CÁLCULO DIFERENCIAL E INTEGRAL I

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$$\int f(x)dx = ?$$

REGRAS DE INTEGRAÇÃO

PARTE 1

$$\int f(u)du = ?$$

ESTUDO DAS INTEGRAIS

Sejam $f, g : I \to \mathbb{R}$ e k uma constante, então:

1. $\int dx = x + C$ $\qquad \frac{d}{dx}[x + C] = 1$

2. $\int \frac{d}{dx} f(x)dx = f(x) + C$ $\qquad \frac{d}{dx}[f(x) + C] = \frac{d}{dx} f(x)$

3. $\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$

4. $\int kf(x)dx = k \int f(x)dx,$ $\qquad k \in \mathbb{R}$

5. $\int x^n dx = \frac{x^{n+1}}{n+1} + C \, , n \neq -1$ $\qquad \frac{d}{dx}\left[\frac{x^{n+1}}{n+1} + C\right] = \frac{(n+1)x^{n+1-1}}{(n+1)} = x^n$

EXEMPLOS

1. $\int 3dx = \quad 3\int dx = 3x + C$

2. $\int x^2 dx = \dfrac{x^{2+1}}{2+1} + \mathrm{C} = \dfrac{x^3}{3} + C$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

3. $\int x^{-1/2} dx = \dfrac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} = \dfrac{x^{1/2}}{1/2} + C = 2\sqrt{x} + C$

$$\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$$

4. $\int \dfrac{x^2+1}{x^2} dx = \int \left(\dfrac{x^2}{x^2} + \dfrac{1}{x^2}\right) dx = \int \left(1 + \dfrac{1}{x^2}\right) dx = \int (1 + x^{-2}) dx = \int dx + \int x^{-2} dx =$

$$= x + \frac{x^{-2+1}}{-2+1} + C = \quad x - \frac{1}{x} + \mathrm{C}$$

EXEMPLOS

5. $\int \left(2x^5 + 8x^3 - 3x^2 + 5 + \sqrt[3]{x^2}\right) dx =$

$$= 2\int x^5 dx + 8\int x^3 dx - 3\int x^2 dx + 5\int dx + \int x^{2/3} dx =$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$= 2\left(\frac{x^6}{6}\right) + 8\left(\frac{x^4}{4}\right) - 3\left(\frac{x^3}{3}\right) + 5x + \frac{x^{\frac{2}{3}+1}}{\frac{2}{3}+1} + C$$

$$= \left(\frac{x^6}{3}\right) + 2x^4 - x^3 + 5x + \frac{3}{5}\sqrt[3]{x^5} + C$$

EXEMPLOS

6. $$\int \left(\frac{x^5 + 2x^{-1/2} + 3}{x^2}\right) dx = \int \frac{x^5}{x^2} dx + 2\int \frac{x^{-1/2}}{x^2} dx + 3\int \frac{dx}{x^2}$$

$$\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$$

$$= \int x^{5-2} dx + 2\int x^{-\frac{1}{2}-2} dx + 3\int x^{-2} dx$$

$$= \int x^3 dx + 2\int x^{-5/2} dx + 3\int x^{-2} dx$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$= \frac{x^4}{4} + 2\frac{x^{-3/2}}{-\frac{3}{2}} + 3\frac{x^{-1}}{-1} + C$$

$$= \frac{x^4}{4} - \frac{4}{3\sqrt{x^3}} - \frac{3}{x} + C$$

EXEMPLOS

7. $\int (3x^6 - 2x^2 + 7x + 1)dx = \quad 3\int x^6 dx - 2\int x^2 dx + 7\int x\,dx + \int dx =$

$$= 3\left(\frac{x^{6+1}}{6+1}\right) - 2\left(\frac{x^{2+1}}{2+1}\right) + 7\left(\frac{x^{1+1}}{1+1}\right) + x + C$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$= 3\left(\frac{x^7}{7}\right) - 2\left(\frac{x^3}{3}\right) + 7\left(\frac{x^2}{2}\right) + x + C$$

