Homework 6

Math 416, Abstract linear algebra, Fall 2019
Instructor: Daesung Kim

Due date: October 18, 2019

Textbooks: In the assignment, the two texts are abbreviated as follows:

- [FIS]: Freidberg, Insel, and Spence, *Linear Algebra*, 4th edition, 2002.
- [Bee]: Beezer, *A First Course in Linear Algebra*, Version 3.5, 2015.

1. Let $A, B \in \mathcal{M}_{n\times n}(\mathbb{R})$ be invertible matrices.
   (a) Prove that AB is invertible and $(AB)^{-1} = B^{-1}A^{-1}$.
   (b) Prove that A^t is invertible and $(A^t)^{-1} = (A^{-1})^t$.
2. Let $A \in \mathcal{M}_{n\times n}(\mathbb{R})$. Prove that A is invertible if and only if L_A is invertible and $(L_A)^{-1} = L_{A^{-1}}$.
3. Let $A, B \in \mathcal{M}_{n\times n}(\mathbb{R})$.
   (a) Prove that if AB is invertible, then A and B are invertible.
   (b) Prove that if $AB = I_n$, then $A = B^{-1}$.
4. Prove that if A and B are similar $n \times n$ matrices, then $\mathrm{tr}(A) = \mathrm{tr}(B)$.
5. Let V be a finite-dimensional vector space over $\mathbb{R}$ with basis β and $\dim(V) = n$. Define $\phi_\beta : V \to \mathbb{R}^n$ by $\phi_\beta(v) = [v]_\beta$. Show that ϕ_β is an isomorphism.
6. Let T be the linear map on $\mathbb{R}^2$ defined by
$$T\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 2a+b \\ a-3b \end{pmatrix}.$$
   Let
$$\beta = \left\{ e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}, \qquad \beta' = \left\{ v_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, v_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\}.$$
   Find $[T]_\beta$, $[I_{\mathbb{R}^2}]_{\beta'}^{\beta}$, $[I_{\mathbb{R}^2}]_{\beta}^{\beta'}$, and $[T]_{\beta'}$.
7. In $\mathbb{R}^2$, let L be the line $y = mx$ where $m \neq 0$. Find expressions for the following linear transformations $T(x, y)$.
   (a) T is the reflection of $\mathbb{R}^2$ about L.
   (b) T is the projection on L along the line perpendicular to L. (That is, for each $(x, y) \in \mathbb{R}^2$, $T(x, y)$ is the closest point on L to (x, y).)