Simulation to assess varying coefficients and guideline

This runs a series of small simulations where we examine varying beta coefficients over time. In these simulations, we generate data and estimate the guidelines based on that data, and then compare these estimated recommendations to the oracle truth (using our general theorem) to see how our guideline works when it is technically misspecified.

In particular, this document produces Figure 4 and Table 2 in Appendix B.

Run simulation across range of sigma_pre

Our initial simulation has varying coefficients for both X and Z, along with θ. The misspecification gives a reduced recommendation to match as shown on the figure to the right.

Here is some sample code showing the default parameters:

```
# Number of simulation replicates per scenario
K = 2 # 25 #1000

cor_Xtheta = c( 0.3, 0.6 )

sigma_pre_tests = seq( 0.3, 1.5, by=0.10 )
names(sigma_pre_tests) = sigma_pre_tests

sim_res_main <- map_df( sigma_pre_tests,
                ~ run_scenario( sigma_pre = .,
                            beta_theta_0 = c( 0.5, 1.0 ),
                            beta_theta_1 = 1.5,
                            beta_x_0 = c( 0.6, 1.1),
                            beta_x_1 = 1.3,
                            beta_z_0 = c( 0.3, 0.7 ),
                            beta_z_1 = 1.0,
                            cor_XZ = 0.5,
                            cor_Xtheta = cor_Xtheta,
                            K = K ),
                .id = "sigma_pre" ) %>%
    mutate( sigma_pre = as.numeric(sigma_pre) )
```

Our table shows, for different residual variation, the proportion of the trials that say "match!", the average estimated reduction in bias, the standard deviation of the estimates across simulation (which is the true SE), and whether the oracle says to match and how much bias would be reduced. The last column is the R^2 for a regression of outcome onto the two observed covariates to get a sense of how much variation is explained by what we can match on.

sigma_pre	per_match	a_tau_xy	SE_tau_xy	match_XY	reduce_XY	R2
0.3	1	0.189	0.006	TRUE	0.239	0.833
0.4	1	0.148	0.008	TRUE	0.200	0.814
0.5	1	0.108	0.007	TRUE	0.157	0.790

| sigma_pre	per_match	a_tau_xy	SE_tau_xy	match_XY	reduce_XY	R2
0.6	1	0.069	0.004	TRUE	0.114	0.761
0.7	1	0.026	0.004	TRUE	0.072	0.735
0.8	0	-0.008	0.005	TRUE	0.033	0.703
0.9	0	-0.042	0.006	FALSE	-0.002	0.673
1.0	0	-0.071	0.007	FALSE	-0.033	0.644
1.1	0	-0.094	0.008	FALSE	-0.061	0.616
1.2	0	-0.116	0.009	FALSE	-0.086	0.586
1.3	0	-0.136	0.012	FALSE	-0.107	0.556
1.4	0	-0.153	0.013	FALSE	-0.126	0.533
1.5	0	-0.168	0.013	FALSE	-0.143	0.507

The plot is as discussed in the appendix:
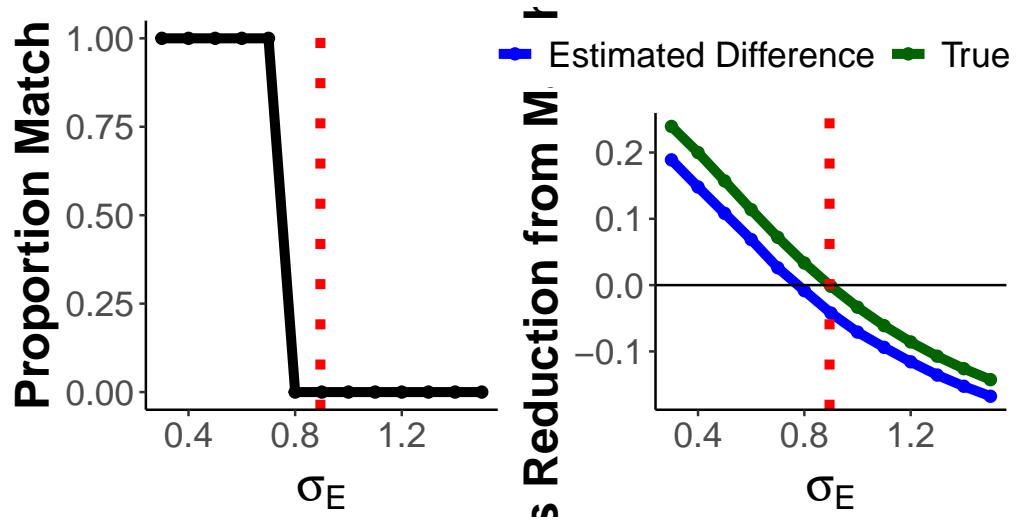
Independent covariates

This is our initial simulation, above, except all three covariates are now independent from each other. The independence does not help us?

