- (c) A is a 6×4 matrix in RREF, and $\text{rank}(A) = 4$. How many different matrices can you construct that meet these criteria?
- (d) $T_A = A\vec{x}$, where $A \in \mathbb{R}^{2 \times 2}$, projects points onto the line $x_1 = x_2$. What is an eigenvalue of A equal to?
- (e) If an eigenvalue of A is $\frac{1}{3}$, what is one eigenvalue of A^{-1} equal to?
- (f) If A is 30×12 and $A\vec{x} = \vec{b}$ has a unique least squares solution \hat{x} for every \vec{b} in \mathbb{R}^{30} , the dimension of $\text{Null}A$ is .

4. A is a 2×2 matrix whose nullspace is the line $x_1 = x_2$, and $C = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$. Sketch the nullspace of $Y = AC$.

5. Construct an SVD of $A = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$. Use your SVD to calculate the condition number of A .