Math 416: Abstract Linear Algebra

Midterm 3, Fall 2019

Date: November 20, 2019

NAME:______________________________

READ THE FOLLOWING INFORMATION.

- This is a 50-minute exam.
- This exam contains 9 pages (including this cover page) and 5 questions. Total of points is 50.
- Books, notes, and other aids are not allowed except for one page of cheat sheet. Collaboration is forbidden.
- Show all steps to earn full credit.
- Do not unstaple pages. Loose pages will be ignored.

Question	Points	Score
1	12	
2	10	
3	10	
4	8	
5	10	
Total:	50	

1. Let $A \in \mathcal{M}_{n\times n}(\mathbb{R})$ with $A^2 = A$.

   (a) (4 points) Show that the only possible eigenvalues for A are 0 and 1.

   (b) (4 points) Show that $\mathcal{R}(L_A) = \mathcal{N}(A - I_n)$.

(c) (4 points) Show that A is diagonalizable.

2. Let $A = \begin{pmatrix} 0 & 4 & -1 & 1 \\ -2 & 6 & -1 & 1 \\ -2 & 8 & -1 & -1 \\ -2 & 8 & -3 & 1 \end{pmatrix}$, then the characteristic polynomial is $f(t) = (t-2)^2(t+2)(t-4)$. Thus, $2, -2$, and 4 are the eigenvalues for A.

   (a) (3 points) Find the algebraic multiplicities $\mathrm{m}_{\mathrm{alg}}(\lambda)$ for $\lambda = 2, -2, 4$.

   $\mathrm{m}_{\mathrm{alg}}(2)=$__________ $\mathrm{m}_{\mathrm{alg}}(-2)=$__________ $\mathrm{m}_{\mathrm{alg}}(4)=$__________

   (b) (3 points) Find the geometric multiplicities $\mathrm{m}_{\mathrm{geo}}(\lambda)$ for $\lambda = 2, -2, 4$.

   $\mathrm{m}_{\mathrm{geo}}(2)=$__________ $\mathrm{m}_{\mathrm{geo}}(-2)=$__________ $\mathrm{m}_{\mathrm{geo}}(4)=$__________

(c) (4 points) Determine whether A is diagonalizable or not. Justify your answer.

3. Let $A = \dfrac{1}{10}\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \end{pmatrix}$.

   (a) (3 points) Write down the definition of a transition matrix and show that A is a transition matrix.

   (b) (4 points) Find the eigenspace E_1 corresponding to $\lambda = 1$.

(c) (3 points) Compute $\lim_{m\to\infty} A^m$.

4. (8 points) Let V be an inner product space over $\mathbb{R}$ and S a subset of V. Show that if $x, y \in \operatorname{Span}(S)$ and $\langle x, z\rangle = \langle y, z\rangle$ for all $z \in S$ then $x = y$.

5. (10 points) Circle True or False. Do not justify your answer.

(a) True False If $A, B \in \mathcal{M}_{3\times 3}(\mathbb{R})$ have the same eigenvalues $\lambda = 1, 6, 7$, then A and B are similar.

(b) True False Let $A \in \mathcal{M}_{n\times n}(\mathbb{R})$ and $\lambda \in \mathbb{R}$. If $A - \lambda I_n$ is onto, then λ is an eigenvalue of A.

(c) True False If $A, B \in \mathcal{M}_{n\times n}(\mathbb{R})$ are transition matrices, then AB is also a transition matrix.

(d) True False Every finite dimensional inner product space has a unique orthonormal basis.

(e) True False Let $V = \mathbb{R}^3$ and define

$$\langle (x_1, x_2, x_3), (y_1, y_2, y_3) \rangle = x_1 y_1 + x_2 y_2$$

for all $(x_1, x_2, x_3), (y_1, y_2, y_3) \in V$. Then, $\langle \cdot, \cdot \rangle$ is an inner product.