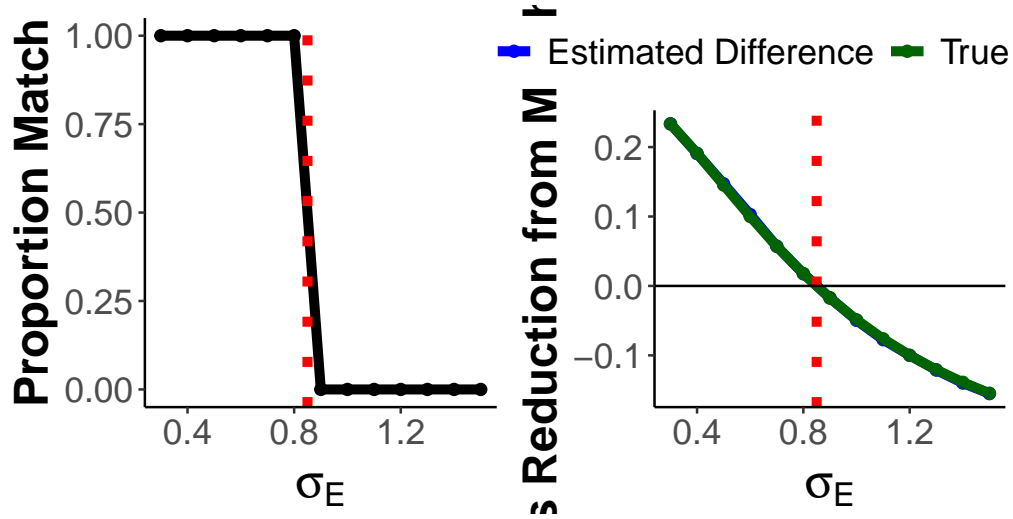| sigma_pre	per_match	a_tau_xy	SE_tau_xy	match_XY	reduce_XY	R2
0.3	1	0.524	0.014	TRUE	0.649	0.566
0.4	1	0.450	0.009	TRUE	0.580	0.544
0.5	1	0.376	0.010	TRUE	0.500	0.516
0.6	1	0.293	0.011	TRUE	0.415	0.487
0.7	1	0.211	0.010	TRUE	0.328	0.460
0.8	1	0.126	0.009	TRUE	0.242	0.431
0.9	1	0.049	0.009	TRUE	0.160	0.402
1.0	0	-0.022	0.011	TRUE	0.083	0.377
1.1	0	-0.086	0.012	TRUE	0.012	0.352
1.2	0	-0.151	0.010	FALSE	-0.053	0.331
1.3	0	-0.201	0.013	FALSE	-0.112	0.307
1.4	0	-0.255	0.017	FALSE	-0.166	0.288
1.5	0	-0.290	0.019	FALSE	-0.214	0.270
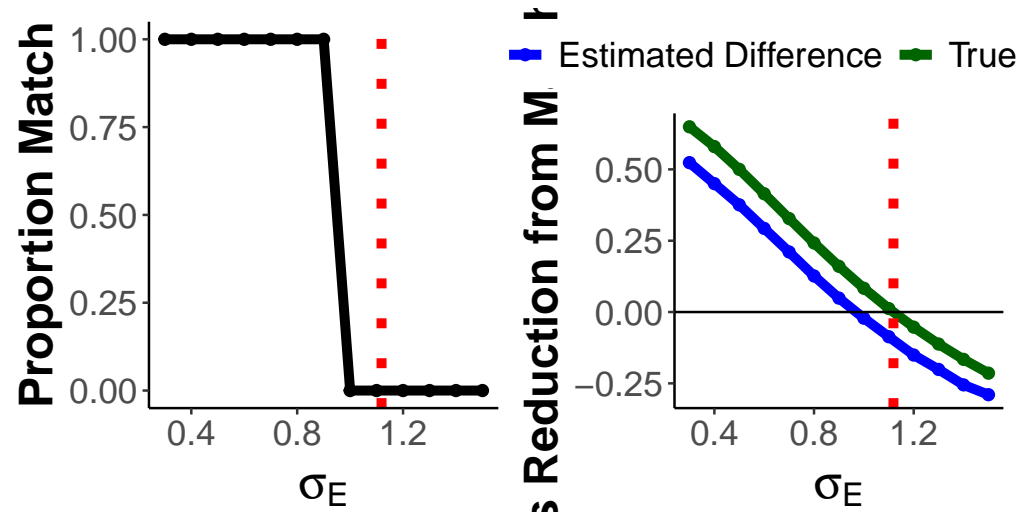
All is parallel

If we have parallel trends for all three covariates (two observed, one latent) then our guideline works as expected.

