EXERCÍCIOS 14.8 ▪ PÁGINA 893

1. $\approx 59,30$

3. Nenhum máximo, mínimos $f(1, 1) = f(-1, -1) = 2$

5. Máximos $f(\pm 2, 1) = 4$, mínimos $f(\pm 2, -1) = -4$

7. Máximo $f(1, 3, 5) = 70$, mínimo $f(-1, -3, -5) = -70$

9. Máximo $2/\sqrt{3}$, mínimo $-2/\sqrt{3}$

11. Máximo $\sqrt{3}$, mínimo 1

13. Máximo $f\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) = 2$, mínimo $f\left(-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right) = -2$

15. Máximo $f(1, \sqrt{2}, -\sqrt{2}) = 1 + 2\sqrt{2}$,
    mínimo $f(1, -\sqrt{2}, \sqrt{2}) = 1 - 2\sqrt{2}$

17. Máximo $\frac{3}{2}$, mínimo $\frac{1}{2}$

19. Máximos $f(\pm 1/\sqrt{2}, \mp 1/(2\sqrt{2})) = e^{-1/4}$,
    mínimos $f(\pm 1/\sqrt{2}, \pm 1/(2\sqrt{2})) = e^{-1/4}$

27-37. Veja os Exercícios 39-49 na Seção 14.7.

39. $L^3/(3\sqrt{3})$

41. Mais próximo $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$, mais longe $(-1, -1, 2)$

43. Máximo $\approx 9,7938$, mínimo $\approx -5,3506$

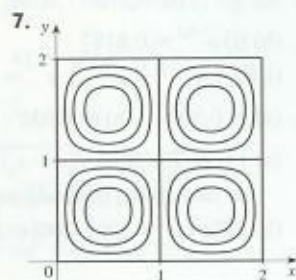45. (a) c/n (b) Quando $x_1 = x_2 = \cdots = x_n$
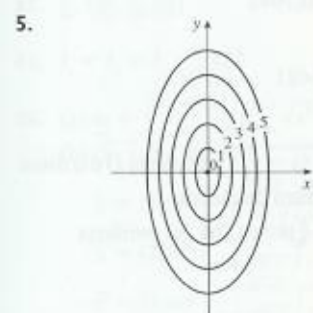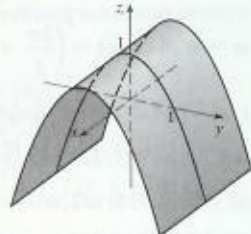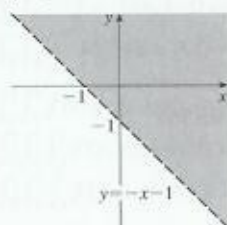
CAPÍTULO 14 REVISÃO ▪ PÁGINA 897

Testes Verdadeiro-Falso

1. Verdadeiro   3. Falso   5. Falso

7. Verdadeiro   9. Falso   11. Verdadeiro

Exercícios

1. $\{(x, y) \mid y > -x - 1\}$ 3.

5.

7.

9. $\frac{2}{3}$

11. (a) $\approx 3,5°C/m$, $-3,0°C/m$

    (b) $\approx 0,35°C/m$ pela Equação 14.6.9 (a Definição 14.6.2 dá
    $\approx 1,1°C/m.$)   (c) $-0,25$

13. $f_x = 1/\sqrt{2x + y^2}, f_y = y/\sqrt{2x + y^2}$

15. $g_u = \text{tg}^{-1} v, g_v = u/(1 + v^2)$

17. $T_p = \ln(q + e^r), T_q = p/(q + e^r), T_r = pe^r/(q + e^r)$

19. $f_{xx} = 24x, f_{yy} = -2y = f_{yx}, f_{yy} = -2x$

21. $f_{xx} = k(k-1)x^{k-2}y^l z^m, f_{xy} = klx^{k-1}y^{l-1}z^m = f_{yx}$,
    $f_{xz} = kmx^{k-1}y^l z^{m-1} = f_{zx}, f_{yy} = l(l-1)x^k y^{l-2} z^m$,
    $f_{yz} = lmx^k y^{l-1} z^{m-1} = f_{zy}, f_{zz} = m(m-1)x^k y^l z^{m-2}$

