Final Exam - MCQ (16656680)

Due: Wed, May 13, 2020 04:00 PM CDT

Question	1	2	3	4	5	6	7	8	9	10	11	12

1. Question Details final-1 [4669896] -

For the linear differential equation

$$y\, t^2 \cos(t) + t^2\, y' = (t^6 + 3)e^{2t},$$

an integrating factor is

- $\mu(t)=e^{-\sin(t)}$
- $\mu(t)=\dfrac{1}{\sin(t)}$
- $\mu(t)=\sin(t)$
- None of these
- $\mu(t)=e^{\sin(t)}$

Solution or Explanation

Since

$$\int \frac{t^2 \cos(t)}{t^2}dt=\int \cos(t)dt=\sin(t),$$

the integrating factor is $\mu(t)=e^{\sin(t)}$.

2. Question Details final-2 [4669902]

Which one of the following is a solution to $y'=(t+2)y^2$?

- $y(t)=0$
- $y(t)=\frac{2}{t^2+4t}$
- None of these
- $y(t)=\frac{1}{2}t^2+2t$
- $y(t)=-\frac{1}{t^2+2t+1}$

Solution or Explanation

First, note that $y(t)=0$ is a solution. Since the given equation is separable, we have $y^{-2}y'=(t+2)$ and so

$$-\frac{1}{y}=\frac{1}{2}t^2+2t+C$$

for arbitrary constant C.

3. Question Details final-3 [4669969]

A certain species has population level $p(t)$ modeled by the initial value problem

$$\frac{dp}{dt}=-3p(p-4)(p-5)$$

with $p(0)=p_0>0$. Then the population will go extinct if

- $p_0=4$
- $p_0=3$
- None of these
- $p_0=4.5$
- $p_0=6$

Solution or Explanation

Since $f(p)=-3p(p-4)(p-5)$ is negative if $0<p<4$, the population will go extinct if $0<p_0<4$.

4. Question Details final-4 [4670043]

The differential equation

$$y'+(\sin t)\,y = (\cos t)\,y^5$$

can be transformed by the substitution $v=y^{-4}$ into the linear equation:

- $v'-4\,(\sin t)\,v = -\,4\cos t$
- None of these
- $4\,v'-(\sin t)\,v = -\cos t$
- $v'-5\,(\sin t)\,v = -\,5\cos t$
- $v'-4\,(\sin t)\,v = \cos t$

Solution or Explanation

First, divide the equation by y^5 to get

$$\frac{y'}{y^5}+(\sin t)\,y^{-4} = (\cos t).$$

Since $v'=-\frac{4\,y'}{y^5}$, we have $v'-4\,(\sin t)\,v = -\,4\cos t$.

5. Question Details final-5 [4670050]

If $y(t)=2\,e^{-3\,t}\cos(2\,t)+2$ is a solution to $y'''+py''+qy'+ry=0$, then what is q?

- $q=4$
- None of these
- $q=5$
- $q=13$
- $q=6$

Solution or Explanation

Since $y(t)=2\,e^{-3\,t}\cos(2\,t)+2$ is a solution, we know that $e^{-3\,t}\cos(2\,t)$ and 1 are fundamental solutions, which means the characteristic equation has the roots $0, -3+2\,i, -3-2\,i$. Therefore, the characteristic equation is $\lambda(\lambda^2+6\,\lambda+13)=0$ and so the differential equation is $y'''+6\,y''+13\,y'=0$.

6. Question Details final-6 [4670064]

Identify the correct form of a particular solution for the following differential equation

$$y''-2y'+y=te^t+4.$$

- ○ $Y(t)=At^2e^t + Bt\,e^t + C$
- ○ $Y(t)=At^2\,e^t + B$
- ○ $Y(t)=Ate^t + B\,e^t + Ct + D$
- ○ None of these
- ○ $Y(t)=At^2e^t + Bt + C$

Solution or Explanation

The fundamental solutions to the homogeneous equation are e^t, te^t. Thus, a particular solution is of the form

$$Y(t)=t(At+B)\,e^t+C.$$

7. Question Details final-7 [4670069]

For which values of m and ω, is the following oscillator in resonance?

$$mu''+48\,u = 2\cos(\omega t).$$

- ○ $m=3$ and $\omega=3$
- ○ $m=4$ and $\omega=5$
- ○ None of these
- ○ $m=3$ and $\omega=4$
- ○ $m=5$ and $\omega=4$

Solution or Explanation

The oscillator is in resonance if $\omega=\sqrt{k/m}$.