Also note that due to large sample size our match recommendation is very precisely estimated because our bias reduction is also very precisely estimated (see the SE column).
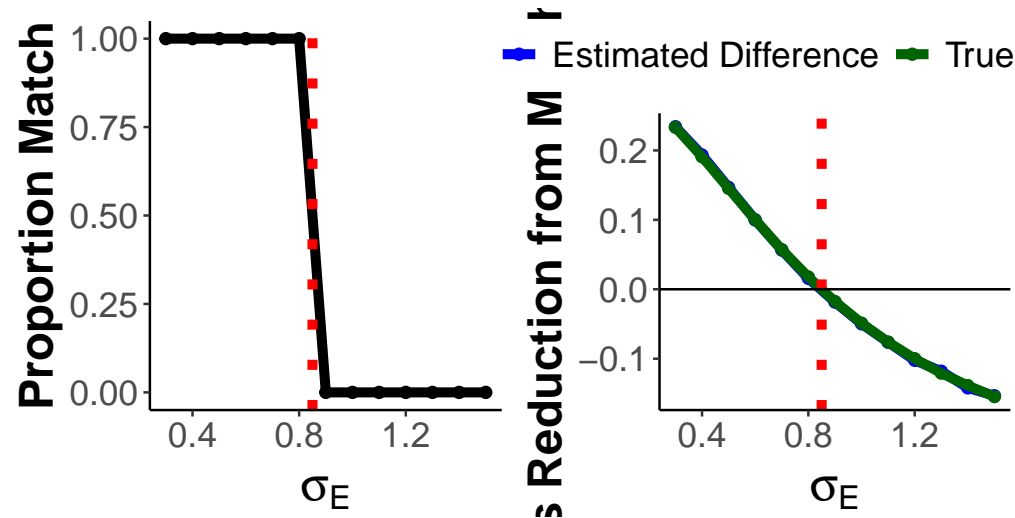
sigma_pre	per_match	a_tau_xy	SE_tau_xy	match_XY	reduce_XY	R2
0.3	1	0.234	0.010	TRUE	0.233	0.840
0.4	1	0.191	0.009	TRUE	0.191	0.825
0.5	1	0.147	0.008	TRUE	0.145	0.804
0.6	1	0.103	0.006	TRUE	0.100	0.783
0.7	1	0.057	0.004	TRUE	0.057	0.759
0.8	1	0.018	0.006	TRUE	0.018	0.733
0.9	0	-0.017	0.004	FALSE	-0.018	0.706
1.0	0	-0.050	0.007	FALSE	-0.049	0.678
1.1	0	-0.077	0.007	FALSE	-0.076	0.650
1.2	0	-0.100	0.007	FALSE	-0.100	0.622
1.3	0	-0.121	0.009	FALSE	-0.121	0.594
1.4	0	-0.140	0.012	FALSE	-0.139	0.568
1.5	0	-0.155	0.010	FALSE	-0.155	0.542

theta parallel, covariates not

Here we have X and Z correlated, but theta is parallel.

sigma__pre	per__match	a__tau__xy	SE__tau__xy	match__XY	reduce__XY	R2
0.3	1	0.234	0.010	TRUE	0.233	0.825
0.4	1	0.193	0.009	TRUE	0.191	0.808
0.5	1	0.147	0.006	TRUE	0.145	0.788
0.6	1	0.100	0.006	TRUE	0.100	0.766
0.7	1	0.057	0.004	TRUE	0.057	0.741
0.8	1	0.016	0.004	TRUE	0.018	0.714
0.9	0	-0.018	0.005	FALSE	-0.018	0.687
1.0	0	-0.050	0.006	FALSE	-0.049	0.657
1.1	0	-0.076	0.008	FALSE	-0.076	0.628
1.2	0	-0.102	0.010	FALSE	-0.100	0.602
1.3	0	-0.118	0.009	FALSE	-0.121	0.573
1.4	0	-0.142	0.010	FALSE	-0.139	0.546
1.5	0	-0.153	0.012	FALSE	-0.155	0.521

small sample size

From our original simulation, if we reduce sample size, estimation error should flatten our curve. It seems like estimation error is very small, which is surprising given all the residualization?