25. (a) $z = 8x + 4y + 1$ (b) $\dfrac{x-1}{8} = \dfrac{y+2}{4} = 1 - z$

27. (a) $2x - 2y - 3z = 3$ (b) $\dfrac{x-2}{4} = \dfrac{y+1}{-4} = \dfrac{z-1}{-6}$

29. (a) $4x - y - 2z = 6$
    (b) $x = 3 + 8t, y = 4 - 2t, z = 1 - 4t$

31. $\left(2, \frac{1}{2}, -1\right), \left(-2, -\frac{1}{2}, 1\right)$

33. $60x + \frac{24}{5}y + \frac{32}{5}z - 120; 38,656$

35. $2xy^3(1 + 6p) + 3x^2 y^2(pe^p + e^p) + 4z^3(p \cos p + \text{sen } p)$

37. $-47, 108$ 43. $ze^{x\sqrt{y}}\langle z\sqrt{y}, xz/(2\sqrt{y}), 2\rangle$ 45. $\frac{43}{5}$

47. $\sqrt{145}/2, \langle 4, \frac{9}{2}\rangle$ 49. $\approx \frac{5}{8}$ nós/mi

51. Mínimo $f(-4, 1) = -11$

53. Máximo $f(1, 1) = 1$; pontos de sela $(0, 0), (0, 3), (3, 0)$

55. Máximo $f(1, 2) = 4$, mínimo $f(2, 4) = -64$

57. Máximo $f(-1, 0) = 2$, mínimo $f(1, \pm 1) = -3$, pontos de
    sela $(-1, \pm 1), (1, 0)$

59. Máximo $f(\pm\sqrt{2/3}, 1/\sqrt{3}) = 2/(3\sqrt{3})$,
    mínimo $f(\pm\sqrt{2/3}, -1/\sqrt{3}) = -2/(3\sqrt{3})$

61. Máximo 1, mínimo -1

63. $(\pm 3^{-1/4}, 3^{-1/4}\sqrt{2}, \pm 3^{1/4}), (\pm 3^{-1/4}, 3^{-1/4}\sqrt{2}, \pm 3^{1/4})$

65. $P(2 - \sqrt{3}), P(3 - \sqrt{3})/6, P(2\sqrt{3} - 3)/3$

PROBLEMAS QUENTES ▪ PÁGINA 902

1. $L^2 W^2, \frac{1}{4}L^2 W^2$ 3. (a) $x = w/3$, base $= w/3$ (b) Sim
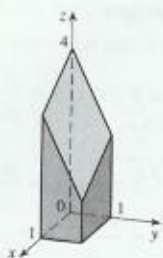
7. $\sqrt{6}/2, 3\sqrt{2}/2$

CAPÍTULO 15

EXERCÍCIOS 15.1 ▪ PÁGINA 912

1. (a) 288   (b) 144

3. (a) $\pi^2/2 \approx 4,935$ (b) 0

5. (a) -6 (b) 3,5

7. $U < V < L$

9. (a) ≈ 248 (b) 15,5

11. 60   13. 3

15. 1,141606, 1,143191, 1,143535, 1,143617, 1,143637, 1,143642

EXERCÍCIOS 15.2 ▪ PÁGINA 917

1. $500y^3, 3x^2$ 3. 10   5. 1   7. 261,632/45   9. $\frac{21}{2}\ln 2$

11. 0   13. π 15. $\frac{21}{2}$ 17. $9 \ln 2$

19. $\frac{1}{2}(\sqrt{3} - 1) - \frac{1}{12}\pi$ 21. $\frac{1}{2}(e^2 - 3)$
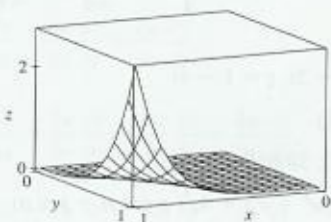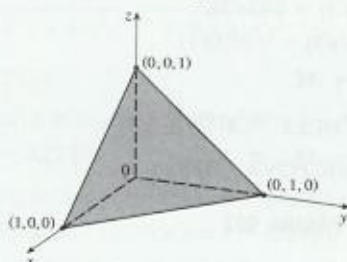
23.

