Como você resolveria cada uma das integrais abaixo? (identifique)

1) $\int \frac{x^2+x+1}{x}\,dx$] manipulação algébrica integral imediata

2) $\int \frac{x+1}{x^2+x+1}\,dx$] $\int \frac{du}{u}$ + $\int$ completar quadrados

3) $\int \frac{dx}{x^2+x+1}$] completar quadrados - formulário

4) $\int \frac{x^2\,dx}{x^3+9}$] $\int \frac{du}{u}$ ⇒ imediata por substituição

5) $\int \frac{dx}{x^2+4}$] substituição $\int \frac{du}{u^2+a^2}$ imediata - formulário

6) $\int \frac{x\,dx}{x^2+4}$] substituição $\int \frac{du}{u}$ formulário

7) $\int \frac{dx}{(x^2-4)(x+1)}$] Racional Método Frações Parciais

8) $\int \frac{x^3-1}{x-1}\,dx$] Racional Divisão Polinômios

9) $\int \frac{x\,dx}{\sqrt{x^2+2}}$] substituição $\int u^n\,du$

10) $\int \ln x\,dx$] por partes

11) $\int x \ln x\,dx$] por partes

12) $\int \frac{(\operatorname{arctg} x)^2}{1+x^2}\,dx$] Substituição $\int u^n\,du$

13) $\int \frac{dx}{(\operatorname{arctg} x)(1+x^2)}$] substituição $\int \frac{du}{u}$

14) $\int \operatorname{arctg} x\,dx$] por partes

15) $\int \frac{\sqrt{x-1}}{x}\,dx$] irracional troca de variável (substitui)

16) $\int \frac{dx}{x\sqrt{1+x}}$] irracional

17) $\sqrt{x^2+2}\,x\,dx$] $\int u^n\,du$ substituição

18) $\int_1^2 \frac{dx}{x-2}$] Integral imprópria 2ª espécie

19) $\int_1^2 \frac{dx}{x+2}$] Integral definida

20) $\int_0^{+\infty} \frac{dx}{x+2}$] Integral imprópria 1ª espécie

1. $\int\underbrace{\left(\frac{x^2}{x}+\frac{x}{x}+\frac{1}{x}\right)}dx = \int\left(x+1+\frac{1}{x}\right)dx = \frac{x^2}{2}+x+\ln|x|+C$

manipulação
algébrica
integral imediata

2. $\int \frac{x+1}{x^2+x+1}\,dx = \frac{1}{2}\int\frac{2x\,dx}{x^2+x+1} + \underbrace{\int\frac{dx}{x^2+x+1}}$

$2bx = x$
$2b = 1$
$b = 1/2$
$b^2 = 1/4$

$u = x^2+x+1$
$du = (2x+1)\,dx$

$\int\frac{du}{u}$

$= \frac{1}{2}\int\frac{2x\,dx}{x^2+x+1} + \int\frac{dx}{x^2+x+\frac{1}{4}-\frac{1}{4}+1}$

$= \frac{1}{2}\ln|x^2+x+1| + \int\frac{dx}{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}}$

$u = x+1/2$
$du = dx$
$a^2 = \frac{3}{4} \Rightarrow a = \frac{\sqrt{3}}{2}$

$\int\frac{du}{u^2+a^2}$

$\int\frac{x+1}{x^2+x+1}\,dx = \frac{1}{2}\ln(x^2+x+1) + \frac{2}{\sqrt{3}}\,\mathrm{arc\,tg}\left(\frac{x+1/2}{\frac{\sqrt{3}}{2}}\right)+C$

$= \frac{1}{2}\ln(x^2+x+1) + \frac{2}{\sqrt{3}}\,\mathrm{arc\,tg}\left(\frac{2x+1}{\sqrt{3}}\right)+C$

completar quadrados

3. $\int\frac{dx}{x^2+x+1} \Rightarrow$ resolvida acima,

4. $\int\frac{x^2\,dx}{x^3+9} = \frac{1}{3}\int\frac{3x^2\,dx}{x^3+9} = \frac{1}{3}\ln(x^3+9)+C$

$u = x^3+9$
$du = 3x^2\,dx$ $\int\frac{du}{u}$

5) $\int\frac{dx}{x^2+4} = \int\frac{du}{u^2+a^2} = \frac{1}{a}\,\mathrm{arc\,tg}\left(\frac{u}{a}\right) = \frac{1}{2}\,\mathrm{arc\,tg}\left(\frac{x}{2}\right)+C$