sigma_pre	per_match	a_tau_xy	SE_tau_xy	match_XY	reduce_XY	R2
0.3	1.0	0.191	0.018	TRUE	0.239	0.831
0.4	1.0	0.158	0.017	TRUE	0.200	0.812
0.5	1.0	0.115	0.019	TRUE	0.157	0.790
0.6	1.0	0.069	0.013	TRUE	0.114	0.758
0.7	1.0	0.023	0.010	TRUE	0.072	0.737
0.8	0.4	-0.002	0.013	TRUE	0.033	0.703
0.9	0.0	-0.039	0.015	FALSE	-0.002	0.673
1.0	0.0	-0.076	0.021	FALSE	-0.033	0.645
1.1	0.0	-0.098	0.018	FALSE	-0.061	0.609
1.2	0.0	-0.122	0.019	FALSE	-0.086	0.586
1.3	0.0	-0.146	0.034	FALSE	-0.107	0.555
1.4	0.0	-0.178	0.033	FALSE	-0.126	0.526
1.5	0.0	-0.182	0.037	FALSE	-0.143	0.508
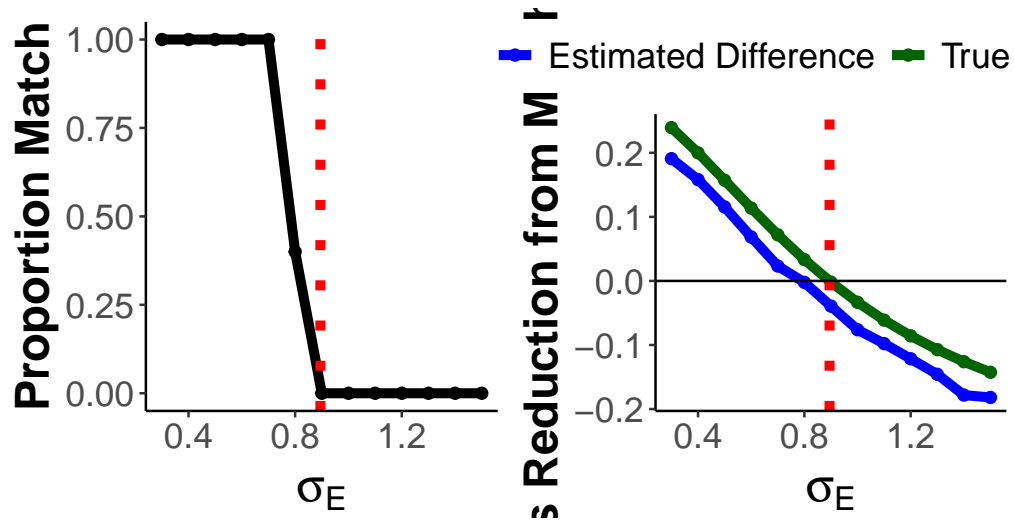
The uncertainty in estimation of the reduction does go up quite a bit:

```
sim_resSS$SE_tau_xy / sim_res_main$SE_tau_xy
```

```
## [1] 2.841846 2.054552 2.740173 3.050978 2.153393 2.660540 2.465954 3.189196
## [9] 2.253256 2.077944 2.787198 2.526709 2.868038
```