25. 47,5   **27.** $\frac{166}{27}$ **29.** 2   **31.** $\frac{64}{3}$

33. $21e - 57$

35. $\frac{5}{6}$

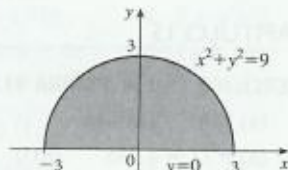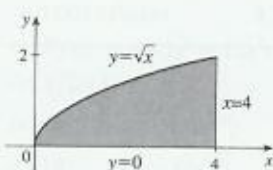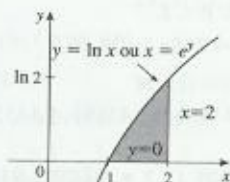**37.** O Teorema de Fubini não se aplica. O integrando tem uma descontinuidade infinita na origem.

EXERCÍCIOS 15.3 ▪ PÁGINA 924

1. $\frac{9}{20}$ **3.** $\frac{3}{10}$ **5.** $e - 1$ **7.** $\frac{256}{21}$ **9.** π **11.** $\frac{1}{2}e^{16} - \frac{17}{2}$

13. $\frac{1}{2}(1 - \cos 1)$ **15.** $\frac{147}{20}$ **17.** 0   **19.** $\frac{6}{35}$ **21.** $\frac{31}{8}$
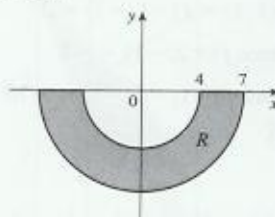
33.

35. 13 984 735 616/14 549 535   **37.** $\pi/2$

39. $\int_0^2 \int_{y^2}^4 f(x, y)\, dx\, dy$ **41.** $\int_{-3}^3 \int_0^{\sqrt{9-x^2}} f(x, y)\, dy\, dx$

43. $\int_0^{\ln 2} \int_{e^y}^2 x^2 f(x, y)\, dx\, dy$

45. $\frac{1}{6}(e^9 - 1)$ **47.** $\frac{1}{3}\ln 9$ **49.** $\frac{1}{3}(2\sqrt{2} - 1)$ **51.** 1

53. $(\pi/16)e^{-1/16} \le \iint_Q e^{-(x^2+y^2)}\, dA \le \pi/16$ **55.** $\frac{3}{4}$

59. 8π **61.** $2\pi/3$

EXERCÍCIOS 15.4 ▪ PÁGINA 930

1. $\int_0^{3\pi/2} \int_0^4 f(r \cos\theta)r\, dr\, d\theta$ **3.** $\int_{-1}^1 \int_0^{(x+1)/2} f(x, y)\, dy\, dx$

5.

$33\pi/2$

7. 0   **9.** $\frac{1}{2}\pi \operatorname{sen} 9$ **11.** $(\pi/2)(1 - e^{-4})$ **13.** $\frac{3}{64}\pi^2$

15. $\pi/12$ **17.** $\frac{1}{8}(\pi - 2)$ **19.** $\frac{16}{3}\pi$ **21.** $\frac{4}{3}\pi$

23. $\frac{4}{3}\pi a^3$ **25.** $(2\pi/3)[1 - (1/\sqrt{2})]$

27. $(8\pi/3)(64 - 24\sqrt{3})$

29. $\frac{1}{2}\pi(1 - \cos 9)$ **31.** $2\sqrt{2}/3$

33. $37,5\pi \text{m}^3$ **35.** $\frac{15}{16}$ **37.** (a)$\sqrt{\pi}/4$ (b)$\sqrt{\pi}/2$

EXERCÍCIOS 15.5 ▪ PÁGINA 939

1. $\frac{64}{3}C$ **3.** $\frac{4}{3}, \left(\frac{4}{3}, 0\right)$ **5.** $6, \left(\frac{3}{4}, \frac{3}{2}\right)$

7. $\frac{1}{4}(e^2 - 1), \left(\dfrac{e^2 - 1}{2(e^2 - 1)}, \dfrac{4(e^3 - 1)}{9(e^2 - 1)}\right)$

