Returns, Risk, and Risk Aversion

Returns and Risk

- People make decisions based on expected returns and risks every day.
  - Should I attend the investment management class today?
    - Return: Hour of leisure time.
    - Risk: You may miss an important topic for the exams.
- Different people have different perceptions of expected returns and risk.
  - Which activity would you prefer?
    - - Shopping.
    - - Golf.
    - - Sky-Diving.

Returns and Risk in Finance

To assess returns and risks in finance we use objective measures:

Returns: $$r_{0,1} = \left[\frac{DIV + (P_1 - P_0)}{P_0}\right]$$

- P_0 is the purchase price at time 0.
- P_1 is the sale price at time 1.
- DIV is the dividend income paid from the date of purchase to the date of sale.

Risk: Variance

- The average squared deviation of stock returns about the average return.

Returns, Variance and Standard Deviation

$$\text{Average Return} = \bar{r} = \frac{\sum_{i=1}^{n} r_i}{n}$$

$$\text{Variance} = \sigma^2 = \frac{\sum_{i=1}^{n} \left(r_i - \bar{r}\right)^2}{(n)}$$

$$\text{Standard Deviation} = \sigma = \sqrt{\frac{\sum_{i=1}^{n} \left(r_i - \bar{r}\right)^2}{(n)}}$$

Simple Example

- Compute the standard deviation of the numbers 1, 2, 3, 4 and 5.
- The average value is 3.
- Compute the deviation and squared deviation from the average:

Value	Deviation	Squared Deviation
1	(1 – 3)	4
2	(2 – 3)	1
3	(3 – 3)	0
4	(4 – 3)	1
5	(5 – 3)	4

- The sum of the squared deviations is 10.
- This sum divided by 5 equals 2.
  - - total variance = 2.
  - - total standard deviation 1.41.

Calculating Stock Returns

= (B4 – B3)/B3

	A	B	C	D	E	F	G	H	I	J	K	L
1				Price Data						Returns		
2	Month	INTC	AEP	AMZN	MRK	XOM		INTC	AEP	AMZN	MRK	XOM
3	1	33.08	31.74	14.19	45.91	34.49						
4	2	26.97	33.84	14.10	47.58	36.69		-0.185	0.066	-0.006	0.036	0.064
5	3	28.72	35.57	14.30	44.92	38.94		0.065	0.051	0.014	-0.056	0.061
6	4	27.02	35.34	16.69	[illegible]	[illegible]		[illegible]	[illegible]	0.167	-0.056	-0.083
7	5	26.11	33.41	18.23	[illegible]	[illegible]		[illegible]	[illegible]	0.092	0.051	0.000
8	6	17.27	31.29	16.25	[illegible]	[illegible]		[illegible]	[illegible]	-0.109	-0.108	0.025
9	7	17.76	25.73	14.45	[illegible]	[illegible]		[illegible]	[illegible]	-0.111	-0.020	-0.102
10	8	15.78	27.17	14.94	39.67	31.88		-0.111	0.056	0.034	0.018	-0.030
11	9	13.14	22.71	15.93	36.16	28.69		-0.167	-0.164	0.066	-0.088	-0.100
12	10	16.37	20.43	19.36	42.91	30.27		0.246	-0.100	0.215	0.187	0.055
13	11	19.78	23.14	23.35	47.00	31.50		0.208	0.133	0.206	0.095	0.041
14	12	14.75	22.25	18.89	45.06	31.63		-0.254	-0.038	-0.191	-0.041	0.004
15	13	14.84	19.23	21.85	44.09	30.92		0.006	-0.136	0.157	-0.022	-0.022
16	14	16.36	18.21	22.01	41.99	31.01		0.102	-0.053	0.007	-0.048	0.003
17	15	15.44	19.11	26.03	43.91	31.85		-0.056	0.049	0.183	0.046	0.027
18	16	17.45	22.06	28.69	46.64	32.08		0.130	0.154	0.102	0.062	0.007
19	17	19.77	24.62	35.89	44.56	33.41		0.133	0.116	0.251	-0.045	0.041
20	18	19.76	25.29	36.32	48.85	32.96		-0.001	0.027	0.012	0.096	-0.013
21	19	23.63	23.79	41.64	44.60	32.66		0.196	-0.059	0.146	-0.087	-0.009
22	20	27.17	24.31	46.32	42.89	34.84		0.150	0.022	0.112	-0.038	0.067
23	21	26.15	25.76	48.43	43.47	33.83		-0.038	0.060	0.046	0.014	-0.029