More time periods with less predictive covariates

If we make covariates less predictive, but have more pre-treatment time periods? (We have also increased variation in theta to cause more trouble.)
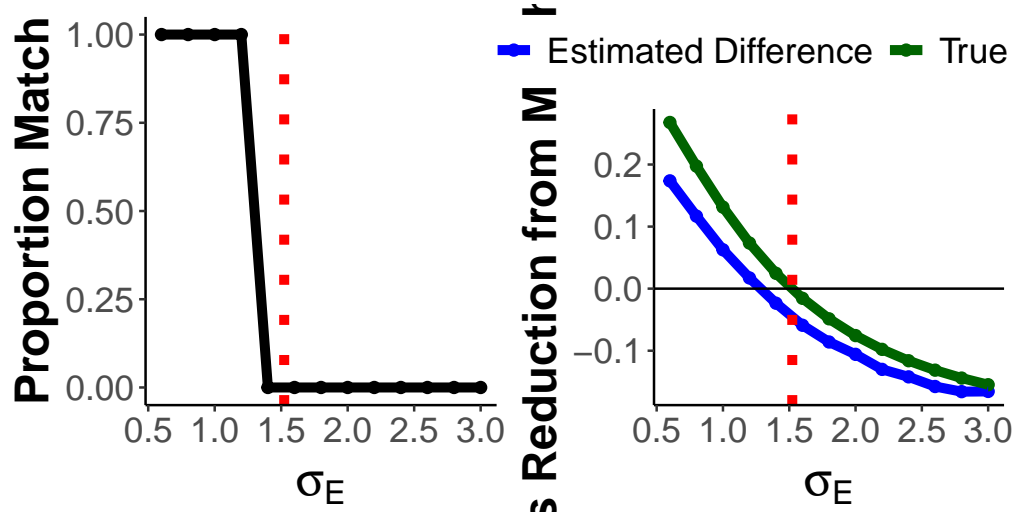
sigma_pre	per_match	a_tau_xy	SE_tau_xy	match_XY	reduce_XY	R2
0.6	1.00	0.171	0.008	TRUE	0.272	0.758
0.8	1.00	0.110	0.006	TRUE	0.202	0.697
1.0	1.00	0.054	0.004	TRUE	0.137	0.632
1.2	0.96	0.006	0.005	TRUE	0.079	0.569
1.4	0.00	-0.035	0.007	TRUE	0.030	0.511
1.6	0.00	-0.065	0.010	FALSE	-0.010	0.458
1.8	0.00	-0.092	0.011	FALSE	-0.044	0.411
2.0	0.00	-0.114	0.015	FALSE	-0.071	0.370
2.2	0.00	-0.132	0.018	FALSE	-0.094	0.332
2.4	0.00	-0.150	0.020	FALSE	-0.112	0.299
2.6	0.00	-0.161	0.018	FALSE	-0.128	0.269
2.8	0.00	-0.166	0.022	FALSE	-0.141	0.244
3.0	0.00	-0.180	0.032	FALSE	-0.152	0.223
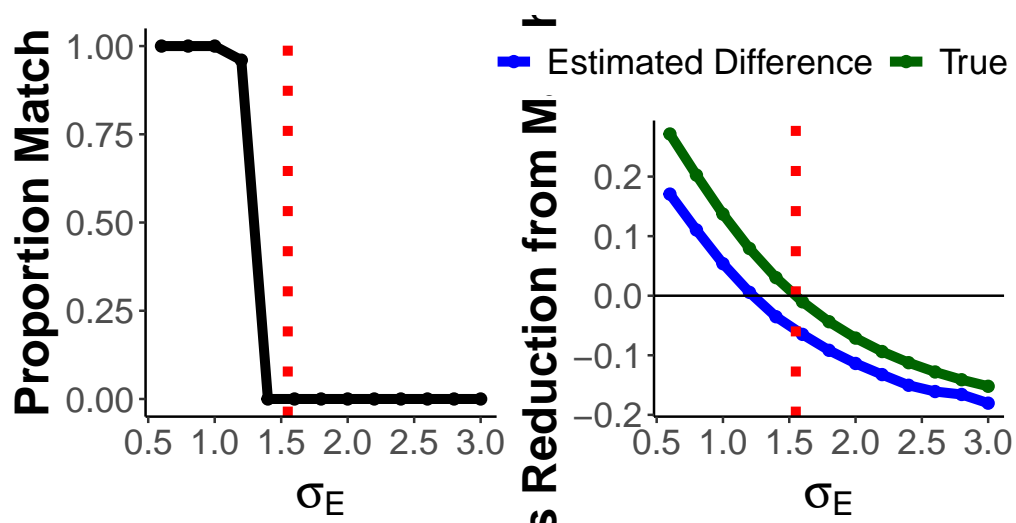
narrow theta assumption only

If all we get is parallel theta in the final two periods, but theta is not parallel before that?

sigma__pre	per__match	a__tau__xy	SE__tau__xy	match__XY	reduce__XY	R2
0.6	1	0.174	0.010	TRUE	0.268	0.758
0.8	1	0.117	0.006	TRUE	0.197	0.697
1.0	1	0.063	0.004	TRUE	0.131	0.631
1.2	1	0.017	0.005	TRUE	0.073	0.568
1.4	0	-0.023	0.005	TRUE	0.025	0.510
1.6	0	-0.059	0.008	FALSE	-0.016	0.457
1.8	0	-0.086	0.008	FALSE	-0.049	0.411
2.0	0	-0.106	0.017	FALSE	-0.076	0.368
2.2	0	-0.130	0.016	FALSE	-0.098	0.329
2.4	0	-0.142	0.015	FALSE	-0.116	0.298
2.6	0	-0.158	0.023	FALSE	-0.131	0.269
2.8	0	-0.166	0.023	FALSE	-0.144	0.242
3.0	0	-0.166	0.024	FALSE	-0.155	0.223