$u = x \rightarrow du = dx$

$a = 2$

6) $\int\frac{x\,dx}{x^2+4} = \frac{1}{2}\int\frac{2x\,dx}{x^2+4} = \frac{1}{2}\ln(x^2+4)+C$

$u = x^2+4$
$du = 2x\,dx$ $\int\frac{du}{u}$

7) $\int \frac{dx}{(x^2-4)(x+1)}$ fatora $(x^2-4)(x+1) = (x+2)(x-2)(x+1)$

$\underbrace{}$

Racional grau $P(x) <$ grau $G(x)$

$$\frac{1}{(x^2-4)(x+1)} = \frac{A}{x+2} + \frac{B}{x-2} + \frac{C}{x+1}$$

$$\frac{1}{(x^2-4)(x+1)} = \frac{A(x-2)(x+1) + B(x+2)(x+1) + C(x^2-4)}{(x+2)(x-2)(x+1)}$$

$$1 = A(x^2 - x - 2) + B(x^2+3x+2) + C(x^2-4)$$

$$1 = (A+B+C)x^2 + (-A+3B)x + (-2A+2B-4C)$$

$A+B+C=0$

$-A+3B=0 \longrightarrow \boxed{A=3B}$

$-2A+2B-4C=1$

$3B+B+C=0$

$-6B+2B-4C=1$

$\begin{cases} 4B+C=0 \Rightarrow C=-4B \\ 4B-4C=1 \longrightarrow 4B+16B=1 \Rightarrow B = 1/20 \end{cases}$

$\Rightarrow C = -1/5$

$\Rightarrow A = 3/20$

$$\int \frac{dx}{(x^2-4)(x+1)} = \frac{3}{20}\int \frac{dx}{x+2} + \frac{1}{20}\int \frac{dx}{x-2} - \frac{1}{5}\int \frac{dx}{x+1}$$

$$= \frac{3}{20}\ln(x+2) + \frac{1}{20}\ln(x-2) - \frac{1}{5}\ln(x+1) + C$$

8) $\underbrace{\int \frac{x^3-1}{x-1}dx}_{\text{Racional}} = \int (x^2+x+1)dx = \frac{x^3}{3} + \frac{x^2}{2} + x + C$

grau $P(x) >$ grau $G(x)$

$$\begin{array}{l|l} x^3-1 & x-1 \\ \hline -x^3+x^2 & x^2+x+1 \\ \hline / \; x^2-1 & \\ -x^2+x & \\ \hline / \; x-1 & \\ -x+1 & \\ \hline / & \end{array}$$

10) $\underbrace{\int \ln x\,dx}_{\text{integral por partes}} = x\ln x - \int \frac{\cancel{x}\,dx}{\cancel{x}} = x\ln x - x + C$

integral por partes $\longrightarrow \int u\,dv = uv - \int v\,du$

$u = \ln x \Rightarrow du = \frac{dx}{x}$

$dv = dx \Rightarrow v = \int dx = x$

11) $\int x\ln x\,dx = \frac{x^2}{2}\ln x - \int \frac{x^{\cancel{2}}}{2}\frac{dx}{\cancel{x}} = \frac{x^2}{2}\ln x - \int \frac{x\,dx}{2}$

$u = \ln x \Rightarrow du = \frac{dx}{x}$

$= \frac{x^2}{2}\ln x - \frac{x^2}{4} + C$

$dv = x\,dx \Rightarrow v = \int x\,dx = \frac{x^2}{2}$

12) $\int \frac{(\operatorname{arctg} x)^2\,dx}{x^2+1} = \int u^n\,du = \frac{(\operatorname{arctg} x)^3}{3} + C$

$u = \operatorname{arctg} x$

$du = \frac{dx}{x^2+1}$

13) $\int \frac{dx}{(\operatorname{arctg} x)(x^2+1)} = \int \frac{du}{u} = \ln(\operatorname{arctg} x)$

$u = \operatorname{arctg} x$

$du = \frac{dx}{x^2+1}$

14) $\int \operatorname{arctg} x\,dx \Rightarrow$ por partes

$u = \operatorname{arctg} x \rightarrow du = \frac{dx}{1+x^2}$

$dv = dx$

$v = \int dx = x$

$\int \operatorname{arctg} x\,dx = x\operatorname{arctg} x - \frac{1}{2}\int \frac{2x}{1+x^2}\,dx$

$u = 1+x^2 \Rightarrow du = 2x\,dx$

$\int \frac{du}{u}$

$\int \operatorname{arctg} x\,dx = x\operatorname{arctg} x - \frac{1}{2}\ln(1+x^2) + C$

