Math 285: Differential Equations

Practice for midterm 2, Spring 2020

READ THE FOLLOWING INFORMATION.

- This is a 90-minute exam.
- No books, notes, calculators, or electronic devices allowed.
- You must not communicate with other students during this test.
- There are several different versions of this exam.
- Do not turn this page until instructed to.

1. Fill in your information:

Full Name:	______________________________
UIN (Student Number):	______________________________
NetID:	______________________________

Circle your section: B1 (9 am) C1 (10 am) E1 (1 pm) F1 (2 pm)

2. Fill in the following answers on the Scantron form:

95. D
96. C

Multiple Choice Questions: Mark answers to questions 1 to 10 on your scantron form.

1. (5 points) If $y_1 = e^{-t} + e^{3t}$ and $y_2 = 2e^{-t} + 3e^{3t}$, then the Wronskian $W[y_1, y_2](t)$ is

(A) $9e^{6t}$

(B) None of these

(C) $e^{2t} - e^{-2t}$

(D) $2e^{2t} + 3e^{6t}$

(E) $4e^{2t}$

2. (5 points) The motion of a certain spring–mass system is governed by

$$u'' + \gamma u + 9 = 0$$

for some constant $\gamma > 0$. The motion is underdamped if

(A) $\gamma \in (0, 6)$

(B) $\gamma \in (6, \infty)$

(C) None of these

(D) $\gamma \in (-6, 6)$

(E) $\gamma = 6$

3. (5 points) What can you say for the method of undetermined coefficients?

$$y'' + p(x)y' + q(x)y = g(x) \qquad (1)$$

(A) None of these

(B) It applies to system (1) with non-constant coefficients p and q

(C) It is more general than the method of variation of parameter.

(D) It helps to find the general solution to the homogeneous equation corresponding to (1)

(E) It provides us a formula for finding a specific solution to (1) provided that we know the general solution of the corresponding homogeneous equation

4. (5 points) Identify the correct form of a particular solution for the following differential equation.

$$y'' - y = t^2 e^{-t} + 10t^3.$$

(A) $Y(t) = (A_0 + A_1 t + A_2 t^2)e^{-t} + (B_0 + B_1 t + B_2 t^2 + B_3 t^3)$

(B) $Y(t) = t(A_0 + A_1 t + A_2 t^2)e^{-t} + (B_0 + B_1 t + B_2 t^2 + B_3 t^3)$

(C) $Y(t) = t^2(A_0 + A_1 t + A_2 t^2)e^{-t} + (B_0 + B_1 t + B_2 t^2 + B_3 t^3)$

(D) None of these

(E) $Y(t) = (A_0 + A_1 t + A_2 t^2 + A_3 t^3)e^{-t}$

5. (5 points) The general solution of $y'' + 4y' + 5y = 0$ is

(A) None of these

(B) $y(t) = e^{-2t}(C_1 \cos t + C_2 \sin t)$

(C) $y(t) = e^{2t}(C_1 + C_2 t)$

(D) $y(t) = C_1 e^{-5t} + C_2 e^{t}$

(E) $y(t) = e^{-t}(C_1 \cos 2t + C_2 \sin 2t)$

6. (5 points) What can you say for the vibration system?

$$ms'' + rs' + ks = F_0 \cos(\omega t) \qquad (1)$$

(A) When $F_0 = 0$ $r^2 \geq 4mk$, the general solution eventually becomes monotone and decays to 0.

(B) When $F_0 = 0$ $r^2 < 4mk$, the general solution oscillates and decays to 0.

(C) Resonance happens when $r = 0$ and $\sqrt{\frac{k}{m}} = \omega$.

(D) When $F_0 = 0$ $r = 0$, the general solution is periodic function.

(E) All of these.

7. (5 points) Which of these is NOT a set of linearly independent solutions?

(A) $\cos x, \sin x, \cos 2x$

(B) $1, x, x^2$

(C) $x, x^2 + x, 2x^2$

(D) e^x, e^{2x}, e^{3x}

(E) None of these

8. (5 points) Find the general solution of the following differential equation.

$$y''' + 8y = 0.$$

(A) $C_1 e^{-2t} + C_2 e^t \cos \sqrt{3}t + C_3 e^t \sin \sqrt{3}t$

(B) $C_1 e^{2t} + C_2 e^t \cos \sqrt{3}t + C_3 e^t \sin \sqrt{3}t$

(C) $C_1 e^{2t} + C_2 e^{-t} \cos \sqrt{3}t + C_3 e^{-t} \sin \sqrt{3}t$

(D) $C_1 e^{-2t} + C_2 e^{-t} \cos \sqrt{3}t + C_3 e^{-t} \sin \sqrt{3}t$

(E) None of these

9. (5 points) The motion of a certain spring–mass system is governed by

$$u'' + 4u = 0$$

with $u(0) = 1$ and $u'(0) = 1$. What are the amplitude and the natural frequency?

(A) $R = \sqrt{2}$ and $\omega_0 = 1$

(B) $R = \sqrt{5}/2$ and $\omega_0 = 1$

(C) $R = \sqrt{5}/2$ and $\omega_0 = 2$

(D) None of these

(E) $R = \sqrt{2}$ and $\omega_0 = 2$

10. (5 points) The equation $t^2 y'' - 4ty' + 6y = 0$ has a solution $y_1(t) = t^2$. If $y_2(t) = v(t)y_1(t)$ is another solution to the equation, then the function $v(t)$ satisfies

(A) None of these

(B) $v'' = 0$

(C) $t^2 v'' + 2t(t-2)v' = 0$

(D) $t^4 v'' + 2t^3(t-2)v' = 0$

(E) $tv'' - 4v' = 0$

11. (5 points) The existence and uniqueness theorem guarantees that there exists a unique solution of the initial value problem $(t-1)(t+3)y'' + (t-4)y' + \sin t y = e^{-3t}$ with $y(-1) = 2$ and $y'(-1) = 0$ on an open interval

(A) $(1, 4)$

(B) $(-3, 1)$

(C) None of these

(D) $(-\infty, -3)$

(E) $(4, \infty)$

12. (5 points) The function $u(t) = -\cos 2t + \sin 2t$ can be written as $u(t) = R\cos(\omega_0 t - \delta)$ where

(A) $R = \sqrt{3}$, $\omega_0 = 2$, $\delta = 3\pi/4$

(B) $R = \sqrt{2}$, $\omega_0 = 2$, $\delta = 3\pi/4$

(C) $R = \sqrt{2}$, $\omega_0 = 2$, $\delta = \pi/4$

(D) None of these

(E) $R = \sqrt{3}$, $\omega_0 = 1$, $\delta = 5\pi/4$

Free Response Questions: Make sure you show your work. You may get partial credit.

13. (10 points) Consider $t^2y'' - t(t+2)y' + (t+2)y = 0$ for $t > 0$.

(i) Verify that $y_1(t) = t$ is a solution to the equation.

(ii) Find the general solution to the equation using the method of reduction of order.

14. (10 points) Find the solution to the following initial value problem

$$y'' - 2y' - 3y = 3te^{2t}, \qquad y(0) = 1, \qquad y'(0) = 0.$$

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