## All covariates independent, four time points

Here we have our theta stable in the final two time periods, and no correlation between any of our three.
covariates.

| sigma__pre	per__match	a__tau__xy	SE__tau__xy	match__XY	reduce__XY	R2
0.6	1.00	0.516	0.015	TRUE	0.771	0.498
0.8	1.00	0.408	0.011	TRUE	0.634	0.438
1.0	1.00	0.295	0.012	TRUE	0.494	0.378
1.2	1.00	0.185	0.012	TRUE	0.360	0.328
1.4	1.00	0.086	0.012	TRUE	0.239	0.283
1.6	0.56	0.002	0.012	TRUE	0.132	0.247
1.8	0.00	-0.072	0.014	TRUE	0.040	0.216
2.0	0.00	-0.146	0.017	FALSE	-0.039	0.189
2.2	0.00	-0.198	0.014	FALSE	-0.106	0.166
2.4	0.00	-0.245	0.033	FALSE	-0.164	0.147
2.6	0.00	-0.294	0.029	FALSE	-0.213	0.130
2.8	0.00	-0.309	0.030	FALSE	-0.254	0.117
3.0	0.00	-0.345	0.032	FALSE	-0.290	0.105
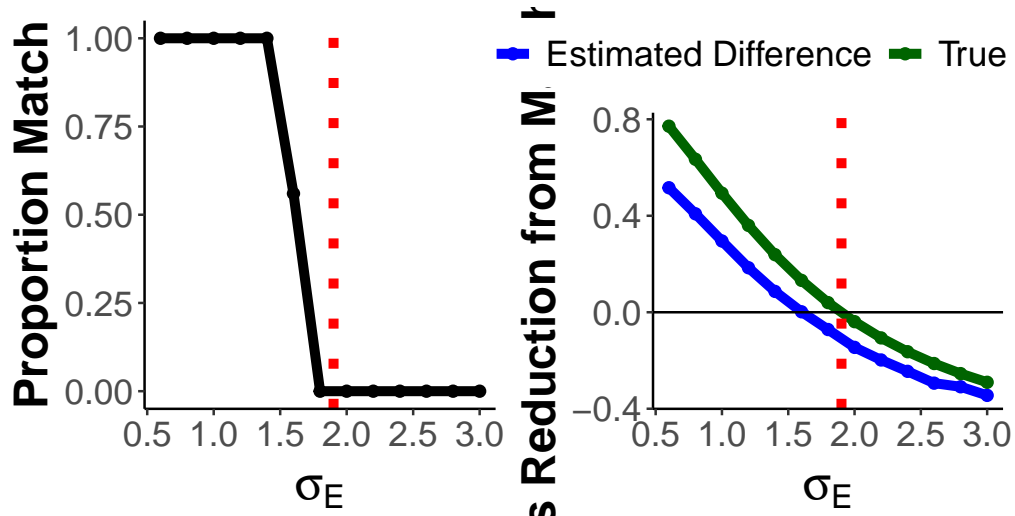
All biases across scenarios

