

Midterm 2 - Free Response Questions (16334775)

Due: Wed, Apr 1, 2020 08:50 PM CDT

Question

1 2

1. Question Details

Mid2-11 [4625505]

Find the solution to the following initial value problem

$$y'' - 6y' + 9y = 2t e^{3t}$$

with $y(0)=1$ and $y'(0)=0$.Upload your solution here no file selected

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Solution or Explanation

The general solution of $y'' - 6y' + 9y = 0$ is $C_1 e^{3t} + C_2 t e^{3t}$ (2 points). By the method of undetermined coefficients, a particular solution has a form

$$Y(t) = t^2(A + B)e^{3t} = (At^3 + Bt^2)e^{3t}$$

(3 points). We then have

$$Y'' - 6Y' + 9Y = (6At + 2B)e^{3t} = 2te^{3t}$$

(2 points), which means $A = 1/3$ and $B = 0$ (1 points). By the initial conditions, we obtain

$$y(t) = e^{3t} - 3te^{3t} + \frac{1}{3}t^3 e^{3t}$$

(2 points).

2. Question Details

Mid2-12 [4625508]

Consider $t^2 y'' - 5ty' + 9y = 0$ for $t > 0$.

(i) (3 points) Find r such that $y_1(t) = t^r$ is a solution to the equation.

(ii) (7 points) Find another solution y_2 to the equation such that $W[y_1, y_2](t) \neq 0$ for all $t > 0$. (Hint: use the method of reduction of order.)

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Solution or Explanation

(i) Since

$$t^2 y_1'' - 5ty_1' + 9y_1 = r(r-1)t^r - 5r t^r + 9t^r = t^r (r^2 - 6r + 9) = 0$$

(2 points) we obtain $r = 3$ (1 points).

(ii) Let $y_2 = v y_1$, then

$$t^2 y_2'' - 5ty_2' + 9y_2 = t^2 (v'' y_1 + 2v' y_1' + v y_1'') - 5t(v' y_1 + v y_1') + 9v y_1 = t^4 (tv'' + v') = 0$$

(2 points). Thus, $tv'' + v' = 0$ (1 point). By solving the equation, we get $v(t) = C_1 \ln t + C_2$ (2 points). Let $y_2 = t^3 \ln t$, then y_2 is a solution and

$$W[y_1, y_2](t) = t^3(3t^2 \ln t + t^2) - 3t^2(t^3 \ln t) = t^5 \neq 0$$

(2 points).

Assignment Details

Name (AID): **Midterm 2 - Free Response Questions (16334775)**

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Code:

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