Problems with Expected Returns

We Have a Problem!

Portfolio weights are sensitive to expected excess returns.

	Expected Excess Returns		Unrestricted Weights		No Short Selling Weights	
Stock	Hist	CAPM	Hist	CAPM	Hist	CAPM
INTC	-0.004	0.014	-0.557	0.279	0.000	0.279
AEP	0.003	0.006	-0.196	0.290	0.000	0.290
AMZN	0.033	0.011	0.670	0.038	0.304	0.038
MRK	0.000	0.006	-0.204	0.079	0.000	0.079
XOM	0.011	0.005	1.286	0.313	0.696	0.313
		Total Weight:	1.000	1.000	1.000	1.000

Models of Expected Returns

Historical Returns: $r_{i,t} = \mu_i + \varepsilon_{i,t}$ where $\varepsilon_{i,t} \sim N(0, \sigma_i^2)$

$$\rightarrow \frac{1}{N}\sum_{t=1}^{N} r_{i,t} = \frac{1}{N}\sum_{t=1}^{N} \mu_i + \frac{1}{N}\sum_{t=1}^{N} \varepsilon_{i,t} \approx \mu_i$$

But past performance does not necessarily predict future performance.

CAPM: $E(r_i) = r_f + \beta_i\left(E(r_m) - r_f\right)$

But CAPM βs measured using historical data.

- βs may change over time.

Models of Expected Returns

Neither historical returns nor the CAPM are forward looking measures of expected returns.

Ideally we want a forward looking measure of expected returns.

Ideally we would like to ascertain the market's estimate of expected returns for each stock.

Utility Maximization

Recall the Investor's utility maximization problem.

$$\text{Max}_{\text{w}} : w^T \left(\mu - r_f\right) - \lambda w^T S w$$

The solution to the problem is:

$$w = \frac{1}{2\lambda} S^{-1}\left(\mu - r_f\right)$$

Assumption:

- At any given time, in equilibrium, the market portfolio is mean-variance efficient.
- We can observe the market weights, w_m.

Reverse the problem

Originally we were solving to calculate weights.

- But we can observe the market weights.
- We don't need to solve for weights, we observe the market weights.

We can solve instead for expected excess returns.

$$\text{Max}_{\text{w}} : w^T \left(\mu - r_f\right) - \lambda w^T S w$$

$$\rightarrow \mu - r_f - 2\lambda S w = 0$$

$$\rightarrow \mu - r_f = 2\lambda S w$$

Reverse the problem

$$\text{Max}_{w} : w^T(\mu - r_f) - \lambda w^T S w$$

$$\rightarrow \mu - r_f - 2\lambda S w = 0$$

$$\rightarrow \mu - r_f = 2\lambda S w$$

We observe the market weights

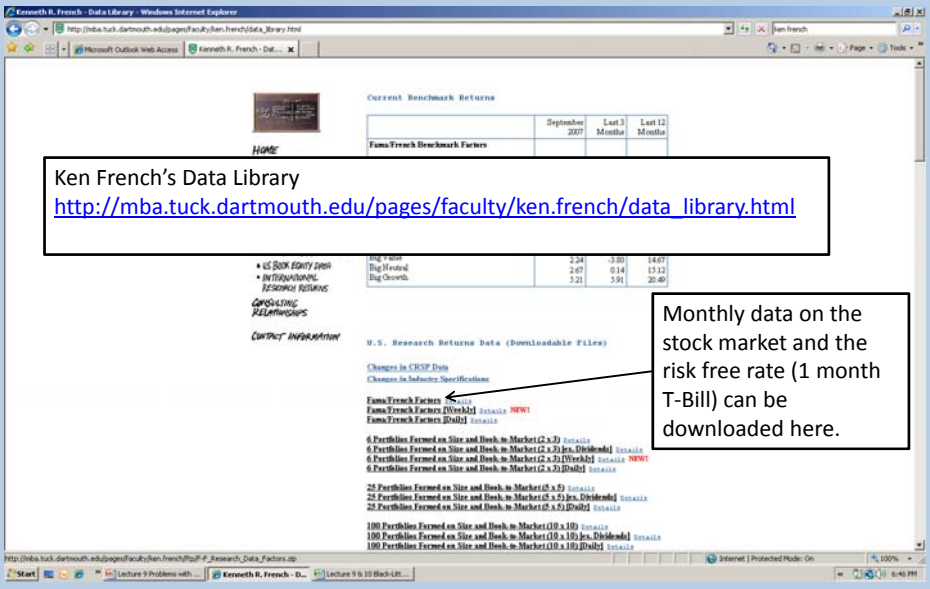
Use historical data for variance-covariance matrix.

What value should be assign to λ?
We know: 1) the average excess returns on the stock market.
2) the risk of investing in the stock market.