15) $\underbrace{\int \frac{\sqrt{x-1}}{x}dx}_{\text{irracional}} = \int \frac{(x-1)^{1/2}dx}{x} = \int \frac{(y^2)^{1/2}\,2y\,dy}{y^2+1}$

$x-1=y^2$
$x=y^2+1$
$dx=2y\,dy$

$= \underbrace{\int \frac{2y^2}{y^2+1}dy}_{\text{racional}} = \int\left(2-\frac{2}{y^2+1}\right)dy$

$\hookrightarrow \int \frac{du}{u^2+a^2} = \frac{1}{a}\operatorname{arctg}\left(\frac{u}{a}\right)$

grau $P(x)$ = grau $G(x)$

$$\begin{array}{r|l} 2y^2 & y^2+1 \\ \hline -2y^2-2 & 2 \\ \hline 1\ \ -2 & \end{array}$$

$\int \frac{\sqrt{x-1}}{x}dx = \int\left(2-\frac{2}{y^2+1}\right)dy = 2y - 2\operatorname{arctg}(y)+C$

Se $y=\sqrt{x-1}$

$\int \frac{\sqrt{x-1}}{x}dx = 2\sqrt{x-1} - 2\operatorname{arctg}(\sqrt{x-1})+C$

16) $\int \frac{dx}{x\sqrt{1+x}} = \underbrace{\int \frac{dx}{x(1+x)^{1/2}}}_{\text{irracional}} = \int \frac{2y\,dy}{(y^2-1)(y^2)^{1/2}} = \int \frac{2\cancel{y}\,dy}{\cancel{y}(y^2-1)}$

$1+x=y^2$
$x=y^2-1$
$dx=2y\,dy$

$= 2\int \frac{dy}{y^2-1}$

$\int \frac{du}{u^2-a^2} = \frac{1}{2a}\ln\left(\frac{u-a}{u+a}\right)+C$

$\int \frac{2dy}{y^2-1} = \frac{2}{2}\ln\left(\frac{y-1}{y+1}\right)+C \Rightarrow \int \frac{dx}{x\sqrt{x-1}} = \ln\left(\frac{\sqrt{1+x}-1}{\sqrt{1+x}+1}\right)+C$

17) $\int \sqrt{x^2+2}\;x\,dx = \frac{1}{2}\int \underbrace{\sqrt{x^2+2}}_{u}\,\underbrace{2x\,dx}_{du} = \frac{1}{2}\frac{(x^2+2)^{1/2+1}}{3/2} = \frac{\cancel{2}}{\cancel{2}}\frac{(x^2+2)^{3/2}}{3}+C$

$\int u^n du$

$u=x^2+2$
$du=2x\,dx$

$= \frac{\sqrt{(x^2+2)^3}}{3}+C$

9) $\underbrace{\int \frac{x\,dx}{\sqrt{x^2+2}}}_{} = \frac{1}{2}\int 2(x^2+2)^{-1/2}x\,dx = \frac{1}{2}\frac{(x^2+2)^{1/2}}{1/2} = \sqrt{x^2+2}+C$

também é $\int u^n du$

$u=x^2+2$
$du=2x\,dx$

→ Faltou no arquivo

18) $\displaystyle\int_1^2 \frac{dx}{x-2}$ ⇒ integral imprópria de 2ª espécie

$D_f = \mathbb{R} - \{2\}$

$$\int_1^2 \frac{dx}{x-2} = \lim_{h \to 0} \int_1^{2-h} \frac{dx}{x-2} = \lim_{h \to 0} \Big(\ln(x-2)\Big)_1^{2-h}$$

$$\int \frac{du}{u}$$

$$= \lim_{h \to 0} \underbrace{\ln|2-2-h|}_{-\infty} - \ln|1-2| = -\infty$$ diverge

19) $\displaystyle\int_1^2 \frac{dx}{x+2}$ ⇒ integral definida, o integrando está definido no intervalo de integração, que é finito.

$D_f = \mathbb{R} - \{-2\}$

$$\int_1^2 \frac{dx}{x+2} = \Big[\ln(x+2)\Big]_1^2 = \ln(4) - \ln(3)$$

20) $\cancel{\displaystyle\int_{-\infty}^{0}}$ $\displaystyle\int_0^{+\infty} \frac{dx}{x+2}$ ⇒ integral imprópria 1ª espécie

↳ mudei os limites de integração

$$= \lim_{b \to +\infty} \int_0^b \frac{dx}{x+2} = \lim_{b \to +\infty} \Big[\ln(x+2)\Big]_0^b =$$

$$= \lim_{b \to +\infty} \Big[\ln(b+2) - \ln(0+2)\Big] = +\infty$$ diverge