Math 285 Final Practice

1. (5 points) What is the correct integrating factor to solve the following ODE for $y(t)$?

$$3t^2y' + t^3y + t\sin t = 0$$

   A. $\mu(t) = t^3$
   B. $\mu(t) = \frac{1}{t^3}$
   C. $\mu(t) = e^{\frac{1}{6}t^2}$
   D. $\mu(t) = 1$
   E. None of these

2. (5 points) Which one of the following is a solution to $ty' = 2y + 1$?
   A. $y = t^2 - 1$
   B. $y = \frac{1}{2}(t - 1)$
   C. $y = 0$
   D. $y = \frac{1}{2}(t^2 - 1)$
   E. $y = 2t - \frac{1}{2}$

3. (5 points) The function $P(t) = P_0$ is a stable solution to the following equation

$$\frac{dP}{dt} = (P-1)(P+3)P^2$$

   if

   A. $P_0 = 1$
   B. $P_0 = 0$
   C. $P_0 = 1$ and $P_0 = -3$
   D. $P_0 = 1$ and $P_0 = 0$
   E. $P_0 = -3$

4. (5 points) The following nonlinear equation for $y(x)$ can be transformed with a substitution into which separable equation for $v(x)$?

$$y' = \frac{x^2 - xy + 2y^2}{x^2}$$

   A. $xv' = 1 - 2v + 2v^2$
   B. $xv' = 1 - v + 2v^2$

C. $v' = 1 - v + 2v^2$

D. $v' = x^2 - 2v + 2v^2$

E. None of these

5. (5 points) Consider

$$y''' - 5y'' + 8y' - 4y = 0$$

Which one of the following is NOT a solution?

A. $y = e^t$

B. $y = te^t + 3e^{2t}$

C. $y = 2e^{2t} - e^t$

D. $y = -te^{2t}$

E. None of these

6. (5 points) Identify the correct form of a particular solution for the following differential equation.

$$y'' + 2y' = 4te^{-2t} + 1.$$

A. $Y(t) = (At + B)e^{-2t} + Ct$

B. $Y(t) = (At^2 + Bt)e^{-2t} + C$

C. $Y(t) = (At^2 + Bt)e^{-2t} + Ct$

D. $Y(t) = (At + B)e^{-2t} + C$

E. $Y(t) = (At + B)e^{-2t}$

7. (5 points) The motion of a certain spring–mass system is governed by

$$u'' + \gamma u' + ku = 0$$

for some constants $\gamma, k > 0$. The motion is overdamped if

A. $\gamma = 1$ and $k = 2$

B. $\gamma = 3$ and $k = 3$

C. $\gamma = 2$ and $k = \frac{3}{2}$

D. $\gamma = 4$ and $k = 3$

E. None of these

8. (5 points) The existence and uniqueness theorem guarantees that there exists a unique solution of the initial value problem $(t-1)(t+2)y'' + y'\sqrt{5-t} + e^t y = 1$ with $y(2) = 1$ and $y'(2) = 3$ on an open interval

A. $(-\infty, 1)$

B. $(-2, 1)$

C. $(1, \infty)$

D. None of these

E. $(1, 5)$

9. (5 points) Consider the following boundary value problem for a variant of the wave equation:

$$\begin{aligned} &u_{tt} = u_{xx} + u, \qquad \text{for } 0 < x < 1, \quad t > 0, \\ &u(0,t) = u_x(1,t) = 0 \qquad \text{for } t \geq 0, \\ &u(x,0) = f(x), \qquad u_t(x,0) = 0 \qquad \text{for } 0 \leq x \leq 1. \end{aligned}$$

Then separated solutions must satisfy which of the following sets of equations?

A. $X'' + \lambda X = 0$ with $X(0) = X'(1) = 0$, and $T'' + (\lambda - 1)T = 0$ with $T'(0) = 0$.

B. $X'' + \lambda X = 0$ with $X(0) = X(1) = 0$, and $T'' + \lambda T = 0$ with $T'(0) = 0$.

C. $X'' + (\lambda - 1)X = 0$ with $X(0) = X'(1) = 0$, and $T'' + \lambda T = 0$ with $T'(0) = 0$.

D. $X'' + (\lambda - 1)X = 0$ with $X(0) = X(1) = 0$, and $T'' + (\lambda + 1)T = 0$ with $T'(0) = 0$.

E. None of these

10. (5 points) Let $X(x) = e^{10x} + e^{-10x}$. Find a nonzero function $Y(y)$ such that the product $u(x,y) = X(x)Y(y)$ satisfies Laplace's equation $u_{xx} + u_{yy} = 0$.

A. $Y(y) = \sinh(10y)$

B. $Y(y) = \cos(10y)$

C. $Y(y) = \sin(5y)$

D. $Y(y) = e^{10y} - e^{-10y}$

E. None of these

11. (5 points) If λ_1 is the smallest eigenvalue of $y'' + \lambda y = 0$ with $y'(0) = y(\pi) = 0$, what is the corresponding eigenfunction?

A. $\cos(\frac{t}{4})$

B. $\sin(\frac{t}{2})$

C. $\cos(\frac{t}{2})$

D. $\sin(\frac{t}{4})$

E. None of these

12. (5 points) The equation

$$y'' - 2xy' + \lambda y = 0$$

can be transformed into the form $(p(x)y')' + q(x)y = 0$ with

A. $p(x) = e^{-2x}$ and $q(x) = \lambda$

B. $p(x) = -x^2$ and $q(x) = -\lambda x^2$

C. $p(x) = 2x$ and $q(x) = \lambda$

D. $p(x) = e^{-x^2}$ and $q(x) = \lambda e^{-x^2}$

E. None of these