If the market is mean-variance efficient we can estimate the price of risk (λ):

$$\lambda = \frac{E(r_p - r_f)}{2\sigma_p^2}$$

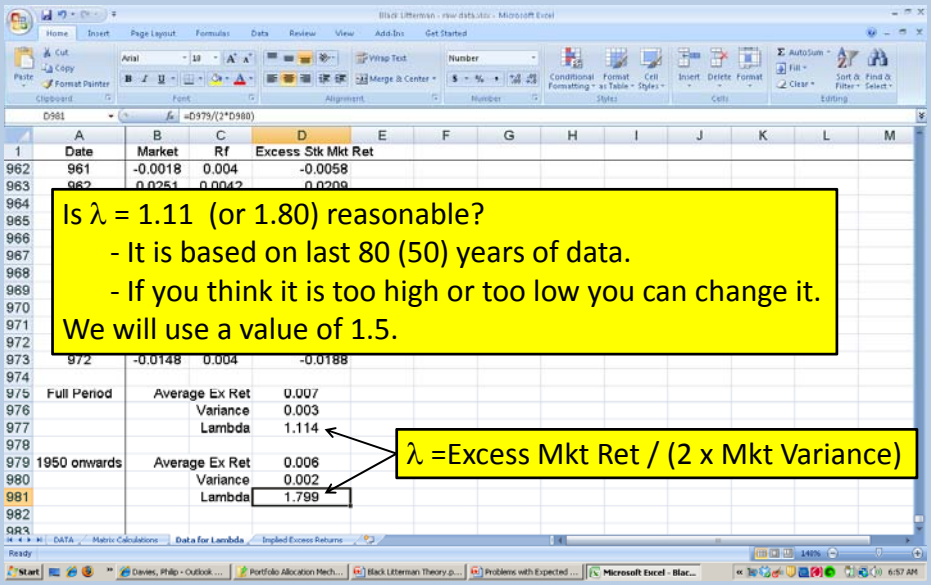
Get Data on Market and Risk Free Returns



Ken French's Data Library
http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

Monthly data on the stock market and the risk free rate (1 month T-Bill) can be downloaded here.

Calculate λ From Historical Data



	A	B	C	D
1	Date	Market	Rf	Excess Stk Mkt Ret
962	961	-0.0018	0.004	-0.0058
973	972	-0.0148	0.004	-0.0188
975	Full Period	Average Ex Ret		0.007
976		Variance		0.003
977		Lambda		1.114
979	1950 onwards	Average Ex Ret		0.006
980		Variance		0.002
981		Lambda		1.799

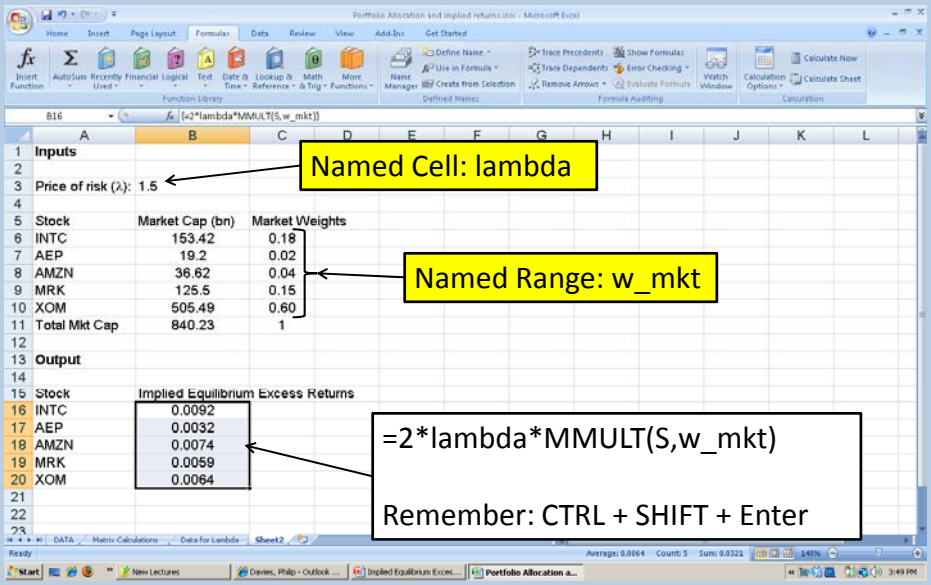
Implied Equilibrium Excess Returns

For this example we will assume that the market consists of only 5 stocks: INTC, AEP, AMZN, MRK, XOM.

We need to calculate the market weights. To do this we need two bits of data. The market capitalization of each stock, and the total market capitalization (assuming the market only consists of the 5 stocks).

Market capitalization data can be obtained from Yahoo!Finance.

