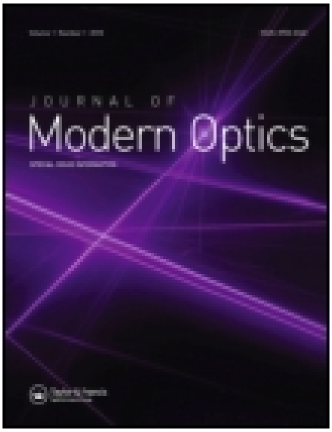

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Computer generated diffractive multi-focal lens

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Abstract. The method has been proposed for computing Fresnel-type multi-focal lenses on the basis of special-type phase nonlinearity. A multi-focal lens is represented as a mathematical superposition of a thin lens and nonlinearity distorted Fresnel lens. Selection of the nonlinearity type is reduced to the problem of the groove form determination for the phase diffraction grating with pre-set energy distribution between orders. In particular, bi-focal lens and seven-focal lens have been investigated.

1. Introduction

Multi-focal optical elements are important in such applications as intraocular lenses with accommodation abilities, laser optical disc heads, 3-D image formation system. In order to solve such a problem one can use segmentized diffractive optical elements [1, 2] whose aperture consists of a proper set of diffractive lens-type segments with different focal lengths. Since the configurations of aperture segments differ from each other, then image resolutions, forms of the exit pupil and the fields of view are different in the foci of segmentized lens. Another method utilizes the well-known multi-focal properties of binary Fresnel zone plates [3]. However, those zone plates do not allow one to control the energy distribution between foci and to form the pre-set number of foci. The main contribution of this paper is the diffraction method for the synthesis of highly effective multi-focal lenses with the required number of foci and pre-set power distribution between them.

2. Multi-focal lens elaboration

Computer generation of phase diffractive lenses gives the opportunity to realize arbitrary phase retardation of light beam. It is well known [3] that zoned phase microrelief with limited height performs a phase retardation equivalent to wide-range-varying phase function φ of thin optical element. In this case microrelief height is proportional to the phase function φ reduced to $[0, 2\pi]$ interval and is really generated by those functions.

This section contains calculations of phase function φ describing a multi-focal diffractive lens. The lens must provide the focusing of illuminating beam (wavelength λ) into N points distributed on the optical axis with predetermined power ratios

$$I_1, \ldots, I_N \left(\sum_{i=1}^{N} I_i = 1 \right)$$

(see figure 1).

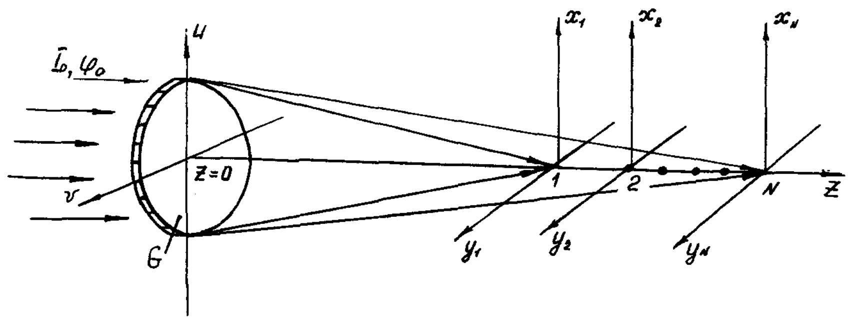

Figure 1. The geometry of focusing.

It is well known that any phase nonlinearity yields additional diffraction orders. Those nonlinear predistortions give several foci if applied to the phase of Fresnel lenses. For example, the binary phase Frensel zone plate, being a result of the hard-clipping-type nonlinearity application to the phase lens, gives an infinite set of foci. However in such a case the illuminating beam power is spent uselessly in undesired foci. It must be emphasized that the nonlinearity type really influences the energy distribution between foci. The authors propose the method for choosing a special-type nonlinearity ϕ, ensuring the required energy distribution between a pre-set number of foci.

