

Midterm 2 - Multiple Choice Questions (16334764)


Current Score: 0/50 Due: Wed, Apr 1, 2020 08:50 PM CDT

Question	1	2	3	4	5	6	7	8	9	10	Total
Points	0/5	0/5	0/5	0/5	0/5	0/5	0/5	0/5	0/5	0/5	0/50

1. 0/5 points

Mid2-1 [4622739]

If $u = 2f + 3g$ and $v = -2f + g$ for some functions $f(t)$ and $g(t)$, then $W[u, v](t) = kW[f, g](t)$. What is k ?

- ☒  $k = 8$
- ☐ $k = 14$
- ☐ None of these
- ☐ $k = 2$
- ☐ $k = -4$

Solution or Explanation

We have

$$W[u, v](t) = (2f' + 3g')(-2f + g) - (2f + 3g)(-2f' + g') = (2 \cdot 1 - 3 \cdot (-2))(f'g - fg') = 8 W[f, g](t).$$


2. 0/5 points

Mid2-2 [4625284]

The existence and uniqueness theorem guarantees that there exists a unique solution of the initial value problem

$$t(t+3)y''' + (t-3)y' + te^{-t^2}y = \ln|6-t|$$

with $y(4) = -1$, $y'(4) = 2$, and $y''(4) = -1$ on an open interval

- ☐ $(0, 3)$
- ☐ $(3, 6)$
- ☒  $(0, 6)$
- ☐ None of these
- ☐ $(-3, 6)$

Solution or Explanation

After dividing by $t(t+3)$, the coefficients are continuous if $t \neq 0, -3, 6$. Since the initial conditions are given at $t=4$, the interval is $(0, 6)$.

3. 0/5 points

Mid2-3 [4625509]

The motion of a certain spring-mass system is governed by

$$u'' + 8u' + 16u = 0$$

with $u(0) = -4$ and $u'(0) = 20$. How many times does the mass pass through the equilibrium position $u=0$? That is, how many positive numbers t are there such that $u(t)=0$?

- ☐ 0
☐ 2
☐ None of these
☐ Infinitely many times
☒ 1

Solution or Explanation

Since the characteristic equation has the repeated roots $\lambda = -4$, the general solution is

$$u(t) = e^{-4t}(At+B).$$

By the initial condition, we get $A=4$ and $B=-4$. Since $u(t)=0$ if and only if $4t-4=0$, the answer is 1.

4. 0/5 points

Mid2-4 [4625354]

The equation $(x-4)y'' - xy' + 4y = 0$ for $x > 4$ has a solution $y_1(x) = e^x$. If $y_2(x) = v(x)y_1(x)$ is another solution to the equation, then the function $v(x)$ satisfies

- ☐ $v''=0$
☒ $(x-4)v'' + (x-8)v' = 0$
☐ $(x-4)v'' - 4v' = 0$
☐ $v'' + v' = 0$
☐ None of these

Solution or Explanation

Since

$$(x-4)y_2'' - xy_2' + 4y_2 = (x-4)(v''y_1 + 2v'y_1' + vy_1'') - x(v'y_1 + vy_1') + 4vy_1 = e^x((x-4)v'' + (x-8)v') = 0,$$


the function $v(x)$ should satisfy $(x-4)v'' + (x-8)v' = 0$

5. 0/5 points

Mid2-5 [4625365]

Identify the correct form of a particular solution for the following differential equation.

$$y'' + 3y' = te^{3t} + 8t^3.$$

- ☒  $Ate^{3t} + Be^{3t} + Ct + Dt^2 + Et^3 + Ft^4$
- ☐ $At^2 e^{3t} + Bt e^{3t} + Ct + Dt + Et^2 + Ft^3$
- ☐ $Ate^{3t} + Be^{3t} + Ct + Dt + Et^2 + Ft^3$
- ☐ None of these
- ☐ $At^2 e^{3t} + Bt e^{3t} + Ct + Dt^2 + Et^3 + Ft^4$

Solution or Explanation

Since the homogeneous equation $y'' + 2y' = 0$ has solutions $1, e^{-3t}$, a particular solution is of the form


$$Ate^{3t} + Be^{3t} + Ct + Dt^2 + Et^3 + Ft^4$$

6. 0/5 points

Mid2-6 [4625450]

For which value of k is the following oscillator in resonance?

$$3u'' + ku = 6\cos(2t)$$

- ☐ 4
- ☐ 18
- ☐ None of these
- ☒  12
- ☐ 6

Solution or Explanation

The oscillator is in resonance if $\sqrt{k/m} = \sqrt{k/3} = 2$.