9. $L/4, (L/2, 16/(9\pi))$ **11.** $\left(\frac{3}{8}, 3\pi/16\right)$ **13.** $(0, 45/(14\pi))$

15. $(2a/5, 2a/5)$ se o vértice for $(0,0)$ e os lados estiverem nos eixos positivos

17. $\frac{1}{16}(e^4 - 1), \frac{1}{8}(e^2 - 1), \frac{1}{16}(e^4 + 2e^2 - 3)$

19. $7ka^6/180, 7ka^6/180, 7ka^6/90$ se o vértice for $(0, 0)$ e os lados estiverem nos eixos positivos

21. $m = \pi^2/8\ (\bar{x}, \bar{y}) = \left(\dfrac{2\pi}{3} - \dfrac{1}{\pi}, \dfrac{16}{9\pi}\right) I_x = 3\pi^2/64$

$I_y = \frac{1}{16}(\pi^4 - 3\pi^2), I_0 = \pi^4/16 - 9\pi^2/64$

23. $\rho bh^3/3, \rho b^3 h/3; b/\sqrt{3}\ h/\sqrt{3}$

25. $\rho a^4/16, \rho a^4/16; a/2, a/2$

27. (a) $\frac{1}{2}$ (b) 0,375   (c) $\frac{5}{48} \approx 0,1042$

29. (b) (i) $e^{-0,2} \approx 0,8187$
(ii) $1 + e^{-1,8} - e^{-0,8} - e^{-1} \approx 0,3481$ (c) 2, 5

31. (a) $\approx 0,500$ (b) $\approx 0,632$

33. (a) $\iint_D (k/20)[20 - \sqrt{(x - x_0)^2 + (y - y_0)^2}]\, dA$, onde D é o disco de raio 10 km centrado no centro da cidade

(b) $200\pi k/3 \approx 209k, 200(\pi/2 - \frac{8}{9})k \approx 136k$, na periferia

EXERCÍCIOS 15.6 ▪ PÁGINA 948

1. $\frac{27}{4}$ **3.** 1   **5.** $\frac{1}{3}(e^3 - 1)$ **7.** $-\frac{1}{3}$ **9.** 4   **11.** $\frac{65}{28}$

13. $8/(3e)$ **15.** $\frac{1}{60}$ **17.** $16\pi/3$ **19.** $\frac{16}{3}$ **21.** $\frac{8}{15}$

23. (a) $\int_0^1 \int_0^x \int_0^{\sqrt{1-y^2}} dz\, dy\, dx$ (b) $\frac{1}{4}\pi - \frac{1}{3}$

25. 60,533

27.
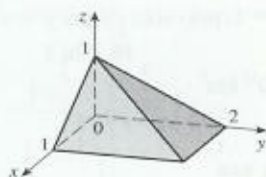
29. $\int_{-2}^{2}\int_{0}^{4-x^2}\int_{-\sqrt{4-x^2-y/2}}^{\sqrt{4-x^2-y/2}} f(x,y,z)\,dz\,dy\,dx$

$= \int_{0}^{4}\int_{-\sqrt{4-y}}^{\sqrt{4-y}}\int_{-\sqrt{4-x^2-y/2}}^{\sqrt{4-x^2-y/2}} f(x,y,z)\,dz\,dy\,dx$

$= \int_{-1}^{1}\int_{0}^{4-4z^2}\int_{-\sqrt{4-y-4z^2}}^{\sqrt{4-y-4z^2}} f(x,y,z)\,dz\,dy\,dx$

$= \int_{0}^{4}\int_{-\sqrt{4-y}/2}^{\sqrt{4-y}/2}\int_{-\sqrt{4-y-4z^2}}^{\sqrt{4-y-4z^2}} f(x,y,z)\,dz\,dy\,dx$

$= \int_{-2}^{2}\int_{-\sqrt{4-x^2}/2}^{\sqrt{4-x^2}/2}\int_{0}^{4-x^2-4z^2} f(x,y,z)\,dz\,dy\,dx$

$= \int_{-1}^{1}\int_{-\sqrt{4-4z^2}}^{\sqrt{4-4z^2}}\int_{0}^{4-x^2-4z^2} f(x,y,z)\,dz\,dy\,dx$