Let us represent a multi-focal lens as the zoned Fresnel-type lens that has the following phase function:

$$\varphi(\mathbf{u}) = \phi[\varphi_1(\mathbf{u})] + \varphi_2(\mathbf{u}) - \varphi_0(\mathbf{u}), \qquad \mathbf{u} \in G. \tag{1}$$

Here $\varphi_0(\mathbf{u})$ is the phase of a beam illuminating the lens with aperture G, $\mathbf{u} = (u, v)$ represents orthogonal Cartesian coordinates in the lens plane, and functions

$$\varphi_1(\mathbf{u}) = \mathrm{mod}_{2\pi}\left(-\frac{k\mathbf{u}^2}{2f_1}\right), \qquad \varphi_2(\mathbf{u}) = -\frac{k\mathbf{u}^2}{2f_2}, \qquad k = \frac{2\pi}{\lambda} \tag{2}$$

are the paraxial phases of the lenses with some focal lengths f_1 and f_2. The term $\varphi_0(\mathbf{u})$ compensates the phase of the illuminating beam. The lens phase $\varphi_2(\mathbf{u})$ insertion into equation (1) yields the additional capabilities in choosing the foci coordinates along the optical axis. The function $\mathrm{mod}_{2\pi}(\cdot)$ reduces the phase to the $[0, 2\pi)$ interval. Function $\phi[\varphi_1] \in [0, 2\pi)$ describes a nonlinear predistortion of the phase $\varphi_1(\mathbf{u})$ (see figure 2). For example this way, any Damman's grating with N_0 slits per period can generate a binary-phase zone plate. Such a zone plate has N_0 rings instead of each ring of a conventional binary phase zone plate. Thus, the multi-focal lens is expressed in the form of mathematical superposition of the conventional lens $\varphi_2(\mathbf{u})$ and complex zone plate $\phi[\varphi_1(\mathbf{u})]$. The zone plate may be the binary one or the multi-level staircase one or even with continuous nonlinearily distorted micro-relief profiles in the zones. We shall call the lens with phase (1) the 'distort-phase' lens.

Let us consider the multi-focal lens's (1) operation for the case of illuminating beam $[I_0(\mathbf{u})]^{1/2} \exp(\mathrm{i}\varphi_0(\mathbf{u}))$ with intensity $I_0(\mathbf{u})$. The thin multi-focal lens's phase is optically added to the illuminating beam's phase $\varphi_0(\mathbf{u})$ (under unchanged amplitude). Thus we can write down the field immediately beyond the lens plane in the form:

$$\boxed{w(\mathbf{u}) = [I_0(\mathbf{u})]^{1/2} \exp\{\mathrm{i}\phi[\varphi_1(\mathbf{u})] + \mathrm{i}\varphi_2(\mathbf{u})\}.} \tag{3}$$

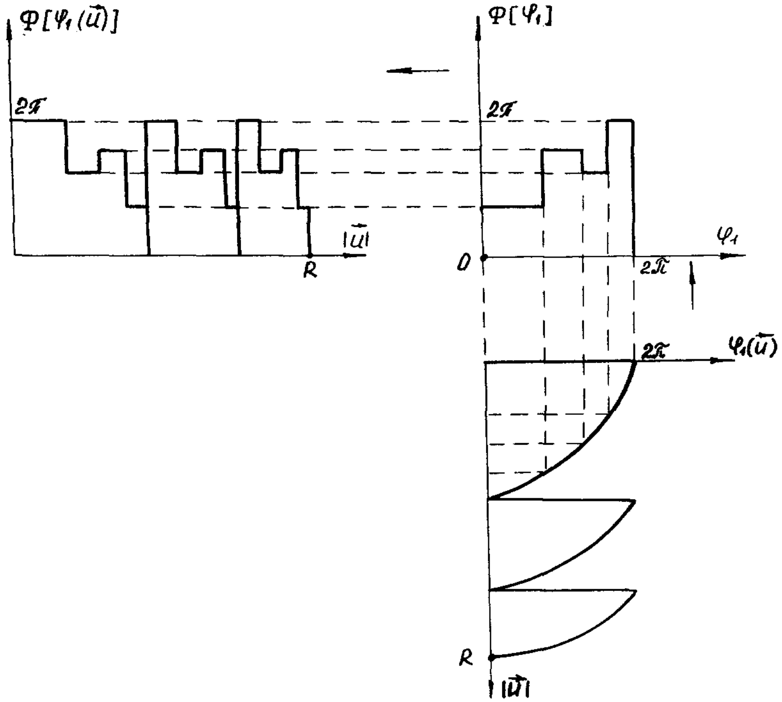

Figure 2. The nonlinear point-to-point phase transformation.