Calculating Average Returns and Risk

=AVERAGE(H4:H69)

	A	B	C	D	E	F	G	H	I	J	K	L
1										Returns		
2	Month	INTC	AEP	AMZN	MRK	XOM		INTC	AEP	AMZN	MRK	XOM
62	60	20.04	41.89	39.46	42.91	75.98		-0.054	0.026	-0.022	-0.012	-0.002
63	61	20.74	42.82	37.67	44.04	73.47		0.035	0.022	-0.045	0.026	-0.033
64	62	19.76	44.55	39.14	43.45	71.37		-0.047	0.040	0.039	-0.013	-0.029
65	63	19.03	48.37	39.79	43.84	75.13		-0.037	0.086	0.017	0.009	0.053
66	64	21.39	49.83	61.33	51.06	79.04		0.124	0.030	0.541	0.165	0.052
67	65	22.18	47.63	69.14	52.06	83.17		0.037	-0.044	0.127	0.020	0.052
68	66	23.74	45.04	68.41	49.80	83.88		0.070	-0.054	-0.011	-0.043	0.009
69	67	23.54	43.19	84.04	50.12	85.59		-0.008	-0.041	0.228	0.006	0.020
							Av Ret	0.001	0.007	0.037	0.004	0.015
							Variance	0.011	0.004	0.020	0.006	0.003
							Std Dev	0.107	0.065	0.142	0.078	0.055

=VAR(H4:H69)

=STDEV(H4:H69)

Risk Aversion

- Even when expected returns and risk are known, people often make different choices about what to invest in.
- Why?
- Different people have different preferences for risk.

Risk Aversion

- Consider the following gamble:
  - - Flip a coin.
    - - If it is "Heads" you win $10,000.
    - - If it is "Tails" you must pay $10,000.

- Would you take the gamble?

- How big a win would you require from "Heads" to take the gamble?

Risk Aversion

- If you took the original gamble, you are "risk neutral".
- If you required a payoff of less than $10,000, you are "risk loving".
- If you required a payoff of more than $10,000, you are risk averse".
- The larger the payoff you required, the more risk averse you are.
  - - The vast majority of people are risk averse.

Utility Functions

- Is there a mathematical way to characterize different people's preferences using a simple mathematical function?

- Utility: a measure of satisfaction or desirability of a particular choice.

- Utility functions: functions that can represent a person's preferences.

A Simple Utility Function

- Consider the utility function:

$$U(x) = E(r_x) - \lambda\sigma_x^2$$

- An investor's utility is a function of the expected returns, $E(r_x)$, on an investment, *x*, and the risk of the investment, σ_x^2.

λ is a measure of an investor's level of risk aversion.