31. $\int_{-2}^{2}\int_{x^2}^{4}\int_{0}^{2-y/2} f(x,y,z)\,dz\,dy\,dx$

$= \int_{0}^{4}\int_{-\sqrt{y}}^{\sqrt{y}}\int_{0}^{2-y/2} f(x,y,z)\,dz\,dx\,dy$

$= \int_{0}^{2}\int_{0}^{4-2z}\int_{-\sqrt{y}}^{\sqrt{y}} f(x,y,z)\,dz\,dx\,dy$

$= \int_{0}^{4}\int_{0}^{2-y/2}\int_{-\sqrt{y}}^{\sqrt{y}} f(x,y,z)\,dz\,dx\,dy$

$= \int_{-2}^{2}\int_{0}^{2-x^2/2}\int_{x^2}^{4-2z} f(x,y,z)\,dz\,dx\,dy$

$= \int_{0}^{2}\int_{-\sqrt{4-2z}}^{\sqrt{4-2z}}\int_{x^2}^{4-2z} f(x,y,z)\,dz\,dx\,dy$

33. $\int_{0}^{1}\int_{\sqrt{x}}^{1}\int_{0}^{1-y} f(x,y,z)\,dz\,dy\,dx$

$= \int_{0}^{1}\int_{0}^{y^2}\int_{0}^{1-y} f(x,y,z)\,dz\,dy\,dx$

$= \int_{0}^{1}\int_{0}^{1-z}\int_{0}^{y^2} f(x,y,z)\,dz\,dy\,dx$

$= \int_{0}^{1}\int_{0}^{1-y}\int_{0}^{y^2} f(x,y,z)\,dz\,dy\,dx$

$= \int_{0}^{1}\int_{0}^{1-\sqrt{x}}\int_{\sqrt{x}}^{1-z} f(x,y,z)\,dz\,dy\,dx$

$= \int_{0}^{1}\int_{0}^{(1-z)^2}\int_{\sqrt{x}}^{1-z} f(x,y,z)\,dz\,dy\,dx$

35. $\int_{0}^{1}\int_{y}^{1}\int_{0}^{y} f(x,y,z)\,dz\,dx\,dy = \int_{0}^{1}\int_{0}^{x}\int_{0}^{y} f(x,y,z)\,dz\,dy\,dx$

$= \int_{0}^{1}\int_{z}^{1}\int_{y}^{1} f(x,y,z)\,dz\,dx\,dy = \int_{0}^{1}\int_{0}^{y}\int_{y}^{1} f(x,y,z)\,dz\,dx\,dy$

$= \int_{0}^{1}\int_{0}^{x}\int_{z}^{x} f(x,y,z)\,dz\,dx\,dy = \int_{0}^{1}\int_{z}^{1}\int_{z}^{x} f(x,y,z)\,dz\,dx\,dy$

37. $\frac{79}{30}, \left(\frac{358}{553}, \frac{33}{79}, \frac{571}{553}\right)$

39. $a, (7a/12, 7a/12, 7a/12)$

41. $I_x = I_y = I_z = \frac{2}{3} kL^5$

43. $\frac{1}{2}\pi kha^4$

45. (a) $m = \int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_{1}^{5-y} \sqrt{x^2+y^2}\,dz\,dy\,dx$

(b) $(\bar{x}, \bar{y}, \bar{z})$, onde

$\bar{x} = (1/m)\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_{1}^{5-y} x\sqrt{x^2+y^2}\,dz\,dy\,dx$

$\bar{y} = (1/m)\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_{1}^{5-y} y\sqrt{x^2+y^2}\,dz\,dy\,dx$

$\bar{z} = (1/m)\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_{1}^{5-y} z\sqrt{x^2+y^2}\,dz\,dy\,dx$

(c) $\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_{1}^{5-y} (x^2+y^2)^{3/2}\,dz\,dy\,dx$

47. (a) $\frac{3}{32}\pi + \frac{11}{24}$

(b) $(\bar{x}, \bar{y}, \bar{z}) = \left(\frac{28}{9\pi+44}, \frac{30\pi+128}{45\pi+220}, \frac{45\pi+208}{135\pi+660}\right)$

