CÁLCULO DIFERENCIAL E INTEGRAL I

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EXERCÍCIOS

RESOLVIDOS

Exemplo 1: $\int \frac{\sqrt{1+\ln x}}{x}dx$, reescrevendo a integral, vem $\int \frac{(1+\ln x)^{1/2}}{x}dx$

Regra

$$6. \int u^n du = \frac{u^{n+1}}{n+1} + C, n \neq -1$$

Fazemos a substituição: $u = (1+\ln x) \Rightarrow du = \frac{1}{x}dx,\ n = 1/2,$

Assim, $$\int \frac{\sqrt{1+\ln x}}{x}dx = \int \sqrt{\underbrace{1+\ln x}_{u}}\ \underbrace{\frac{1}{x}dx}_{du} =$$

$$= \frac{(1+\ln x)^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = \frac{(1+\ln x)^{3/2}}{3/2} + C = \frac{2(1+\ln x)^{3/2}}{3} + C = \frac{2\sqrt{(1+\ln x)^3}}{3} + C$$

Exemplo 2: $\int x\sqrt{x^2+1}\,dx$ Substituição: $u = x^2+1 \Rightarrow du = 2xdx,\ n = 1/2$

$$\int x\sqrt{x^2+1}\,dx = \frac{1}{2}\int 2x\sqrt{x^2+1}\,dx = \frac{1}{2}\frac{(x^2+1)^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = \frac{1}{2}\frac{(x^2+1)^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{\sqrt{(x^2+1)^3}}{3} + C$$

Regra

$$6.\ \int u^n du = \frac{u^{n+1}}{n+1} + C\,, n \neq -1$$

Exemplo 3. $\int \frac{\text{sen}(2x)}{\sqrt{sen^2(x)+1}}dx$

Substituição: $u = sen^2(x) + 1 \Rightarrow du = 2sen(x)cos(x)\,dx,\ n = -\frac{1}{2}\quad e\quad 2sen(x)cos(x) = sen(2x)$

$$\int \frac{\text{sen}(2x)}{\sqrt{sen^2(x)+1}}dx = \int \underbrace{(sen^2(x)+1)}_{u}{}^{-1/2}\underbrace{sen(2x)dx}_{du} = \frac{(sen^2(x)+1)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C\ = 2\sqrt{sen^2(x)+1} + C$$

Exemplo 4. $\int \frac{ln^2(x)}{x}dx$

Substituição: $u = \ln(x) \Rightarrow du = \frac{dx}{x},\ n = 2.$

$$\int \frac{ln^2(x)}{x}dx = \int \frac{(ln(x))^2}{x}dx = \frac{(\ln(x))^{2+1}}{2+1} + C = \frac{(\ln(x))^3}{3} + C$$

Exemplo 5: $\int \frac{\cos(x)\,dx}{sen(x)+10}$

fazemos a seguinte substituição $u = sen(x) + 10 \Rightarrow du = \cos(x)\,dx\ \ e$ obtemos uma integral $\int \frac{du}{u} = \ln|u| + c$

$$\int \frac{\cos(x)\,dx}{sen(x)+10} = \int \frac{du}{u} = \ln(sen(x)+10) + C$$

Regra

$7.\ \int \frac{du}{u} = ln|u| + C, u > 0$

Exemplo 6: $\int 6^{2x} dx$

fazendo $u = 2x \Rightarrow du = 2xdx$ e, para completar o du, multiplicamos e dividimos a integral por 2, assim

$$\int 6^{2x} dx = \frac{1}{2}\int 6^{2x} 2dx = \frac{6^{2x}}{2ln6} + C$$

Regra

$8.\ \int a^u du = \frac{a^u}{lna} + C, a > 0, a \neq 1,$

Exemplo 7. $\int e^{x^3} x^2 dx$

fazemos a substituição $u = x^3 \Rightarrow du = 3x^2 dx$, como temos no integrando x^2, simplesmente multiplicamos por 3 e dividimos por 3 o integrando. Agora, podemos calcular:

Regra

9. $\int e^u du = e^u + C$

$$\int e^{x^3} x^2 dx = \frac{1}{3} \int e^{x^3} 3x^2 dx = \frac{1}{3} e^{x^3} + C$$

Exemplo 8. $\int x^3 e^{x^4+2} dx$

fazemos a substituição $u = x^4 + 2 \Rightarrow du = 4x^3 dx$, como temos no integrando x^3 , simplesmente multiplicamos por 4 e dividimos por 4 o integrando. Agora, podemos calcular:

$$\int e^{x^4+2} x^3 dx = \int e^u \frac{du}{4} = \frac{1}{4} \int e^{x^4+2} 4x^3 dx = \frac{1}{4} e^{x^4+2} + C$$

Regra

9. $\int e^u du = e^u + C$

9. Resolva a integral: $\int \left(x^2 sen(2x^3) + \frac{cos(\sqrt{x})}{\sqrt{x}} - \frac{2x}{1+x^2} + \frac{tg(1/x)}{x^2} + \frac{a^{\frac{2}{x}}}{x^2} \right) dx =$

$$\int x^2 sen(2x^3)dx + \int \frac{cos(\sqrt{x})}{\sqrt{x}} dx - \int \frac{2x}{1+x^2} dx + \int \frac{tg(1/x)}{x^2} dx + \int \frac{a^{\frac{2}{x}}}{x^2} dx$$