8. $\int \left(\frac{t^2 - 2t^4}{t^4}\right) dt = \int \left(\frac{t^2}{t^4} - 2\frac{t^4}{t^4}\right) dt = \int \left(\frac{1}{t^2} - 2\right) dt = \int t^{-2} dt - 2\int dt =$

$$= \frac{t^{-2+1}}{-2+1} - 2t + C = -\frac{1}{t} - 2t + C$$

$$8.\ \int \left(\frac{x^{2/3}+2}{x^{4/3}}\right)dx = \int \frac{x^{2/3}}{x^{4/3}}dx + \int \frac{2}{x^{4/3}}dx = \int x^{\frac{2}{3}-\frac{4}{3}}dx + 2\int x^{-\frac{4}{3}}\,dx = \int x^{-\frac{2}{3}}dx + 2\int x^{-4/3}dx$$

$$= \frac{x^{-\frac{2}{3}+1}}{-\frac{2}{3}+1} + 2\,\frac{x^{-4/3+1}}{-\frac{4}{3}+1} + C = \frac{x^{1/3}}{\frac{1}{3}} + 2\,\frac{x^{-1/3}}{-\frac{1}{3}} + C = 3x^{1/3} - \frac{6}{x^{1/3}} + C = 3\sqrt[3]{x} - \frac{6}{\sqrt[3]{x}} + C$$

$$9.\ \int (3x^{-3} + 4\sqrt[4]{x} + 7x^2 + 10)dx = 3\int x^{-3}dx + 4\int x^{1/4}dx + 7\int x^2\,dx + 10\int dx =$$

$$= 3\left(\frac{x^{-3+1}}{-3+1}\right) + 4\left(\frac{x^{1/4+1}}{1/4+1}\right) + 7\left(\frac{x^{2+1}}{2+1}\right) + 10x + C = 3\left(\frac{x^{-2}}{-2}\right) + 4\left(\frac{x^{5/4}}{5/4}\right) + 7\left(\frac{x^3}{3}\right) + 10x + C$$

$$= -\frac{3}{2}\left(\frac{1}{x^2}\right) + \frac{16}{5}x\sqrt[4]{x} + \frac{7}{3}x^3 + 10x + C$$

10. $\int \left(\frac{1}{\sqrt{x}} - 3\sqrt[5]{x^7} + \frac{1}{9}\right) dx = \int x^{-1/2} dx - 3 \int x^{7/5} dx + \frac{1}{9} \int dx$

$$= \left(\frac{x^{1/2}}{1/2}\right) - 3\left(\frac{x^{12/5}}{12/5}\right) + \frac{1}{9}(x) + C \; = 2\sqrt{x} - \frac{15}{12} x^{2}\sqrt[5]{x^3} + \frac{1}{9}x + C$$

11. $\int (1-x)\sqrt{x})dx = \int \left(x^{1/2} - x^{3/2}\right) dx = \int x^{1/2}\, dx - \int x^{3/2} dx = \frac{2}{3} x^{3/2} - \frac{2}{5} x^{\frac{5}{2}} + C$

12. $\int (3s+4)^2 ds = \int (9s^2 + 24s + 16) ds = 9 \int s^2\, ds + 24 \int sds + 16 \int\, ds = \frac{9s^3}{3} + \frac{24s^2}{2} + 16s + C$

Quais das seguintes funções são primitivas (integrais) de:

$$1.\ f(x) = -\frac{sen(x)}{2}$$

$$a)\frac{\cos(x)}{2} \qquad b)\frac{cos(x)}{2} + 4 \qquad c)\frac{cos(x)}{2} + sen(x) \qquad d)\frac{\cos(x)}{4}$$

Sabe-se que $\int f(x)dx = F(x) + C$ é equivalente a $\frac{d}{dx}[F(x) + c] = f(x)$

Resposta: a) e b) pois $\frac{d}{dx}\frac{\cos(x)}{2} = -\frac{sen(x)}{2}$ $\qquad \frac{d}{dx}\left(\frac{\cos(x)}{2} + 4\right) = -\frac{sen(x)}{2}$

$$2.\ f(x) = e^{3x}$$

$$a)e^{3x} + \mathrm{C} \qquad b)3e^{3x} + C \qquad c)\frac{e^{3x}}{3} + C$$

Resposta: c) pois $\frac{d}{dx}\left(\frac{1}{3}e^{3x} + C\right) = \frac{3}{3}e^{3x} = e^{3x}$