In order to describe series of foci let us expand the function exp $(i\phi(\xi)]$ into a Fourier series on the $[0, 2\pi)$ interval [4, 5]

$$\exp[i\phi(\xi)] = \sum_{l=-\infty}^{+\infty} a_l \exp(il\xi). \tag{4}$$

where

$$a_l = \frac{1}{2\pi}\int_0^{2\pi} \exp[i\phi(\xi) - il\xi]\, d\xi \tag{5}$$

are the Fourier-coefficients and

$$\sum_{l=-\infty}^{+\infty} |a_l|^2 = 1.$$

Substituting $\xi = \varphi_1(\mathbf{u})$ into equation (4) and using phase 2π periodicity we shall write down equation (3) in the form:

$$w(\mathbf{u}) = [I_0(\mathbf{u})]^{1/2} \exp\left(-\frac{ik\mathbf{u}^2}{2f_2}\right) \sum_{l=-\infty}^{+\infty} a_l \exp\left[il\left(-\frac{k\mathbf{u}^2}{2f_1}\right)\right]. \tag{6}$$

We can interpret the expression (6) as a superposition of a set of paraxial spherical beams with focal lengths

$$\boxed{F_l = \frac{f_2}{1 + \dfrac{lf_2}{f_1}}}, \tag{7}$$

and power distribution $|a_l|^2$ between them.

Let us suppose $l_1, \ldots, l_N$ to be indexes corresponding to the required focal lengths in multi-focal lens design. So it is sufficient to determine a specific function $\exp[i\phi(\xi)]$ on the period $\xi \in [0, 2\pi]$ whose Fourier coefficients (5) obey the equations:

$$|a_{l_i}|^2 = I_i, \qquad i = 1, N. \tag{8}$$

It has been proposed by the authors to interpret the predistortion $\phi(\xi)$ as the phase function of the Nth-order diffraction grating that has the period 2π and the intensities $I_1, \ldots, I_N$ in the orders $l_1, \ldots, l_N$. This interpretation reduces the problem of the multi-focal lens computation to the well known problem of the phase diffraction grating synthesis with pre-set power distribution between diffraction orders [6, 7, 8]. It should be mentioned that the diffraction grating is the only virtual one here. It is not superimposed on a lens, but serves only as a mathematical point-to-point nonlinearity.

3. Computation and investigation of bi-focal lens

As an important example we shall consider bi-focal lens synthesis with focal lengths of f_a, f_b and with equal powers in each focus. Aperture G is circular with radius R. In this case we shall put $F_1 = f_a$, $F_{-1} = f_b$ in the general expression (7) for the focal lengths of 'distort phase' multi-focal lens and get

$$f_1 = \frac{2f_a f_b}{f_b - f_a}, \qquad f_2 = \frac{2f_a f_b}{f_a + f_b}. \tag{9}$$

Furthermore we shall define the nonlinear predistortion function $\phi(\xi)$ as the phase function of two-order diffraction grating:

$$\phi(\xi) = \begin{cases} 0, & \xi \in [0, \pi), \\ \pi, & \xi \in [\pi, 2\pi). \end{cases} \tag{10}$$

The Fourier coefficients in expansion (4) of function $\exp[i\phi(\xi)]$ are the following:

$$a_l = \begin{cases} 0, & l = 0 \\ \dfrac{1 - (-1)^l}{\pi i l}, & l \neq 0, \end{cases} \tag{11}$$

and $|a_1|^2 = |a_{-1}|^2 = 0{\cdot}4053$. Thus more than 81% of illuminating beam power concentrates in the orders with numbers $l = 1$ and $l = -1$. The substitution of equations (9), (11) into equation (6) with $l = \pm 1$ non-zero terms yields the following expression for output field under the plane illuminating beam:

$$w(\mathbf{u}) = a_1 \exp\left(-\frac{ik\mathbf{u}^2}{2f_a}\right) + a_{-1} \exp\left(-\frac{ik\mathbf{u}^2}{2f_b}\right). \tag{12}$$

Equation (12) describes the required process of focusing into the two points (in a paraxial approximation). We have made computer simulations for the field intensity formed by 'distort-phase' bi-focal lens with the phase function of equations (1), (2), (9), (10) and microrelief height $\sim \mathrm{mod}_{2\pi}(\varphi)$. The segmentized lens with phase function

$$\varphi_S(\mathbf{u}) = \begin{cases} -\dfrac{k\mathbf{u}^2}{2f_a}, & |\mathbf{u}| \leqslant \dfrac{R}{\sqrt{2}}, \\ -\dfrac{k\mathbf{u}^2}{2f_b}, & \dfrac{R}{\sqrt{2}} < |\mathbf{u}| \leqslant R, \end{cases} \tag{13}$$

