W subspace in $\mathbb{R}^n$

$\dim(W) + \dim(W^{\perp}) = n$

Midterm 3 Lecture Review Activity, Math 1554

1. Indicate **true** if the statement is true, otherwise, indicate **false**.

	true	false
a) If S is a two-dimensional subspace of $\mathbb{R}^{50}$, then the dimension of $S^{\perp}$ is 48.	☒	○
b) An eigenspace ($= \mathrm{Null}(A - \lambda I)$) is a subspace spanned by a single eigenvector. eigenvectors corresponding to a single eigenvalue	○	☒
c) The $n \times n$ zero matrix can be diagonalized.	☒	○
d) A least-squares line that best fits the data points $(0, y_1), (1, y_2), (2, y_3)$ is unique for any values y_1, y_2, y_3.	☒	○

2. If possible, give an example of the following.

2.1) A matrix, A, that is in echelon form, and $\dim\left((\mathrm{Row} A)^{\perp}\right) = 2$, $\dim\left((\mathrm{Col}\, A)^{\perp}\right) = 1$

$\lambda = 0$

2.2) A singular 2×2 matrix whose eigenspace corresponding to eigenvalue $\lambda = 2$ is the line $x_1 = 2x_2$. The other eigenspace of the matrix is the x_2 axis.

$$A = P \cdot D P^{-1} = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}^{-1}$$

2.3) A subspace S, of $\mathbb{R}^4$, that satisfies $\dim(S) = \dim(S^{\perp}) = 3$.

2.4) A 2×3 matrix, A, that is in RREF. $(\mathrm{Row}\, A)^{\perp}$ is spanned by $\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$.

$$\begin{cases} (0, y_1) \quad (1, y_2) \quad (2, y_3) \\ y = \beta_0 + \beta_1 x \end{cases}$$

$$\Rightarrow \begin{cases} y_1 = \beta_0 + \beta_1 \cdot 0 \\ y_2 = \beta_0 + \beta_1 \cdot 1 \\ y_3 = \beta_0 + \beta_1 \cdot 2 \end{cases}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix}$$

$$\mathrm{Row}(A)^{\perp} = \mathrm{Col}(A^T)^{\perp} = \mathrm{Nul}((A^T)^T) = \mathrm{Nul}(A)$$

How to find $\dim(\mathrm{Row}(A))$

$$A \longrightarrow \cdots \longrightarrow \begin{bmatrix} 1 & \text{———} \\ 0 & - - - \\ & \end{bmatrix}$$

$\dim(\mathrm{Row}(A)^{\perp}) = 2$

$\dim(\mathrm{Row}(A)) = 1 = \dim(\mathrm{Col}(A))$

$$\begin{bmatrix} 1 & * & * \\ 0 & 0 & 0 \end{bmatrix}$$

$\dim(\mathrm{Col}(A)^{\perp}) = 1$

$\mathrm{Col}(A)$ in $\mathbb{R}^2$

3. Circle **possible** if the set of conditions are create a situation that is possible, otherwise, circle **impossible**. For the situations that are possible give an example.

   3.1) A is $n \times n$, $A\vec{x} = A\vec{y}$ for a particular $\vec{x} \neq \vec{y}$, $\vec{x}$ and $\vec{y}$ are in $\mathbb{R}^n$, and $\dim((\text{Row}\,A)^{\perp}) \neq 0$.

   possible **impossible**

   3.2) A is $n \times n$, $\lambda \in \mathbb{R}$ is an eigenvalue of A, and $\dim((\text{Col}(A - \lambda I))^{\perp}) = 0$.

   possible **impossible**

   3.3) $\text{proj}_{\vec{v}}\vec{u} = \text{proj}_{\vec{u}}\vec{v}$, $\vec{v} \neq \vec{u}$, and $\vec{u} \neq \vec{0}$, $\vec{v} \neq \vec{0}$.

   possible **impossible**

4. Consider the matrix A.

$$A = \begin{pmatrix} 1 & -3 & 0 & 2 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

   Construct a basis for the following subspaces and state the dimension of each space.

   4.1) $(\text{Row}A)^{\perp}$

   4.2) $\text{Col}\,A$

   4.3) $(\text{Col}A)^{\perp}$