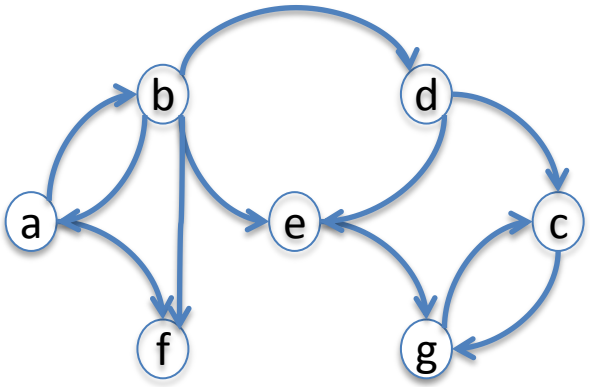
Traversal Tasks

- Minimum Spanning Tree
  - Find the smallest subset of edges that (weakly) connect the graph

What is a minimum spanning tree of this graph?

Why might you want to compute this?



Traversal Tasks: Euler's Bridges of Konigsberg

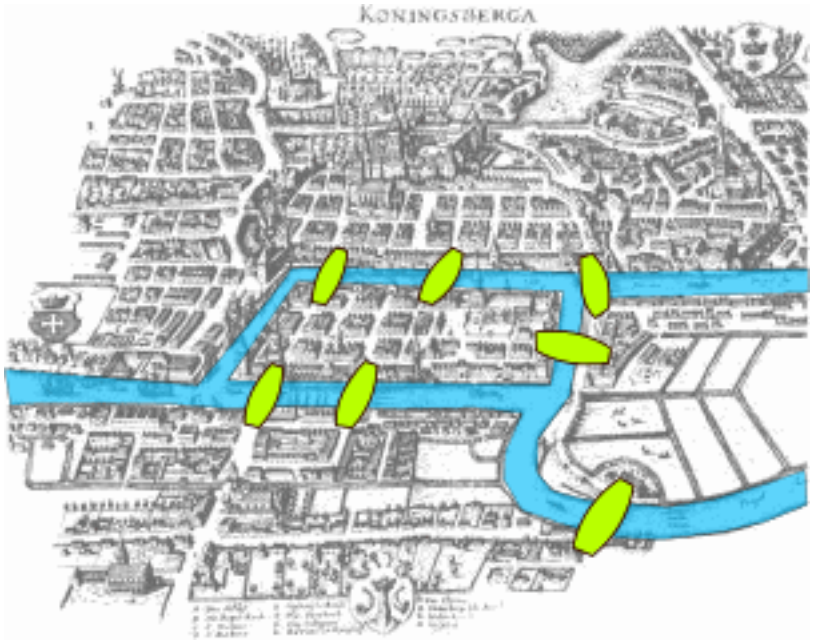
Can we visit every vertex and cross each bridge only once?

Observation: if you enter by a bridge, you must leave by a bridge

Result: If you don't care where you start and end, then at most two vertices can have an odd degree.

Result: If you want to start and end in the same place, every vertex must have an even degree.

Very easy to check!



Traversal Tasks: Hamiltonian Paths

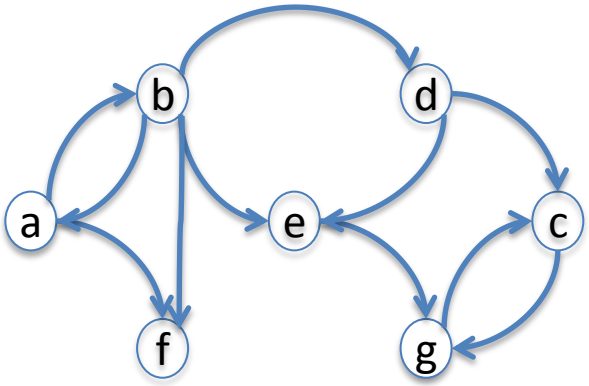
Can we create a path that visits every vertex only once?

A lot harder! Why?

Intuition: Each vertex is involved in a lot of possible paths. But we can only "use" the vertex once. Which is the "best" way to use the vertex?

Extension: Assume there is a cost to traversing each edge. Can we find the path that visits every vertex with the minimum cost?

No efficient algorithm can exist. Heuristics and approximations are the best we can do.



Traversal Tasks

- Maximum Flow
  - Input: A graph with labeled edges indicating the "capacity" of that edge, and special vertices called sources and sinks
  - Find a subgraph that maximizes flow between sources and sinks

Condition: for each vertex, incoming flow must equal outgoing flow