A Simple Utility Function

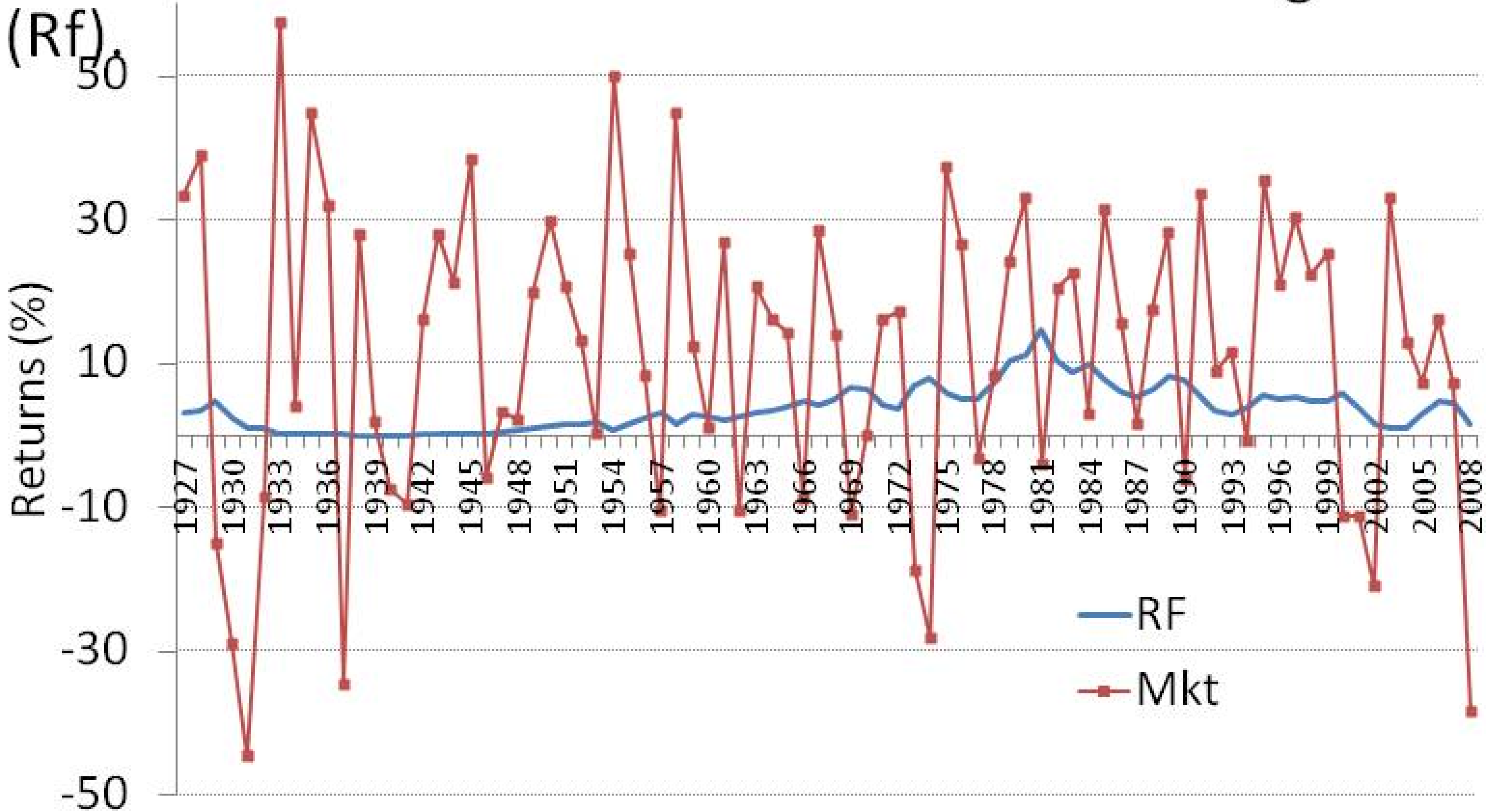
- Consider the utility function:

$$U(x) = E(r_x) - \lambda\sigma_x^2$$

- λ is a measure of an investor's level of risk aversion.
- If $\lambda > 0$ the investor is risk averse.
  - - Investor is more risk averse the higher the values of λ
  - - The penalty he/she attaches to risk is higher.
- If $\lambda = 0$ the investor is risk neutral.
  - - He/She only cares about expected returns.
- If $\lambda < 0$ the investor is risk loving.