(c) $\frac{1}{240}(68 + 15\pi)$

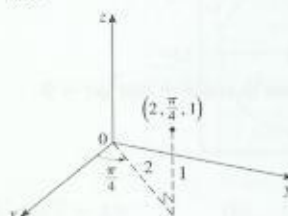**49.** (a) $\frac{1}{8}$ (b) $\frac{1}{64}$ (c) $\frac{1}{5760}$

51. $L^3/8$

53. A região limitada pelo elipsoide $x^2 + 2y^2 + 3z^2 = 1$

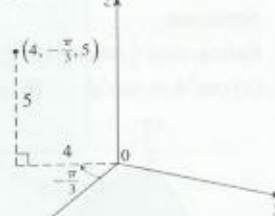## EXERCÍCIOS 15.7 ■ PÁGINA 953

1. (a)

(b)

$(\sqrt{2}, \sqrt{2}, 1)$ $(2, -2\sqrt{3}, 5)$
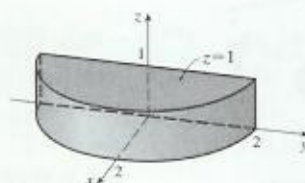
3. (a) $(\sqrt{2}, 7\pi/4, 4)$ (b) $(2, 4\pi/3, 2)$

5. Semiplano vertical pelo eixo z **7.** Paraboloide circular

9. (a) $z = r^2$ (b) $r = 2\operatorname{sen}\theta$

11.

13. Coordenadas cilíndricas: $6 \le r \le 7, 0 \le \theta \le 2\pi, 0 \le z \le 20$

15.

$64\pi/3$
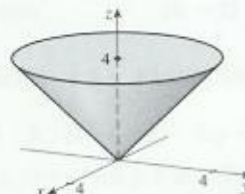
17. 384π **19.** 0 **21.** $2\pi/5$

23. (a) 162π (b) $(0, 0, 15)$

25. $\pi Ka^2/8, (0, 0, 2a/3)$ **27.** 0

29. (a) $\iiint_C h(P)\,dV$, onde C é o cone   (b) $\approx 4.4 \times 10^{18}$ J

EXERCÍCIOS 15.8 ■ PÁGINA 959

1. (a)

$(0, 0, 1)$

(b)

$\left(\frac{1}{2}\sqrt{2}, \frac{1}{2}\sqrt{6}, \sqrt{2}\right)$
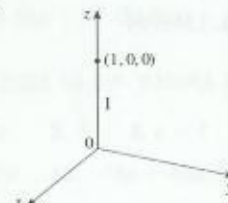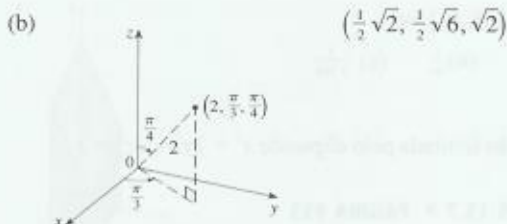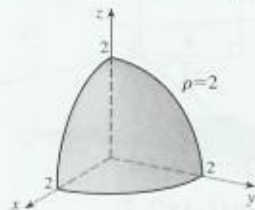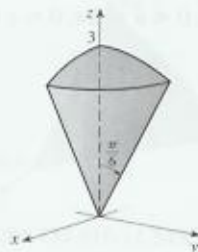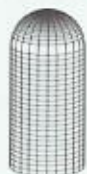
3. (a) $(4, \pi/3, \pi/6)$ (b) $(\sqrt{2}, 3\pi/2, 3\pi/4)$

5. Semicone

7. Esfera, raio $\frac{1}{2}$, centro $(0, \frac{1}{2}, 0)$

9. (a) $\cos^2\phi = \sen^2\phi$ (b) $\rho^2(\sen^2\phi \cos^2\theta + \cos^2\phi) = 9$

11.

13.
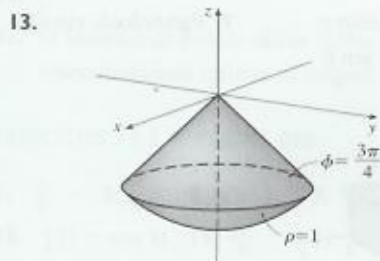
15. $0 \le \phi \le \pi/4, 0 \le \rho \le \cos\phi$

17.