7. 0/5 points

mid2-7 [4625463]

The differential equation

$$u'' + 6u' + 11 = 0$$

corresponds to an oscillator that is

- ☐  underdamped
- ☐ critically damped
- ☐ overdamped
- ☐ undamped
- ☐ None of these


Solution or Explanation

The characteristic equation has complex roots.

8. 0/5 points

Mid2-8 [4625468]

The function $u(t) = -4\sqrt{3}\cos(3t) + 4\sin(3t)$ can be written as $u(t) = R\cos(\omega_0 t - \delta)$ where

- ☐ $R=4, \quad \omega_0=1, \quad \delta=11\pi/6$
- ☐ $R=8, \quad \omega_0=1, \quad \delta=\pi/6$
- ☐ None of these
- ☐  $R=8, \quad \omega_0=3, \quad \delta=5\pi/6$
- ☐ $R=4, \quad \omega_0=3, \quad \delta=7\pi/6$

Solution or Explanation

We have $R = \sqrt{(-4\sqrt{3})^2 + (4)^2} = 8$ and $\omega_0 = 3$. Since $\cos(\delta) = -\sqrt{3}/2$ and $\sin(\delta) = 1/2$, $\delta = 5\pi/6$.

9. 0/5 points


Mid2-9 [4625483]

Determine whether

$$3t - 5, t^2 + 1, 10t^2 + 7t$$

are linearly dependent or linearly independent. If they are linearly dependent, find k_1, k_2, k_3 such that

$$k_1(3t - 5) + k_2(t^2 + 1) + k_3(10t^2 + 7t) = 0.$$

- ☐  linearly independent
- ☐ linearly dependent and $k_1 = 1, \quad k_2 = 5, \quad k_3 = 1$
- ☐ linearly dependent and $k_1 = 2, \quad k_2 = 10, \quad k_3 = 1$
- ☐ None of these

10. 0/5 points

Mid2-10 [4625504]

The third order differential equation

$$y''' + py'' + qy' + ry = 0$$

has the characteristic equation $(\lambda + 2)(\lambda^2 + 6\lambda + 18) = 0$. What is the general solution to the differential equation?

- ☐ $C_1 e^{-2t} + C_2 e^{-3t} + C_3 e^{3t}$
- ☒ $C_1 e^{-2t} + C_2 e^{-3t} \cos(3t) + C_3 e^{-3t} \sin(3t)$
- ☐ None of these
- ☐ $C_1 e^{-2t} + C_2 e^{3t} \cos(3t) + C_3 e^{3t} \sin(3t)$
- ☐ $C_1 e^{2t} + C_2 e^{-3t} \cos(3t) + C_3 e^{-3t} \sin(3t)$

Solution or Explanation

Since the roots are $\lambda = -2, -3 \pm 3i$, the general solution is

$$C_1 e^{-2t} + C_2 e^{-3t} \cos(3t) + C_3 e^{-3t} \sin(3t).$$

Assignment Details

Name (AID): **Midterm 2 - Multiple Choice Questions (16334764)**

Submissions Allowed: **3**

Category: **Exam**

Code:

Locked: **No**

Author: **Kim, Daesung** (daesungk@illinois.edu)

Last Saved: **Mar 19, 2020 02:36 AM CDT**

Permission: **Protected**

Randomization: **Person**

Which graded: **Last**

Feedback Settings

Before due date

Save Work

After due date

Question Score

Assignment Score

Publish Essay Scores

Question Part Score

Mark

Add Practice Button

Help/Hints

Response