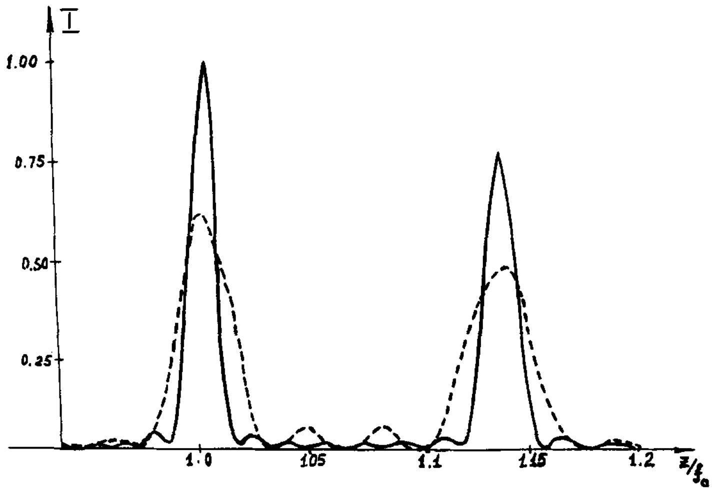

Figure 3. The normalized intensity distribution along an optical axis for the distort-phase and segmentized bi-focal lenses under $f_a = 30$ mm, $f_b = 34$ mm ($f_1 = 510$ mm, $f_2 = 31{\cdot}875$ mm), $\lambda = 0{\cdot}555$ µm, $R = 1{\cdot}5$ mm.

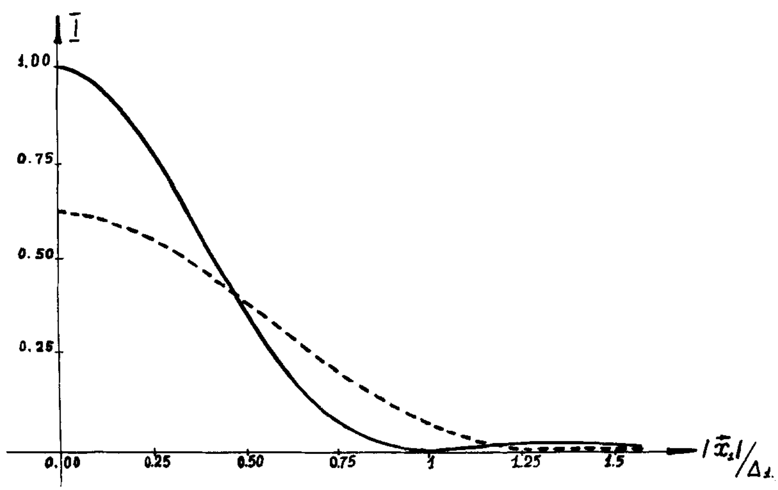

Figure 4. The normalized intensity distribution in the first focal plane ($Z = 30$ mm) for the distort-phase and segmentized bi-focal lenses (where $\Delta_1 = 0{\cdot}61\lambda f_a/R$ is the diffraction-limited spot-size for a lens).

was also under consideration for the sake of comparison. Figure 3 illustrates the plots of normalized intensity I distribution along the optical axis Z for the 'distort-phase' lens (solid curve) and segmentized lens (13) (dashed curve). The plots have been obtained by the paraxial Kirchhoff integral numerical computations. Figure 3 illustrates that the depth of focus for the 'distort-phase' lens is less than for a segmentized lens since the entire aperture operates in each focus of 'distort-phase' lens (1). Figures 4 and 5 display the computer-simulated point-spread function on two focal planes. The spot-size corresponds to 0·1 intensity fall for lens (1). Energy