Em primeiro lugar, devemos identificar a substituição a ser feita, temos que escolher, convenientemente, o u e o du, para, após, determinar a regra a ser usada:

$$= \frac{1}{6} \int sen\left(\underbrace{2x^3}_{u}\right) \underbrace{6x^2dx}_{du} + 2 \int cos\left(\overbrace{\sqrt{x}}^{u}\right) \underbrace{\frac{dx}{2\sqrt{x}}}_{du} - \int \frac{\overbrace{2xdx}^{du}}{\underbrace{1+x^2}_{u}} - \int tg(1/x) \underbrace{\frac{dx}{-x^2}}_{du} \quad - \frac{1}{2} \int a^{\overbrace{\frac{2}{x}}^{u}} \underbrace{\frac{-2dx}{x^2}}_{du}$$

$u = 2x^3$
$du = 2x^2$

$u = \sqrt{x}$
$du = \frac{1}{2} x^{\frac{1}{2}-1} = \frac{1}{2\sqrt{x}}$

$u = 1 + x^2$
$du = 2xdx$

$u = x^{-1}$
$du = -x^{-2}dx$

$u = 2x^{-1}$
$du = -2x^{-2}dx$

$$= -\frac{1}{6}\cos(2x^3) + 2\mathrm{sen}(\sqrt{x}) - \ln(1+x^2) - \ln\left(\sec\left(\frac{1}{x}\right)\right) - \frac{1}{2}\frac{a^{\frac{2}{x}}}{\ln(a)} + C$$

10. Resolva a integral: $\displaystyle\int\left(\overbrace{\sqrt{4-x^2}x}^{1}+\overbrace{\sqrt{4-x^2}}^{2}-\overbrace{\sqrt{x^2-16}}^{3}+\overbrace{\frac{1}{\sqrt{2x^2+3}}}^{4}\right)dx=$

$$-\frac{1}{2}\int\sqrt{4-x^2}\underbrace{(-2)x\,dx}_{du}+\int\sqrt{4-x^2}\,dx-\int\sqrt{x^2-16}\,dx+\frac{1}{\sqrt{2}}\int\frac{\sqrt{2}1}{\sqrt{2x^2+3}}dx=$$

$u=4-x^2$
$du=-2xdx$

$u=x$
$du=2xdx \quad a=2$

$u=x$
$du=2xdx \quad a=4$

$u=\sqrt{2}x$
$du=\sqrt{2}dx \quad a=\sqrt{3}$

$\int u^n du$ $\qquad$ $\int\sqrt{a^2-u^2}du$ $\qquad$ $\int\sqrt{u^2-a^2}du$ $\qquad$ $\int\frac{1}{\sqrt{u^2+a^2}}du$

$$=\underbrace{-\frac{1}{3}(4-x^2)^{\frac{3}{2}}}_{1}+\underbrace{\frac{x}{2}\sqrt{4-x^2}+2arcsen\left(\frac{x}{2}\right)}_{2}-\underbrace{\frac{x}{2}\sqrt{x^2-16}-8ln\left(x+\sqrt{x^2-16}\right)}_{3}+\underbrace{\frac{\sqrt{2}}{2}ln(\sqrt{2})x+\sqrt{2x^2+3}}_{4}+C$$

EXERCÍCIOS

PROPOSTOS

Resolva as seguintes integrais:

1. $\int (x+\sqrt{x})dx = \frac{x^2}{2} + \frac{2x\sqrt{x}}{3} + C$

2. $\int \left(3x\sqrt{x} - \frac{4}{x} + 5\cos(x) - 7e^{-x}\right) dx {=} \frac{6}{5}\sqrt{x^5} - 4\ln(x) + 5sen(x) + 7e^{-x}$

3. $\int x\sqrt{x^2+1}\, dx = \frac{1}{3}\sqrt{(x^2+1)^3} + C$

4. $\int \frac{(1+2x)\, dx}{1+x^2} = \text{arc tg}\, x + \ln\,(1+x^2) + C$

5. $\int \left[\frac{x}{x^2+1} + \frac{1}{\sqrt{9-x^2}}\right] dx = \frac{1}{2}\ \text{ln}\ (x^2+1) + \text{arc sen}\ \left(\frac{x}{3}\right) + C$

6. $\int \frac{5x\,dx}{\sqrt{1-x^4}} = \frac{5}{2}\text{arc sen}\, x^2 + C$

7. $\int e^{sen^2x} sen2x\,dx \;\; = e^{sen^2x} + C$

8. $\int \frac{1}{3\cos\left(5x-\frac{\pi}{4}\right)}dx \;\; = \frac{1}{15}\ln\left[\operatorname{tg}\left(5x-\frac{\pi}{4}\right)+\sec\left(5x-\frac{\pi}{4}\right)\right]+C$

9. $\int \frac{1}{\cos^2(x+1)3\operatorname{tg}(x+1)}dx \;\; = \frac{1}{3}\ln(\operatorname{tg}\,(x+1))+C$

10. $\int \frac{\cos x}{\operatorname{sen}^2 x}\,dx = -\frac{1}{\operatorname{sen} x} + C$