Numerical Linear Algebra for Financial Engineering

The Pre-MFE Program at Baruch College

Homework 1

Assigned: February 3; Due: February 10

(1) Find the value of x such that the matrix below has rank 3:

$$\begin{pmatrix} x & 1 & 1 & 1 \\ 1 & x & 1 & 1 \\ 1 & 1 & x & 1 \\ 1 & 1 & 1 & x \end{pmatrix}.$$

(2) Show that the product of two symmetric matrices is not necessarily a symmetric matrix.

Hint: Find 2×2 matrices A and B such that $A = A^t$, $B = B^t$, and $(AB)^t \neq AB$.

(3) Let A be a square matrix of size n. Show that $AD = DA$ for any diagonal matrix D of size n if and only if A is a diagonal matrix.

(4) Let M_1, M_2, M_3, and M_4 be square matrices of the same size such that $M_1 M_2 M_3 M_4 = I$. Show that

$$M_1 M_2 M_3 M_4 \;=\; M_2 M_3 M_4 M_1 \;=\; M_3 M_4 M_1 M_2 \;=\; M_4 M_1 M_2 M_3 \;=\; I.$$

(5) Let A be a nonsingular matrix. If $I + A$ and $I + A^{-1}$ are also nonsingular matrices, show that

$$(I + A)^{-1} \;=\; I - (I + A^{-1})^{-1}.$$

(6) Show that an $n \times n$ matrix A has rank 1 if and only if there exist an $n \times 1$ nonzero column vector v and an $1 \times n$ nonzero row vector w^t such that $A = vw^t$.

(7) Let x and y be column vectors of size n, and let I be the identity matrix of size n. Show that the matrix $I - xy^t$ is nonsingular if and only if $y^t x \neq 1$.

(8) The covariance matrix of five random variables is

$$\Sigma \;=\; \begin{pmatrix} 1 & -0.525 & 1.375 & -0.075 & -0.75 \\ -0.525 & 4 & 0.1875 & 0.1875 & -0.675 \\ 1.375 & 0.1875 & 12.25 & 0.4375 & -1.875 \\ -0.075 & 0.1875 & 0.4375 & 6.25 & 0.3 \\ -0.75 & -0.675 & -1.875 & 0.3 & 4.41 \end{pmatrix}$$

Find the correlation matrix of these random variables.

(9) The correlation matrix of five random variables is

$$\Omega \;=\; \begin{pmatrix} 1 & -0.25 & 0.15 & -0.05 & -0.30 \\ -0.25 & 1 & -0.10 & -0.25 & 0.10 \\ 0.15 & -0.10 & 1 & 0.20 & 0.05 \\ -0.05 & -0.25 & 0.20 & 1 & 0.10 \\ -0.30 & 0.10 & 0.05 & 0.10 & 1 \end{pmatrix}$$

(i) Compute the covariance matrix of these random variables if their standard deviations are 0.1, 0.2, 0.5, 1, and 2, in this order.

(ii) Compute the covariance matrix of these random variables if their standard deviations are 2, 1, 0.5, 0.2, and 0.1, in this order.

(10) The file *indeces-close-jan3-jan31-2017.xlsx* contains the January 3, 2017 – January 31, 2017 end of day values of Dow Jones, Nasdaq, and S&P 500.

(i) Compute the log daily returns of the three indices over the given time period.

(ii) Compute the sample covariance matrix of the log daily returns of the three indices over the given time period.

(iii) Compute the percentage daily returns of the three indices over the given time period.

(iv) Compute the sample covariance matrix of the percentage daily returns of the three indices over the given time period.

(11) The file *indices-july2016.csv* contains the January 2016 – July 2016 end of day values of nine major US indeces.

(i) Compute the sample covariance matrix of the daily percentage returns of the indeces, and the corresponding sample corelation matrix.

Compute the sample covariance and correlation matrices for daily log returns, and compare them with the corresponding matrices for daily percentage returns.

(ii) Compute the sample covariance matrix of the weekly percentage returns of the indeces, and the corresponding sample corelation matrix.

Compute the sample covariance and correlation matrices for weekly log returns, and compare them with the corresponding matrices for weekly percentage returns.

(iii) Compute the sample covariance matrix of the monthly percentage returns of the indeces, and the corresponding sample corelation matrix.

Compute the sample covariance and correlation matrices for monthly log returns, and compare them with the corresponding matrices for monthly percentage returns.

(iv) Comment on the differences between the sample covariance and correlation matrices for daily, weekly, and monthly returns.

(12) Consider a market made of cash and an asset with spot price \$40. The continuously compounded risk–free rate is 2%. In four months, the price of the stock will be either \$45 or \$35.

(i) What is the payoff matrix M in four months?

(ii) Is the market complete, i.e., is the matrix M nonsingular?

(iii) How do you replicate a four months at–the–money call option on this asset, using the cash and the underlying asset?

(13) In six months, the price of an asset with spot price \$42 will be either \$35, \$38, \$40, \$43, \$45, or \$50. Consider a market made of cash, the stock, and a six months at-the-money call and a six months at-the-money put option on one share of the stock. The risk–free interest rate is constant equal to 2%.

(i) What is the payoff matrix in six months?

(ii) Is this market complete?

(iii) Are all the securities in this market non–redundant?