All dose-response relations considered are twice continuously differentiable, have d=0 when E=0 and have positive slope. It follows that they can equally well be considered response-dose relations using the compositional inverse. Over the years, many, many dose-response relations of the form E=f(d) have been developed. For example in S3.1.1 of the condensed supplement the Hill (i.e. Fisk) function is given as

 (S3.1)

Here Emax and M are merely scale parameters and the shape is governed by 1 shape parameter, H. Most applications use more than 1 shape parameter. For generalizations and alternatives having more shape parameters see Wiki.

For reasons we discussed I want to have a similar zoo of response-dose relations of the form:

dE/dd=g(E); g(E=0) = 0 which we may think of as giving d(E).

A simple candidate set is given by

 (2)

This can be solved using partial fractions; get their coefficients ck by using the simplest version of the method of residues I talked about (see Wiki); integrate to get d as a sum of terms involving ln(1+ak), i.e.

 (3)

Program this in R and graph some examples. I attach an R program which I wrote and got streamlined by a student, Binglin Song. Start with N= 3 or 4 and exactly one ak, say aN, negative. Make your own version of this program using code you understand completely.

Working directly with Eq. (2) find the conditions on the ak that d2E/dd2(d=0) is positive (convexity near the origin), zero (linearity near the origin), or negative (concavity near the origin) (Hint: differentiate the equation and then set E=0; while you’re at it give a relation between d2E/dd2(d=0) and d2d/dE2(E=0) ). Check your conditions with your R program. Check them again using your explicit solution (3). Can you get E(d) to be sigmoidal. What is the behavior if instead of starting with E= 0 at d=0 you start with E larger than its maximum 1/aN?

