

Math 285 Midterm 3 Free Response Questions

Due: 4/29 (Wed) at 8 pm

1. Consider the two point boundary problem

$$x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + 2y = 0 \quad (\text{A})$$

with $y(1) = -1$ and $y(2) = 1$.

- (a) (5 points) Let $x = e^t$. Show that the equation can be written as

$$\frac{d^2 y}{dt^2} - 3 \frac{dy}{dt} + 2y = 0.$$

- (b) (5 points) Find the general solution to the equation (A).

- (c) (5 points) Find the solution to the two point boundary problem.

2. Consider the heat conduction equation $4u_{xx} = u_t$ with $u(0, t) = u_x(\pi, t) = 0$ and $u(x, 0) = x$.

- (a) (3 points) Find a pair of two ordinary differential equations using the method of separation of variables $u(x, t) = X(x)T(t)$.

- (b) (3 points) Find the boundary condition for $X(x)$.

- (c) (9 points) Show that

$$u(x, t) = \sum_{m=1}^{\infty} C_m e^{-(2m-1)^2 t} \sin\left(\frac{(2m-1)x}{2}\right)$$

is a solution for some C_m .