Implied Equilibrium Excess Returns



	A	B	C
1	**Inputs**		
2			
3	**Price of risk (λ):**	1.5	
4			
5	**Stock**	**Market Cap (bn)**	**Market Weights**
6	INTC	153.42	0.18
7	AEP	19.2	0.02
8	AMZN	36.62	0.04
9	MRK	125.5	0.15
10	XOM	505.49	0.60
11	**Total Mkt Cap**	840.23	1
12			
13	**Output**		
14			
15	**Stock**	**Implied Equilibrium Excess Returns**	
16	INTC	0.0092	
17	AEP	0.0032	
18	AMZN	0.0074	
19	MRK	0.0059	
20	XOM	0.0064	

Comparing Estimates of Expected Returns

Stock	Historical Excess Returns	CAPM Excess Returns	Implied Excess Returns
INTC	-0.0035	0.0135	0.0092
AEP	0.0026	0.0064	0.0032
AMZN	0.0326	0.0111	0.0074
MRK	0.0002	0.0057	0.0059
XOM	0.0111	0.0052	0.0064

The Black-Litterman Asset Allocation Model

The Black-Litterman Model - Overview

Combine information from two sources to create an estimate of expected returns.

Source 1: What does the current market tell us about expected excess returns?

- Implied excess equilibrium returns.

Source 2: What views does the investment manager have about particular stocks, sectors, asset classes, or countries?

The Black-Litterman model combines these different sources to produce estimates of expected returns.

Combining Predicted and Implied Returns

$$E(r) = \left[(\tau S)^{-1} + P^T \Omega^{-1} P\right]^{-1} \left[(\tau S)^{-1} \Pi + P^T \Omega^{-1} Q\right]$$

Basic Notation:

τ = A scalar (Assume τ = 1).

S = Variance-Covariance matrix for all assets under consideration.

Ω = Uncertainty surrounding your views.

Π = Implied Excess Returns.

Q = Views on expected excess returns for some or all assets.

P = Matrix identifying which assets you have views about.

Understanding the Formula

Consider the second part first:

Implied Excess Returns

$$(\tau S)^{-1} \Pi + P^T \Omega^{-1} Q$$

Our Views

We are combining implied excess returns with our own views on excess returns.

- A weighted average.

What are the weights?

- How confident the investor about his/her views relative to the implied excess returns.

Understanding the formula

$$E(r) = \left[(\tau S)^{-1} + P^T \Omega^{-1} P\right]^{-1} \left[(\tau S)^{-1} \Pi + P^T \Omega^{-1} Q\right]$$

The second part of the formula combines implied excess returns with our views about different assets.

What does the first part do?

- The first term is there to ensure that the weights assigned to implied excess returns and our views add up to 1.

The formula is just a weighted average!

An example

Consider 1 stock, AMZN.

- The implied excess returns are 0.74% per month and the variance is 2.015%^{2}.
- We predict excess returns of 2% per month. The uncertainty surrounding this view is reflected by a variance of 0.50%^{2}.
- Assume $\tau = 1$. P = 1

A more complicated example

Stock	Historical Excess Returns	CAPM Excess Returns	Implied Excess Returns
INTC	-0.0035	0.0135	0.0092
AEP	0.0026	0.0064	0.0032
AMZN	0.0326	0.0111	0.0074
MRK	0.0002	0.0057	0.0059
XOM	0.0111	0.0052	0.0064

Incorporating Investor Views

Views:

1) Analysts tell you that AEP has found a way to store electricity. Based on this breakthrough, they expect AEP to outperform XOM by 1% per month.
2) Given the current economic conditions we think that INTC will outperform AMZN by 1.75% per month.

The views expressed above are relative views of assets.

- Relative views are common in reality.
- Absolute views, such as AEP having returns of 2% per month, are much less common.

Views vs Implied Excess Returns

View 1) AEP outperforms XOM by 1% per month.

- The difference in implied excess returns is -0.32% per month.
- We would expect that incorporating our view will lead to an increase in our holdings of AEP and a decrease in XOM.

View 2) INTC will outperform AMZN by 1.75% per month.

- The difference in implied excess returns is 0.18% per month.
- We would expect that incorporating our view will increase our holdings of INTC and reduce our holdings of AMZN.

Incorporating Our Views

To link our views to implied excess returns we need a link matrix, P.

Matrix P is constructed in the following way:

Each row represents a view, each column represents a company:

$$P = \begin{array}{c} \\ \text{View 1} \\ \text{View 2} \end{array} \begin{array}{c} \begin{array}{ccccc} \text{INTC} & \text{AEP} & \text{AMZN} & \text{MRK} & \text{XOM} \end{array} \\ \begin{bmatrix} & & & & \\ & & & & \end{bmatrix} \end{array}$$

The View Vector and Uncertainty

What do Q and Ω look like?

$$Q + \varepsilon = \begin{bmatrix} Q_1 \\ \vdots \\ Q_k \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_k \end{bmatrix} \rightarrow Q + \varepsilon = \begin{bmatrix} 0.01 \\ 0.0175 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix}$$

where

$$\begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_k \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \omega_{11} & \cdots & \omega_{1k} \\ \vdots & \ddots & \vdots \\ \omega_{k1} & \cdots & \omega_{kk} \end{bmatrix} \right)$$

Ω

Calculating Ω

There is no best way to calculate Ω. It will depend on how confident you are of your predictions.