Investing in the Stock Market vs T-Bills

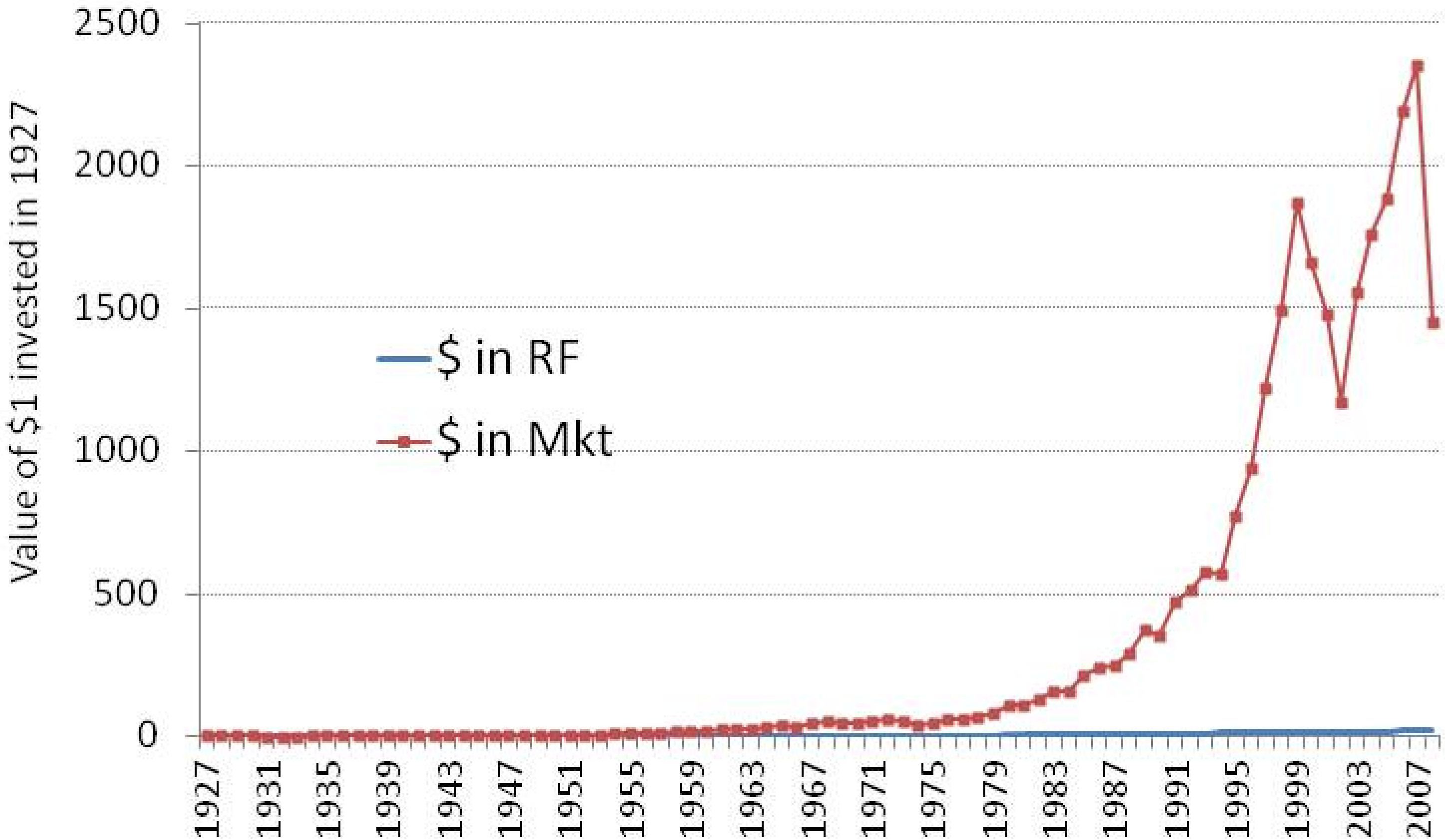
- The stock market is much riskier than investing in T-Bills (Rf).



- If individuals are risk averse we should expect the stock market to have higher average returns.

Investing in the Stock Market vs T-Bills

- The risk premium for investing in the stock market is 7.83% per year over the last 80 years.
  - - This is the premium necessary to compensate investors for bearing stock market risk.



You have two job offers:

1) Research Assistant to a Portfolio Manager.

Pay: $50,000 per year.

2) Sales position in an Investment Bank.

Pay is dependent on your sales performance:

If the market is booming you will make $85,000.

If the market is in recession you will make $15,000.

When will you take the sales job?

1. Prob of Boom = 0.1
2. Prob of Boom = 0.5
3. Prob of Boom = 0.75
4. Prob of Boom = 0.9
5. Prob of Boom = 1

Expected Earnings and Risk Aversion

Are you risk seeking, risk neutral, or risk averse?

Prob of Boom	Prob of Recession	RA Wealth	Sales Job expected wealth	Behavior if you prefer Sales to RA
0.1	0.90	$50,000	0.1 x 85 + 0.9 x 15 = $22,000	Risk Seeking
0.50	0.50	$50,000	0.5 x 85 + 0.5 x 15 = $50,000	Risk Neutral
0.75	0.25	$50,000	0.75 x 85 + 0.25 x 15 = $67,500	Risk Averse
0.90	0.10	$50,000	0.9 x 85 + 0.1 x 15 = $78,000	Risk Averse

- In reality, the majority of humans are risk averse.

You have two possible investment opportunities:

1) Risk Free T-Bills earn 5%.

2) High Tech Stocks. If the market is going up you earn 60%. If the market is falling you will earn -30%.

When will you invest in High Tech Stocks?