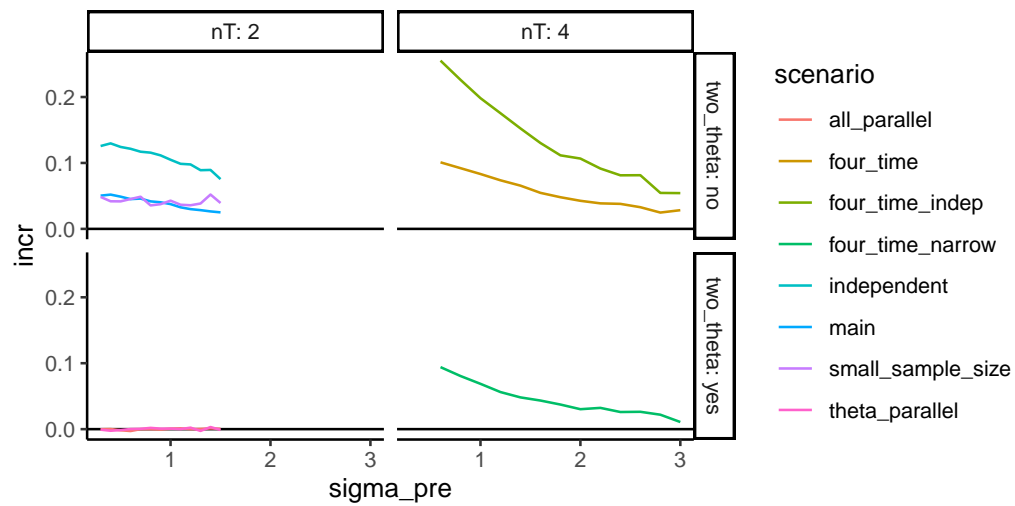These plots aggregate the above to compare trends and sizes of biases, etc.
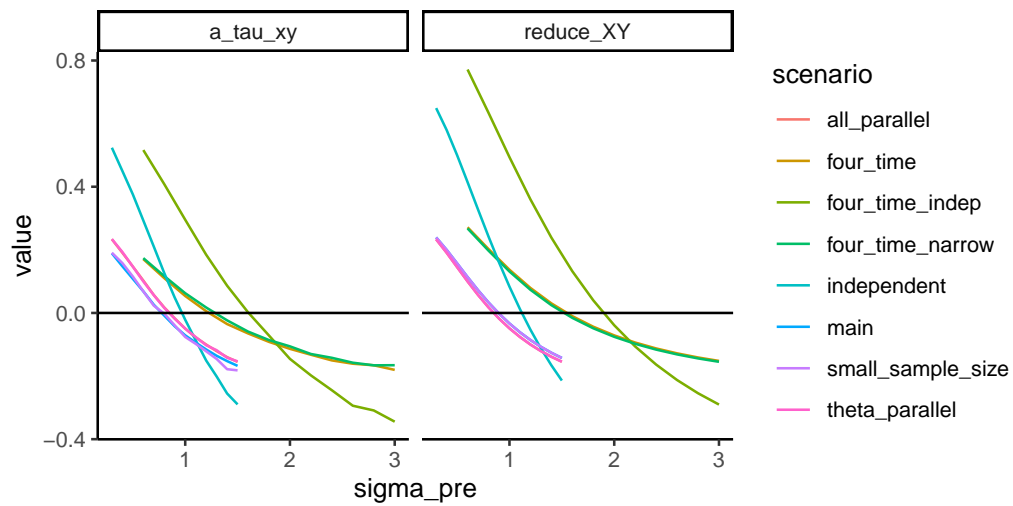
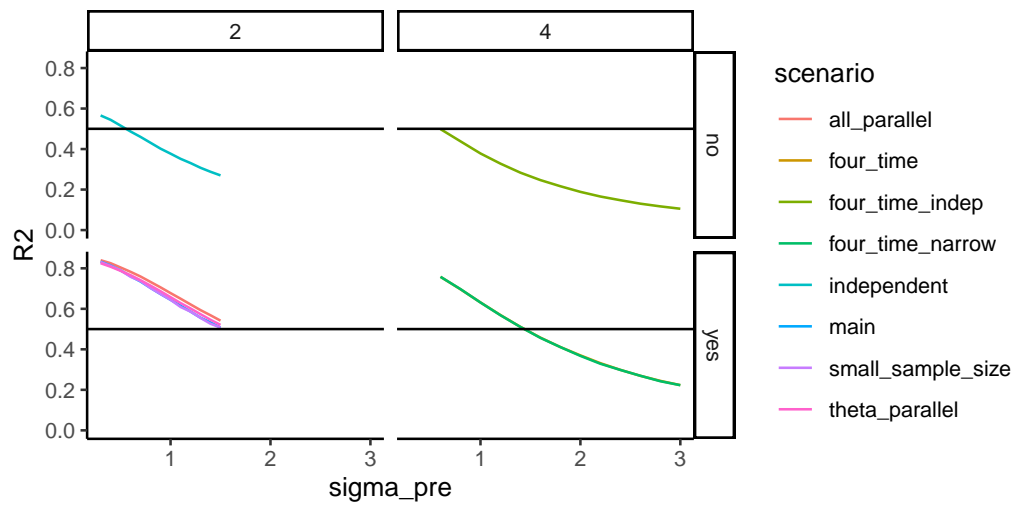Comparing the changeover of the guideline to match vs. oracle guideline:
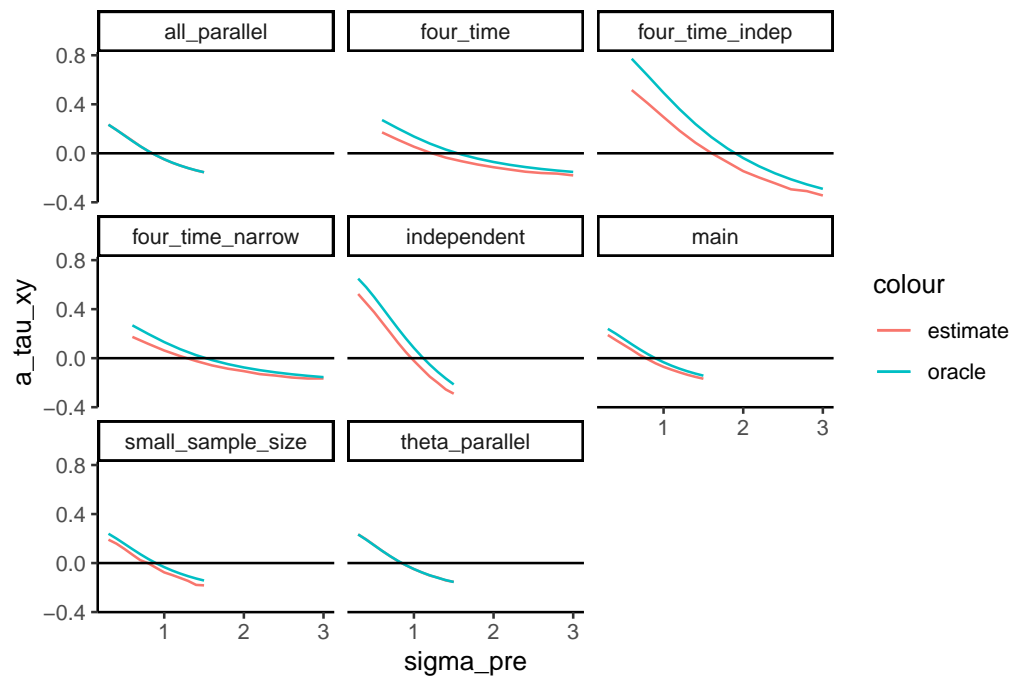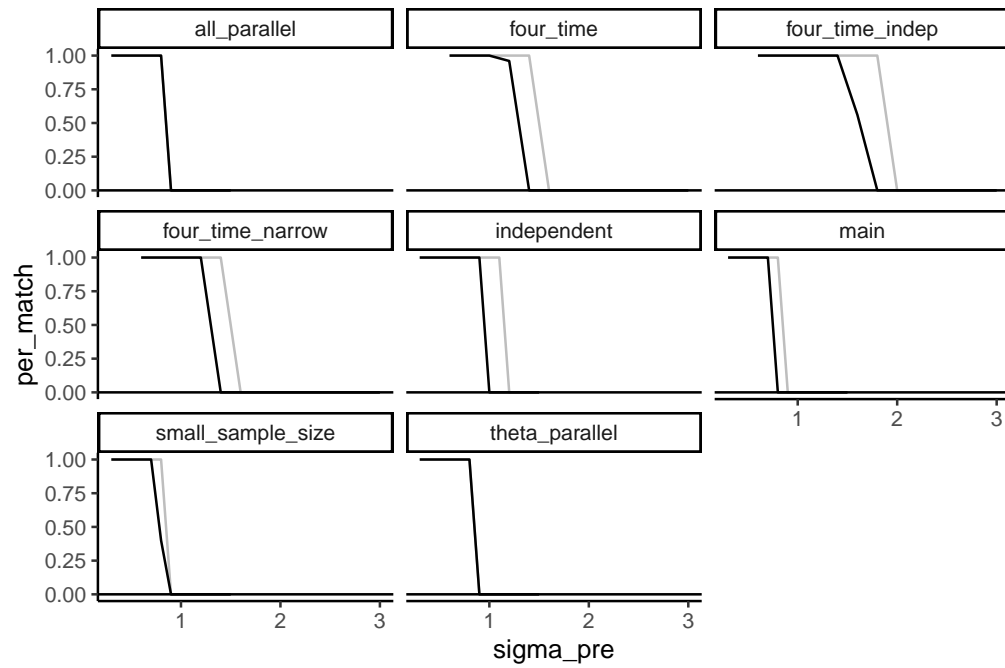
Looking at shift of estimated bias to actual (grouping by whether the final two time periods have parallel theta or not, and number of time periods):

Looking at the R^2 mesures of the scenarios

Looking at the estimated sigma pre values