Math 285 Midterm 3 Practice Exam

1. (5 points) Find a solution to $y^{(4)} + y'' = 24t$.

   A. $y = 4t^3 + 1$

   B. $y = 6t^3 + 5t^2$

   C. $y = 4t^2 + t$

   D. $y = 4t^3 + t^2$

   E. None of these

2. (5 points) Find the smallest number λ such that $y'' + \lambda y = 0$ with $y'(0) = y'(\pi) = 0$ has a nontrivial solution.

   A. 1

   B. 0

   C. $\frac{1}{2}$

   D. π^2

   E. None of these

3. (5 points) Suppose that a function $f(t)$ which is periodic of period 2π has the Fourier series

$$f(t) = \sum_{m=1}^{\infty} \frac{(-1)^m}{m^2 + 3m} \cos mt.$$

   Evaluate the integral

$$\int_{-\pi}^{\pi} f(t) \cos 6t \, dt.$$

   A. $\frac{1}{54}$

   B. $\frac{1}{27}$

   C. $\frac{\pi}{54}$

   D. $\frac{\pi}{27}$

   E. None of these

4. (5 points) Consider the function $f(t)$ defined on $\mathbb{R}$ such that $f(t) = f(t + 2\pi)$ and

$$f(t) = \begin{cases} 3, & -\pi \leq t < 0, \\ e^{\pi^2}, & t = 0, \\ -1, & 0 < t < \pi. \end{cases}$$

   Let $S(t)$ be the Fourier series of $f(t)$. What is $S(0)$?

A. e^{π^2}

B. 2

C. 0

D. 1

E. None of these

5. (5 points) Let $f(t)$ be a function on $[0, 2]$ given by $f(t) = 2t$. Find the Fourier sine series for $f(t)$ of period 4.

A. $\displaystyle\sum_{m=1}^{\infty} a_m \sin\left(\frac{m\pi}{2}t\right)$ where $a_m = \frac{1}{2}\int_0^2 t\sin\left(\frac{m\pi}{2}t\right)\,dt.$

B. $\displaystyle\sum_{m=1}^{\infty} a_m \sin\left(\frac{m\pi}{2}t\right)$ where $a_m = \frac{1}{2}\int_0^2 t\sin\left(\frac{m\pi}{2}t\right)\,dt.$

C. $\displaystyle\sum_{m=1}^{\infty} a_m \sin\left(\frac{m\pi}{4}t\right)$ where $a_m = \int_0^2 t\sin\left(\frac{m\pi}{4}t\right)\,dt.$

D. $\displaystyle\sum_{m=1}^{\infty} a_m \sin\left(\frac{m\pi}{2}t\right)$ where $a_m = 2\int_0^2 t\sin\left(\frac{m\pi}{2}t\right)\,dt.$

E. $\displaystyle\sum_{m=1}^{\infty} a_m \sin\left(\frac{m\pi}{4}t\right)$ where $a_m = \int_{-2}^2 t\sin\left(\frac{m\pi}{4}t\right)\,dt.$

6. (5 points) Let f and g be functions defined on $\mathbb{R}$. Which one of the followings is NOT correct?

A. If f is even, then f' is odd.

B. The function $\sin 3t + \cos 2t$ is periodic with period 2π.

C. If f is even and g is odd, then $f(x) + g(x)$ is even.

D. If f is even and g is odd, then $\displaystyle\int_{-4}^{4} f(x)g(x)\,dx = 0$.

E. If f is periodic with period 4 and $f(x) = x$ for $0 < x < 2$, then $f(x) = x - 4$ for $4 < x < 6$.

7. (5 points) Find a pair of ordinary differential equations from the partial differential equation $xu_{xx} + u_t = 0$ using the method of separation of variables.

A. $X''(x) + \lambda X(x) = 0$ and $T'(t) + \lambda x T(t) = 0$

B. $xX''(x) + \lambda X(x) = 0$ and $T'(t) - \lambda T(t) = 0$

C. $X''(x) + \lambda x X(x) = 0$ and $\lambda T'(t) - T(t) = 0$

D. $X''(x) - \lambda x X(x) = 0$ and $T'(t) - \lambda T(t) = 0$

E. None of these

8. (5 points) Consider the heat conduction problem

$$\begin{aligned} &5u_{xx} = u_t, \quad 0 < x < 3, \\ &u(0,t) = u(3,t) = 0, \quad u(x,0) = f(x) \end{aligned}$$

for some function f defined on $[0,3]$. Which one of the followings is correct?

A. If $f(x) = \sin \pi x$, then the solution is $u(x,t) = e^{-5\pi^2 t} \sin \pi x$.

B. If $u(x,t)$ and $v(x,t)$ are solutions, then $u(x,t) + v(x,t)$ is also a solution.

C. The thermal diffusivity is 3.

D. The solution is

$$u(x,t) = \sum_{m=1}^{\infty} C_m e^{-\frac{5m^2\pi^2}{3}t} \sin\left(\frac{m\pi}{3}x\right)$$

for some C_m.

E. None of these.

9. (5 points) What is the steady state solution $v(x)$ for the following problem?

$$\begin{aligned} 5u_{xx} &= u_t, \qquad 0 < x < 6, \quad t \geq 0, \\ u(0,t) &= 10, \qquad u(6,t) = 2. \end{aligned}$$

A. $v(x) = \frac{5}{2}x - 1$

B. $v(x) = 0$

C. $v(x) = x + 5$

D. $v(x) = x - 10$

E. $v(x) = 10 - \frac{4}{3}x$