



Cálculo Diferencial e Integral I - T: A

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Cálculo Diferencial e Integral I – T: C – Dúvidas

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Calcule os limites:

a) $\lim\limits_{x \to -1} \dfrac{x^2+1}{x-1} = \dfrac{(-1)^2+1}{-1-1} = \dfrac{1+1}{-2} = \dfrac{2}{-2} = -1$

b) $\lim\limits_{x \to -1} \dfrac{x^2-1}{x+1} = \dfrac{(-1)^2-1}{-1+1} = \dfrac{0}{0} \rightarrow$ Forma Indeterminada

Fatorar o numerador: $x^2-1 = (x-1)(x+1)$

$$\lim_{x \to -1} \frac{x^2-1}{x+1} = \lim_{x \to -1} \frac{(x-1)\cancel{(x+1)}}{\cancel{x+1}} = \lim_{x \to -1} (x-1)$$

$$= -1 - 1 = -2$$

$$\boxed{\lim_{x \to -1} \frac{x^2 - 1}{x + 1} = -2}$$

$$c)\ \lim_{x \to 1} \frac{x - 1}{\sqrt[3]{x} - 1} = \frac{1 - 1}{\sqrt[3]{1} - 1} = \frac{0}{0}$$

Forma Indeterminada

$$\lim_{x \to 1} \frac{x - 1}{\sqrt[3]{x} - 1}$$

Mudança de variável:

$$x = t^3 \longrightarrow \lim_{t \to 1} \frac{t^3 - 1}{\sqrt[3]{t^3} - 1} = \lim_{t \to 1} \frac{t^3 - 1}{t - 1}$$

$$x \to 1 \Rightarrow t^3 \to 1$$

$$t \to 1$$

$$= \lim_{t \to 1} t^2 + t + 1 = 1^2 + 1 + 1$$

$$= 3$$

$$
\begin{array}{l|l}
\cancel{t^3} - 1 & t - 1 \\ \cline{2-2}
-\cancel{t^3} + t^2 & t^2 + t + 1 \\ \cline{1-1}
\cancel{t^2} - 1 & \\
-\cancel{t^2} + t & \\ \cline{1-1}
t - 1 & \\
-t + 1 & \\ \cline{1-1}
0 &
\end{array}
$$

$$
\boxed{\text{Portanto,} \quad \lim_{x \to 1} \frac{x-1}{\sqrt[3]{x} - 1} = 3}
$$

d) $\lim\limits_{x \to +\infty} \dfrac{x^2+1}{x-1} = \lim\limits_{x \to +\infty} \dfrac{x^2}{x} = \lim\limits_{x \to +\infty} x = +\infty$

↳ Fator dominante

e) $\lim\limits_{x \to -\infty} \dfrac{x^2+1}{x^2+4} = \lim\limits_{x \to -\infty} \dfrac{x^2}{x^2} = 1$ //

Se houver, Determine as assíntotas horizontais de $f(x) = \dfrac{x^2+1}{x^2+4}$.

$$\lim_{x \to +\infty} \frac{x^2+1}{x^2+4} = \lim_{x \to +\infty} \frac{x^2}{x^2} = 1$$

$$\lim_{x \to -\infty} \frac{x^2+1}{x^2+4} = \lim_{x \to -\infty} \frac{x^2}{x^2} = 1$$

$y = 1$ é assíntota horizontal de $f(x)$.

Determine, se houver, a(s) assíntota(s) vertical(is) do gráfico de $f(x) = \dfrac{2x+1}{x+3}$.

$D(f) = \mathbb{R} - \{-3\}$

$x = -2,9999\ldots$

$$\lim_{x \to -3^+} \frac{2x+1}{x+3} = \frac{2(-3)+1}{-2,999\ldots+3} = \frac{-5}{0,0\ldots1} = -\infty$$

$x = -3,0000\ldots1$

$$\lim_{x \to -3^-} \frac{2x+1}{x+3} = \frac{2(-3)+1}{-3,0000\ldots1+3} = \frac{-5}{-0,00\ldots1} = +\infty$$

$x = -3$ é assíntota vertical de $f(x)$.

$$\lim_{x \to 1} \frac{\sqrt{x} - 1}{\sqrt[3]{x} - 1}$$

$$x = t^6 \qquad \rightarrow \lim_{t \to 1} \frac{\sqrt{t^6} - 1}{\sqrt[3]{t^6} - 1}$$

$$x \to 1 \Rightarrow t^6 \to 1$$

$$t \to \sqrt[6]{1} = 1$$

$$= \lim_{t \to 1} \frac{t^3 - 1}{t^2 - 1}$$