8. Question Details final-8 [4670088]

The existence and uniqueness theorem guarantees that there exists a unique solution to the initial value problem

$$(t+3)(t-2)y'' + (\ln(t^2+1))\, y' + t^2 e^t\, y = \cos(t-5)$$

with $y(4)=-8$ and $y'(4)=-9$ on an open interval

- $(2, \infty)$
- $(1, 5)$
- $(-3, 2)$
- $(-1, 2)$
- None of these

Solution or Explanation

Dividing $(t+3)(t-2)$, the coefficients are continuous if $x \neq -3, 2$. Since the initial conditions are given at $t=4$, the answer is $(2, \infty)$.

9. Question Details final-9 [4670101]

If you were to solve the variant of wave equation $u_{tt}=u_{xx}+u$ for $0<x<6$ and $t>0$ with

$$u(0,t)=u(3,t)=0,\ u(x,0)=2x,\ u_t(x,0)=0$$

using separation of variables, what would be the correct form of $X_n(x)$?

- $X_n(x)=\sin(\frac{n\pi}{3}x)$
- None of these
- $X_n(x)=\cos(\frac{n\pi}{3}x)$
- $X_n(x)=\sin(\frac{n^2\pi^2}{9}x)$
- $X_n(x)=\cosh(\frac{n\pi}{9}x)$

Solution or Explanation

Using the method of separation of variables, we see that $X(x)$ satisfies $X''+\lambda X=0$ with $X(0)=X(3)=0$. Thus, the eigenvalues are $\lambda_n=\frac{n^2\pi^2}{9}$ and $X_n(x)=\sin(\frac{n\pi}{3}x)$

10. Question Details final-10 [4670109]

Let $X(x)=5\cos(4x)+\sin(4x)$, then $u(x,y)=X(x)Y(y)$ is a solution to Laplace's equation $u_{xx}+u_{yy}=0$ if

- $Y(y)=\cosh(4y)$
- $Y(y)=\sinh(16y)$
- $Y(y)=\cos(4y)$
- $Y(y)=16\sin(y)$
- None of these

Solution or Explanation

Since $u_{xx}+u_{yy}=0$ and $\frac{X''}{X}=-16$, we have $Y''-16Y=0$. Thus, $Y(y)=c_1e^{4y}+c_2e^{-4y}$.

11. Question Details final-11 [4670158]

If λ is a nonnegative eigenvalue of $y''+\lambda y=0$ with $y(0)+y'(0)=0$ and $y(2)=0$, then λ satisfies

- $2\sqrt{\lambda}=\tan(\sqrt{\lambda})$
- $\sqrt{\lambda}=\tan(2\sqrt{\lambda})$
- $\lambda=\cot(2\lambda)$
- $\sqrt{\lambda}=-\tan(2\sqrt{\lambda})$
- None of these

Solution or Explanation

If $\lambda=0$, then $y(x)=c_1+c_2x$. The boundary conditions imply $c_1=c_2=0$. If $\lambda=\mu^2>0$, then $y(x)=c_1\cos(\mu x)+c_2\sin(\mu x)$. The boundary conditions imply $c_1+c_2\mu=0$ and $c_1\cos(2\mu)+c_2\sin(2\mu)=0$. Solving the system of equations, we get $\mu=\tan(2\mu)$.

12. Question Details final-12 [4670156] -

The equation

$$xy''+(1-x)y'+\lambda y=0$$

can be transformed into the form $(p(x)y)'+q(x)y=0$ with

- $p(x)=e^{-x^2}$ and $q(x)=\lambda e^{-x^2}$
- None of these
- $p(x)=e^{-x}$ and $q(x)=\lambda x$
- $p(x)=1-\frac{1}{x}$ and $q(x)=-\lambda x(1-x)$
- $p(x)=xe^{-x}$ and $q(x)=\lambda e^{-x}$

Solution or Explanation

Multiplying by $\mu(x)$, we have

$$x\mu(x)y''+(1-x)\mu(x)y'+\lambda\mu(x)y=0.$$

Then, solving the equation $(x\mu(x))'=(1-x)\mu$, we get $\mu(x)=e^{-x}$. Thus, $p(x)=xe^{-x}$ and $q(x)=\lambda e^{-x}$.

Assignment Details

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