$(9\pi/4)(2 - \sqrt{3})$

19. $\int_0^{\pi/2}\int_0^3\int_0^2 f(r\cos\theta, r\sen\theta, z)\, r\, dz\, dr\, d\theta$

21. $312\,500\pi/7$ **23.** $15\pi/16$ **25.** $1562\pi/15$

27. $(\sqrt{3} - 1)\pi a^3/3$ **29.** (a) 10π (b) $(0, 0, 2,1)$

31. $\left(0, \frac{525}{296}, 0\right)$

33. (a) $\left(0, 0, \frac{3}{8}a\right)$ (b) $4K\pi a^5/15$

35. $(2\pi/3)[1 - (1/\sqrt{2})], (0, 0, 3/[8(2 - \sqrt{2})])$

37. $5\pi/6$ **39.** $(4\sqrt{2} - 5)/15$

41.

43. $136\pi/99$

EXERCÍCIOS 15.9 ■ PÁGINA 968

1. 16   **3.** 0   **5.** $2uvw$

7. O paralelogramo com vértices $(0, 0), (6, 3), (12, 1), (6, -2)$

9. A região limitada pela reta $y = 1$, pelo eixo y e por $y = \sqrt{x}$

11. -3 **13.** 6π **15.** $2\ln 3$

17. (a) $\frac{4}{3}\pi abc$ (b) $1\,083 \times 10^{12}$ km^3

19. $\frac{8}{5}\ln 8$ **21.** $\frac{3}{2}\sen 1$ **23.** $e - e^{-1}$

CAPÍTULO 15 REVISÃO ■ PÁGINA 969

Testes Verdadeiro-Falso

1. Verdadeiro   **3.** Verdadeiro   **5.** Verdadeiro   **7.** Falso

Exercícios

1. $\approx 64,0$ **3.** $4e^2 - 4e + 3$ **5.** $\frac{1}{2}\sen 1$ **7.** $\frac{2}{3}$

9. $\int_0^\pi\int_2^4 f(r\cos\theta, r\sen\theta)\, r\, dr\, d\theta$

11. A região dentro do laço da rosácea de quatro pétalas $r = \sen 2\theta$ no primeiro quadrante

13. $\frac{1}{2}\sen 1$ **15.** $\frac{1}{2}e^6 - \frac{7}{2}$ **17.** $\frac{1}{4}\ln 2$ **19.** 8

21. $81\pi/5$ **23.** $40,5$ **25.** $\pi/96$ **27.** $\frac{64}{15}$

29. 176   **31.** $\frac{2}{3}$ **33.** $2ma^3/9$

35. (a) $\frac{1}{4}$ (b) $\left(\frac{1}{3}, \frac{8}{15}\right)$

   (c) $I_x = \frac{1}{12}, I_y = \frac{1}{24}, \overline{\overline{y}} = 1/\sqrt{3}, \overline{\overline{x}} = 1/\sqrt{6}$

37. $(0, 0, h/4)$

39. $97,2$ **41.** $0,0512$

43. (a) $\frac{1}{15}$ (b) $\frac{1}{3}$ (c) $\frac{1}{45}$

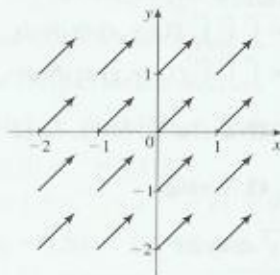**45.** $\int_0^1\int_0^{1-z}\int_{-\sqrt{y}}^{\sqrt{y}} f(x, y, z)\, dx\, dy\, dz$ **47.** $-\ln 2$ **49.** 0

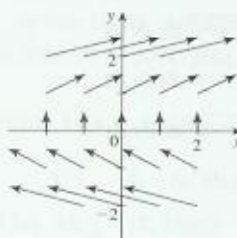**PROBLEMAS QUENTES ■ PÁGINA 972**

1. 30   **3.** $\frac{1}{2}\sen 1$ **7.** (b) $0,90$

CAPÍTULO 16

EXERCÍCIOS 16.1 ■ PÁGINA 980

1.

3.

5.