1. Up = 0.25
2. Up = 0.5
3. Up = 0.75
4. Up = 0.9

Expected returns and risk

$$E(\text{Ret}_i) = \Pr(B) \times \text{Ret}_{i,\text{Boom}} + \Pr(R) \times \text{Ret}_{i,\text{Rec}}$$

- $$\text{Risk (Variance)} = \Pr(B) \times \left(\text{Ret}_{i,\text{Boom}} - E(\text{Ret}_i)\right)^2 + \Pr(R) \times \left(\text{Ret}_{i,\text{Rec}} - E(\text{Ret}_i)\right)^2$$

Prob of Boom	Prob of Recession	$E(\text{Ret}_{\text{T-Bill}})$	$\text{Risk}_{\text{T-Bill}}$	$E(\text{Ret}_{\text{High Tech}})$	$\text{Risk}_{\text{High Tech}}$
0.25	0.75	0.05	0.00	-0.08	0.15
0.50	0.50	0.05	0.00	0.15	0.20
0.75	0.25	0.05	0.00	0.38	0.15
0.90	0.10	0.05	0.00	0.51	0.07

- When deciding between T-Bills or High Tech Stocks you automatically made a trade-off between risk and return.

Applying the utility function

$$U(x) = E(r_x) - \lambda \sigma_x^2$$

Prob of Upmkt	Utility from T-Bill	Utility from Tech Stocks for different values of λ			
		-3	0	1	5
0.25	0.05	0.38	-0.08	-0.23	-0.83
0.50	0.05	0.76	0.15	-0.05	-0.86
0.75	0.05	0.83	0.38	0.22	-0.38
0.90	0.05	0.73	0.51	0.44	0.15

- Are your own preferences captured by one of the values for λ?

The Characteristics of Stock Returns

Back to utility functions

$$U(x) = E(r_x) - \lambda\sigma_x^2$$

- Investors are risk averse.
- Investors only consider expected returns and risk, as characterized by variance, when maximizing expected utility.
- Portfolio theory and the CAPM are based on this type of utility function.

S&P 500 Returns

- The average monthly returns on the S&P 500 index since 1996 are 0.70%. The risk (variance) is $0.18\%^2$.
-
- What can we say about any given month's return?
  - - Is it equally likely to be above or below the average?
- Does the S&P 500 index experience large jumps in its price?

Skewness

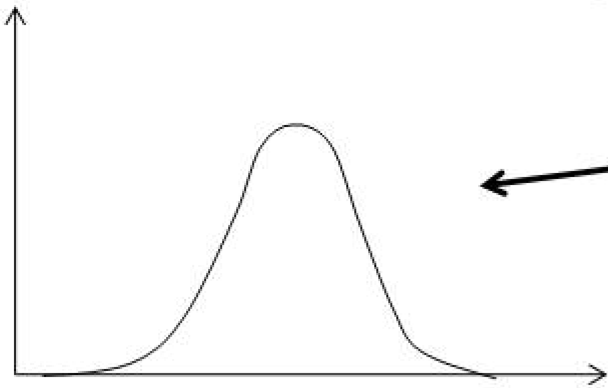
- We have characterized the distribution of stock returns using:
- - Mean (First Moment)
- - Variance (Second Moment)
- Skewness is the third moment. It is a measure of the asymmetry of a probability distribution.

$$\text{Skew} = \frac{n}{(n-1)(n-2)} \sum_{i=1}^{I} \left(\frac{x_i - \bar{x}}{s} \right)^3$$

where:

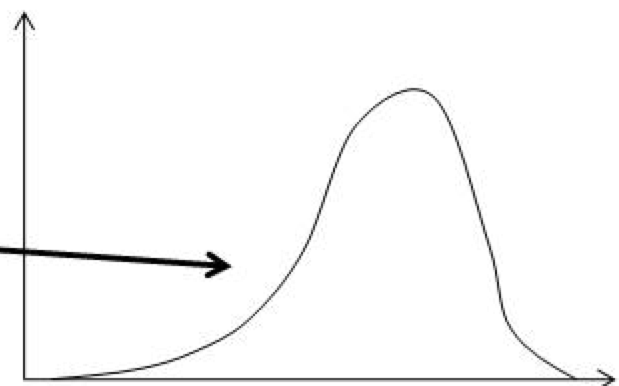
- $$\bar{x} = \frac{1}{n} \sum_{i=1}^{I} x_i \text{ and } s = \sqrt{\frac{\sum_{i=1}^{I} (x_i - \bar{x})^2}{n-1}}$$

