10-65. Consider the hypothesis test $H_0: \sigma_1^2 = \sigma_2^2$ against $H_1: \sigma_1^2 < \sigma_2^2$ respectively. Suppose that the sample sizes are $n_1 = 5$ and $n_2 = 10$, and that $s_1^2 = 23.2$ and $s_2^2 = 28.8$. Use $\alpha = 0.05$. Test the hypothesis and explain how the test could be conducted with a confidence interval on σ_1 / σ_2.

1) The parameters of interest are the standard deviations σ_1, σ_2

2) H₀ : $\sigma_1^2 = \sigma_2^2$

3) H₁ : $\sigma_1^2 < \sigma_2^2$

4) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

5) Reject the null hypothesis if $f_0 < f_{.0.95,4,9} = 1/f_{.0.05,9,4} = 1/6 = 0.1666$ for $\alpha = 0.05$

6) $n_1 = 5$ $n_2 = 10$ $\qquad s_1^2 = 23.2 \qquad s_2^2 = 28.8$

$$f_0 = \frac{(23.2)}{(28.8)} = 0.806$$

7) Conclusion: Because $0.1666 < 0.806$ do not reject the null hypothesis.

95% confidence interval:

$$\frac{\sigma_1^2}{\sigma_2^2} \le \left(\frac{s_1^2}{s_2^2}\right) f_{1-\alpha,n_2-1,n_1-1}$$

Typo: should be f_{α,n_2-1,n_1-1}

$\frac{\sigma_1^2}{\sigma_2^2} \le (\frac{23.2}{28.8}) f_{0.05,9,4}$ where $f_{.05,9,4} = 6.00$ $\qquad \frac{\sigma_1^2}{\sigma_2^2} \le 4.83$ or $\frac{\sigma_1}{\sigma_2} \le 2.20$

Because the value one is contained within this interval, there is no significant difference in the variances.

10-27. ⊕ Two companies manufacture a rubber material intended for use in an automotive application. The part will be subjected to abrasive wear in the field application, so you decide to compare the material produced by each company in a test. Twenty-five samples of material from each company are tested in an abrasion test, and the amount of wear after 1000 cycles is observed. For company 1, the sample mean and standard deviation of wear are $\bar{x}_1 = 20$ milligrams/1000 cycles and $s_1 = 2$ milligrams/1000 cycles, and for company 2, you obtain $\bar{x}_2 = 15$ milligrams/1000 cycles and $s_2 = 8$ milligrams/1000 cycles.

(a) Do the data support the claim that the two companies produce material with different mean wear? Use $\alpha = 0.05$, and assume that each population is normally distributed but that their variances are not equal. What is the P-value for this test?

(b) Do the data support a claim that the material from company 1 has higher mean wear than the material from company 2? Use the same assumptions as in part (a).

(c) Construct confidence intervals that will address the questions in parts (a) and (b) above.

1) The parameter of interest is the difference in mean wear amount, $\mu_1 - \mu_2$, with $\Delta_0 = 0$

2) H_0: $\mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) H_1: $\mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

4) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

5) Reject the null hypothesis if $t_0 < -t_{0.025,26}$ or $t_0 > t_{0.025,26}$ where $t_{0.025,26} = 2.056$ for $\alpha = 0.05$ because

$$\nu = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}} = 26.98$$

$$\nu \cong 26$$

6) $\bar{x}_1 = 20$ $\quad \bar{x}_2 = 15$

$s_1 = 2$ $\quad s_2 = 8$

$n_1 = 25$ $\quad n_2 = 25$

$$t_0 = \frac{(20 - 15)}{\sqrt{\frac{(2)^2}{25} + \frac{(8)^2}{25}}} = 3.03$$

7) Conclusion: Because 3.03 > 2.056 reject the null hypothesis. The data support the claim that the two companies produce material with significantly different wear at the 0.05 level of significance.

P-value = $2P(t > 3.03)$, $2(0.0025) < P\text{-value} < 2(0.005)$, $0.005 < P\text{-value} < 0.010$

10-53. Fifteen adult males between the ages of 35 and 50 participated in a study to evaluate the effect of diet and exercise on blood cholesterol levels. The total cholesterol was measured in each subject initially and then three months after participating in an aerobic exercise program and switching to a low-fat diet. The data are shown in the following table.