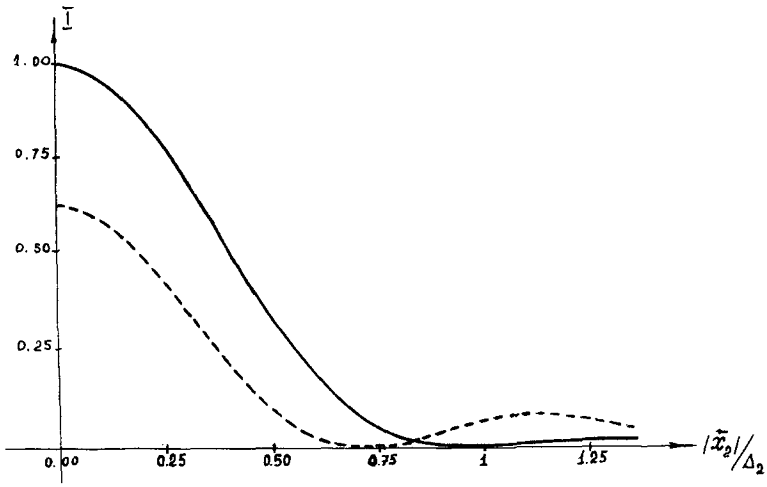

Figure 5. The normalized intensity distribution in the second focal plane ($Z = 34$ mm for the distort-phase and segmentized bi-focal lenses (where $\Delta_2 = 0{\cdot}61\ \lambda f_b/R$ is the diffraction-limited spot-size for a lens).

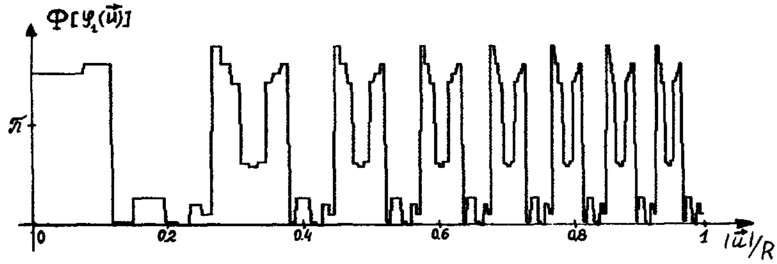

Figure 6. The staircase profile of the seven-focal zone plate.

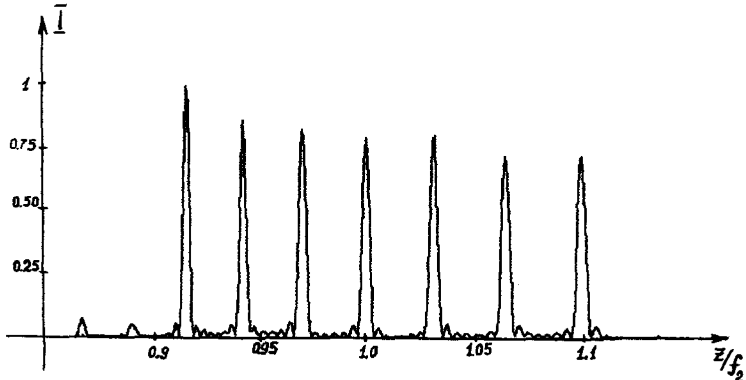

Figure 7. The intensity distribution along an optical axis for the seven-focal distort-phase lens under $f_1 = 1000$ mm, $f_2 = 34$ mm, $\lambda = 0{\cdot}106$ μm, $R = 4$ mm.

efficiency ε is introduced as the power inside the spot-size, normalized by incident beam power. On the first focal plane ($Z=30$ mm) the value of ε equals 32·8% and 32·6% for 'distort-phase' and segmentized lens respectively. On the second focal plane ($Z=34$ mm) ε is equal to 34·2% and 14·5% respectively. Only the central segment of the segmentized lens contributes to the first focal plane with a spot-size that is 1·5–2 times greater than the spot-size of the proposed 'distort-phase' lens (see figure 4). Figure 5 displays a similar effect in the second focal plane where a double-maxima point-spread function occurs for segmentized lens due to its ring-like pupil [9]. So we can conclude that the proposed 'distort-phase' bifocal lens operates similarly to a conventional lens in each of two foci. Furthermore it has two times greater the energy efficiency in the second focal plane as compared with a segmentized lens.

A bi-focal lens is not the top of the method. We have not met any fundamental difficulties of multi-focal lens calculation. This way the seven-focal lens was calculated as the superposition of a complicated staircase zoned plate $\phi[\varphi_1(\mathbf{u})]$ (see figure 6) and the usual lens phase according to equation (1). All the foci have approximately equal powers (see figure 7). Not less than 85% of illuminating beam power is directed into the desired foci.

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