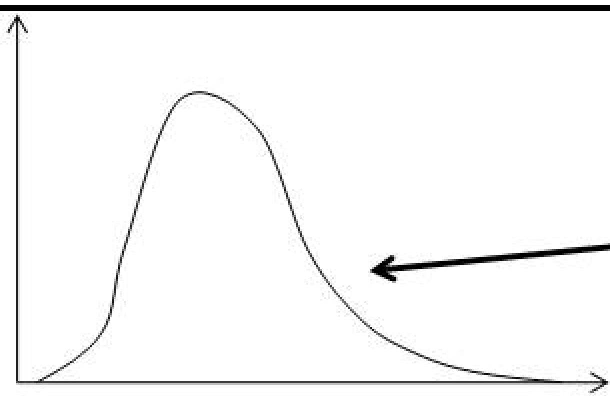
Skewness



Case 1: No Skewness
Distribution is symmetric around the mean.

Case 2: Negative Skewness
The left tail is longest; the mass of the distribution is focused to the right of the figure





Case 3: Positive Skewness
The right tail is longest; the mass of the distribution is focused to the left of the figure

Kurtosis

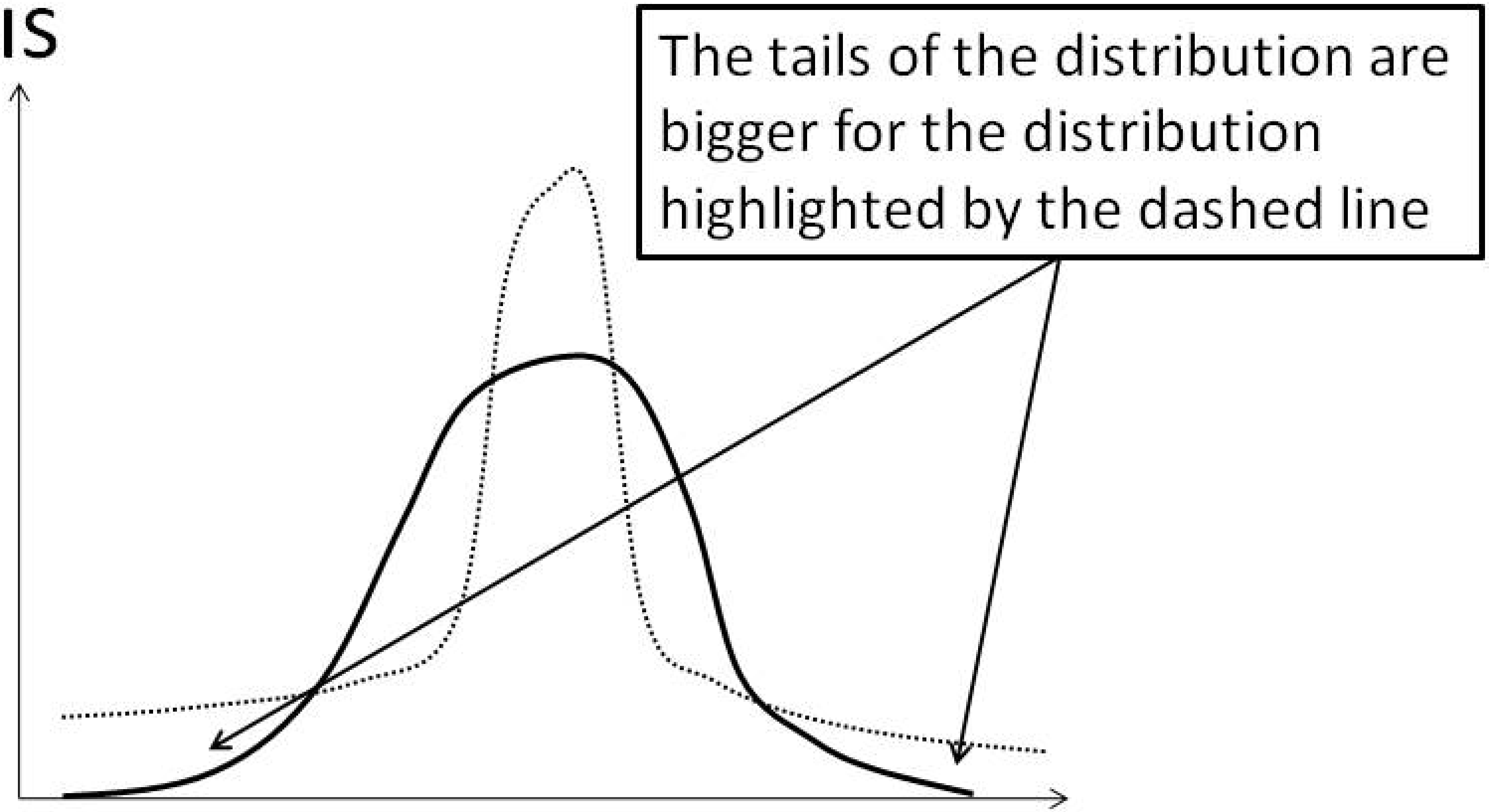
- Kurtosis is the fourth moment. It is a measure of the fatness of the tails of a probability distribution.
  - \- Higher kurtosis means more of the standard deviation is due to infrequent extreme deviations.