Blood Cholesterol Level		
Subject	**Before**	**After**
1	265	229
2	240	231
3	258	227
4	295	240
5	251	238
6	245	241
7	287	234
8	314	256
9	260	247
10	279	239
11	283	246
12	240	218
13	238	219
14	225	226
15	247	233

(a) Do the data support the claim that low-fat diet and aerobic exercise are of value in producing a mean reduction in blood cholesterol levels? Use $\alpha = 0.05$. Find the P-value.

(b) Calculate a one-sided confidence limit that can be used to answer the question in part (a).

a)

1) The parameter of interest is the difference in blood cholesterol level, μ_d where d_i = Before – After.

2) $H_0: \mu_d = 0$

3) $H_1: \mu_d > 0$

4) The test statistic is

$$t_0 = \frac{\bar{d}}{s_d / \sqrt{n}}$$

5) Reject the null hypothesis if $t_0 > t_{0.05,14}$ where $t_{0.05,14} = 1.761$ for $\alpha = 0.05$

6) $\bar{d} = 26.867$
$s_d = 19.04$
$n = 15$

$$t_0 = \frac{26.867}{19.04/\sqrt{15}} = 5.465$$

7) Conclusion: Because 5.465 > 1.761, reject the null hypothesis. The data support the claim that the mean difference in cholesterol levels is significantly less after diet and an aerobic exercise program at the 0.05 level of significance.

P-value = $P(t > 5.565) \cong 0$

b) 95% confidence interval:

$$\bar{d} - t_{\alpha,n-1}\left(\frac{s_d}{\sqrt{n}}\right) \le \mu_d$$

$$26.867 - 1.761\left(\frac{19.04}{\sqrt{15}}\right) \le \mu_d$$

$$18.20 \le \mu_d$$

Because the lower bound is positive, the mean difference in blood cholesterol level is significantly less after the diet and aerobic exercise program.

b)

1) The parameter of interest is the difference in mean wear amount, $\mu_1 - \mu_2$

2) $H_0: \mu_1 - \mu_2 = 0$

3) $H_1: \mu_1 - \mu_2 > 0$

4) The test statistic is

$$t_0 = \frac{(\overline{x}_1 - \overline{x}_2) - \Delta_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

5) Reject the null hypothesis if $t_0 > t_{0.05,27}$ where $t_{0.05,26} = 1.706$ for $\alpha = 0.05$ since

Typo: should be 26

6) $\overline{x}_1 = 20$ $\overline{x}_2 = 15$

$s_1 = 2$ $s_2 = 8$

$n_1 = 25$ $n_2 = 25$

$$t_0 = \frac{(20-15)}{\sqrt{\frac{(2)^2}{25} + \frac{(8)^2}{25}}} = 3.03$$

7) Conclusion: Because 3.03 > 1.706 reject the null hypothesis. The data support the claim that the material from company 1 has a higher mean wear than the material from company 2 at a 0.05 level of significance.

c) For part (a) use a 95% two-sided confidence interval:
$t_{0.025,26} = 2.056$

$$\left(\bar{x}_1-\bar{x}_2\right)-t_{\alpha,v}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}} \le \mu_1-\mu_2 \le \left(\bar{x}_1-\bar{x}_2\right)+t_{\alpha,v}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}$$

$$(20-15)-2.056\sqrt{\frac{(2)^2}{25}+\frac{(8)^2}{25}} \le \mu_1-\mu_2 \le (20-15)+2.056\sqrt{\frac{(2)^2}{25}+\frac{(8)^2}{25}}$$

$$1.609 \le \mu_1-\mu_2 \le 8.391$$

For part (b) use a 95% lower one-sided confidence interval:
$t_{0.05,26} = 1.706$

$$\left(\bar{x}_1-\bar{x}_2\right)-t_{\alpha,v}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}} \le \mu_1-\mu_2$$

$$(20-15)-1.706\sqrt{\frac{(2)^2}{25}+\frac{(8)^2}{25}} \le \mu_1-\mu_2$$

$$2.186 \le \mu_1-\mu_2$$

For part a) we are 95% confident the mean abrasive wear from company 1 exceeds the mean abrasive wear from company 2 by between 1.609 and 8.391 mg/1000.

For part b) we are 95% confident the mean abrasive wear from company 1 exceeds the mean abrasive wear from company 2 by at least 2.186 mg/1000.