-
$$\text{Kurt} = \frac{n(n+1)}{(n-1)(n-2)(n-3)}\sum_{i=1}^{I}\left(\frac{x_i-\bar{x}}{s}\right)^4$$

where :

-
-
$$\bar{x} = \frac{1}{n}\sum_{i=1}^{I} x_i \text{ and } s = \sqrt{\frac{\sum_{i=1}^{I}(x_i-\bar{x})^2}{n-1}}$$

Kurtosis

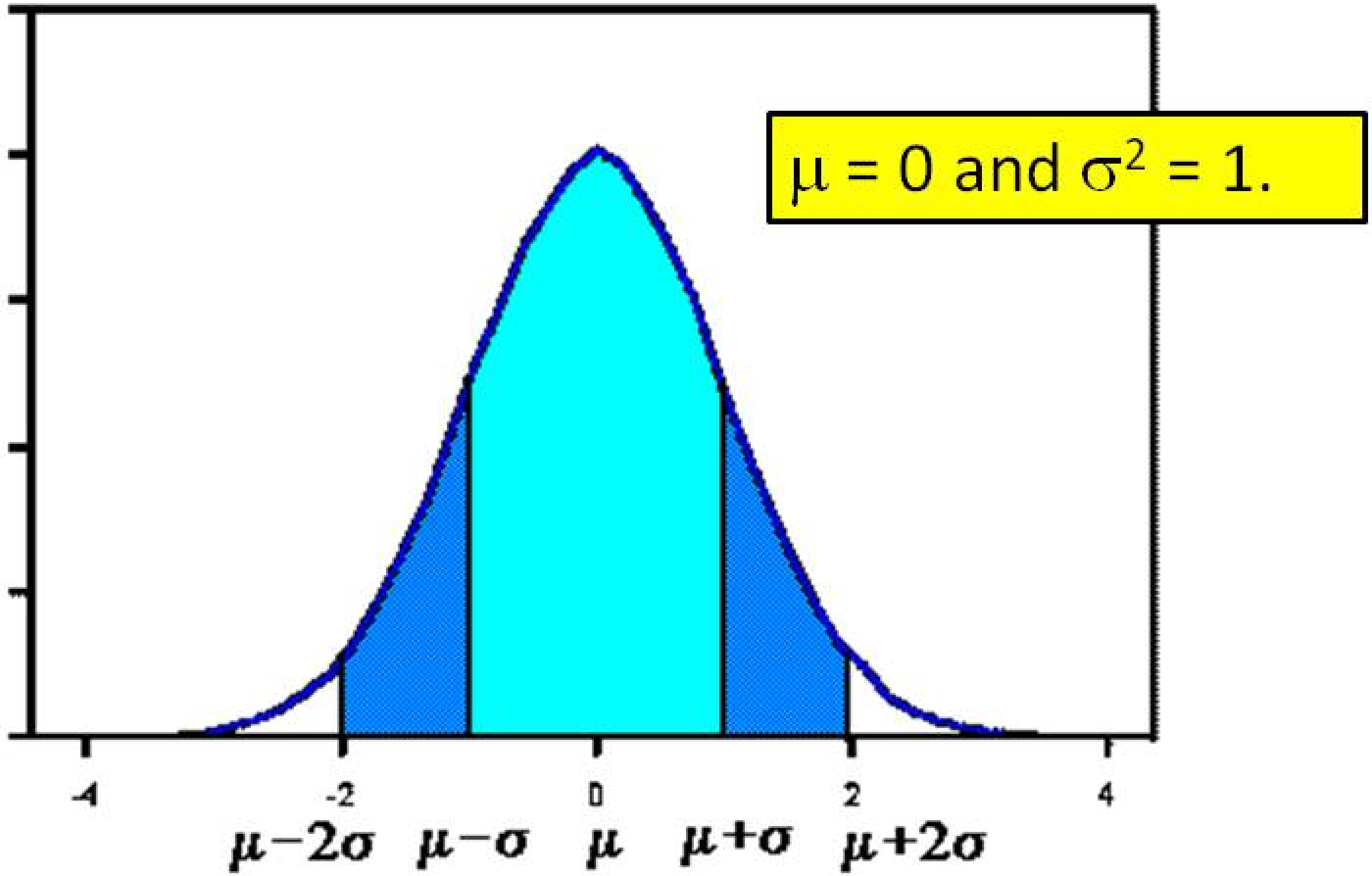


- The probability distribution highlighted by the dashed lines has a higher level of kurtosis than the distribution highlighted by the continuous line.

The Normal Distribution

The normal distribution is characterized by two moments

- mean, μ.
- variance σ^2.



The Normal Distribution

- The normal distribution is symmetric around the mean.
  - - Skewness = 0.
- The normal distributions has a kurtosis of 3.
  - - Typically we compare distributions to the normal distribution.
    - - Excess kurtosis is a measure of kurtosis relative to the normal distribution.
      - - Ex Kurt = Kurt – 3.

Skewness and Kurtosis in Excel

C143 =AVERAGE(C3:C140)

	A	B	C
1	Month	S&P 500	Returns
133	132	1418.30	0.01261575
134	133	1438.24	0.01405908
135	134	1406.82	-0.02184615
136	135	1420.86	0.00997995
137	136	1482.37	0.04329068
138	137	1530.62	0.03254923
139	138	1503.35	-0.01781631
140	139	1473.91	-0.01958293
141			
142			
143		Mean	0.0070
144		Variance	0.0018
145		Skewness	-0.5387
146		Ex Kurtosis	0.6867
147		Min	-0.1458
148		Max	0.0967

=SKEW(C3:C140)

=KURT(C3:C140)

NB: Excel calculates "Excess Kurtosis" = Kurtosis – 3.

There is some evidence that the distribution of S&P 500 returns is negatively skewed, and has slightly more kurtosis than the normal distribution.

Preferences for skewness and kurtosis?

$$U(x) = E(r_x) - \lambda\sigma_x^2$$

- If stock returns are normally distributed:
  - - The standard utility function is appropriate.
    - - Investors only care about mean and variance.

- If stock returns are not normally distributed AND investors have preferences for skewness and/or kurtosis:
  - - The standard utility function is not appropriate.

The Distribution of Stock Returns

- Fama (1965) conducts an extensive analysis of return distributions.

- He concludes that at a monthly horizon stock return distributions can be characterized well by a normal distribution.

- However, at a daily horizon, Fama argues the return distributions are definitely not normal, as they exhibit too many large deviations from the mean.

Mean-Variance is a good approximation

- Paul Samuelson (1970) showed that:
  - \- The importance of additional moments beyond the variance is much smaller than the importance of the expected returns and variance.
  - \- Variance is as important as expected returns to investors.
- Samuelson relied on the assumption of compactness.
  - \- The risk of a stock holding will decline if you hold it for a shorter period.
  - \- If you hold the stock for only an instant, the risk is zero.

- Samuelson, P., 1970, The Fundamental Approximation Theorem of Portfolio Analysis in Terms of Means, Variances, and Higher Moments, *Review of Economic Studies* 37

US Return horizons and risk

	Monthly	Weekly	Daily
Mean	0.0070	0.0017	0.0004
Variance	0.0018	0.0005	0.0001
Skewness	-0.5387	-0.3072	-0.0237
Ex Kurtosis	0.6867	2.1400	3.2328
Min	-0.1458	-0.1105	-0.0687
Max	0.0967	0.0778	0.0573

Chinese Return horizons and risk

- Let's investigate the monthly and daily return of Shanghai and Shenzhen Composite A-index.

Summary

- Four moments provide an effective characterization of a stock return distribution.

- 1) Mean – What is the average return?
- 2) Variance – How large are average squared deviations from the mean?
- 3) Skewness – Is the return distribution symmetric?
- 4) Kurtosis – To what extent is the variance driven by large infrequent deviations?

- At a monthly return horizon return distributions are well approximated by the normal distribution.