Black and Litterman recommend:

$$\Omega = \tau PSP^T$$

We have assumed the τ = 1 so we can ignore τ.

Incorporating Investor Views

Views:

1) Analysts tell you that AEP has found a way to store electricity. Based on this breakthrough, they expect AEP to outperform XOM by 1% per month.
2) Given the current economic conditions we think that INTC will outperform AMZN by 1.75% per month.

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- Relative views are common in reality.
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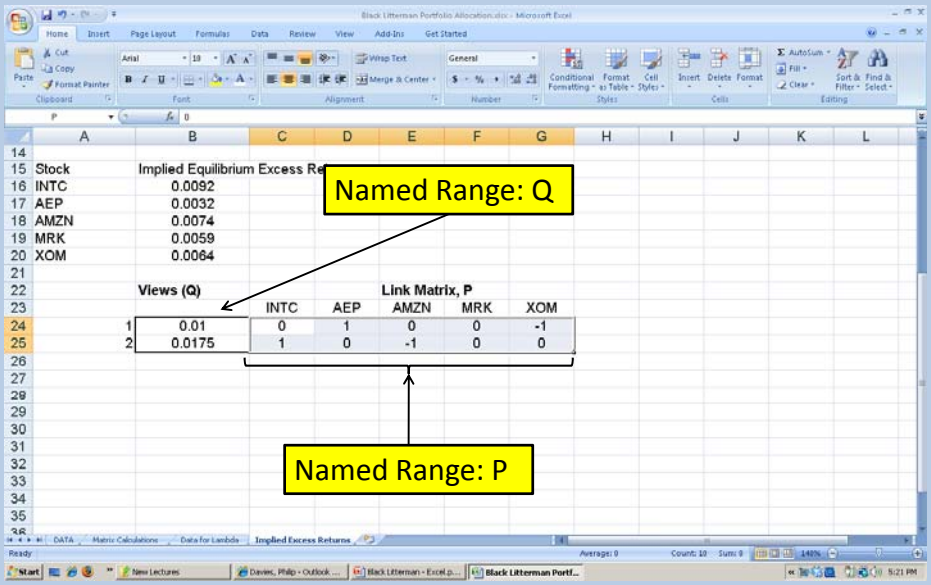
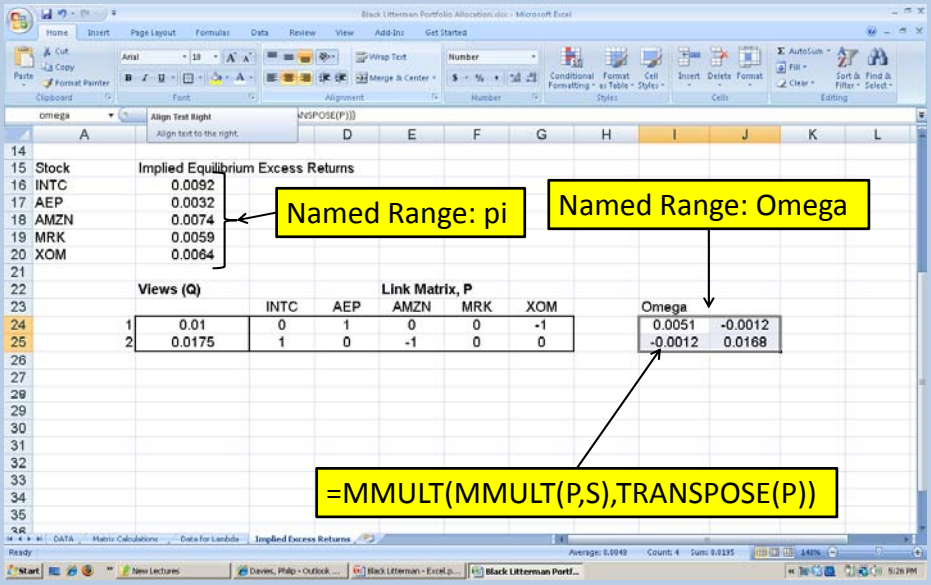
Views and Links, Q and P



Uncertainty about views



Stock	Implied Equilibrium Excess Returns
INTC	0.0092
AEP	0.0032
AMZN	0.0074
MRK	0.0059
XOM	0.0064

Views (Q)

1	0.01
2	0.0175

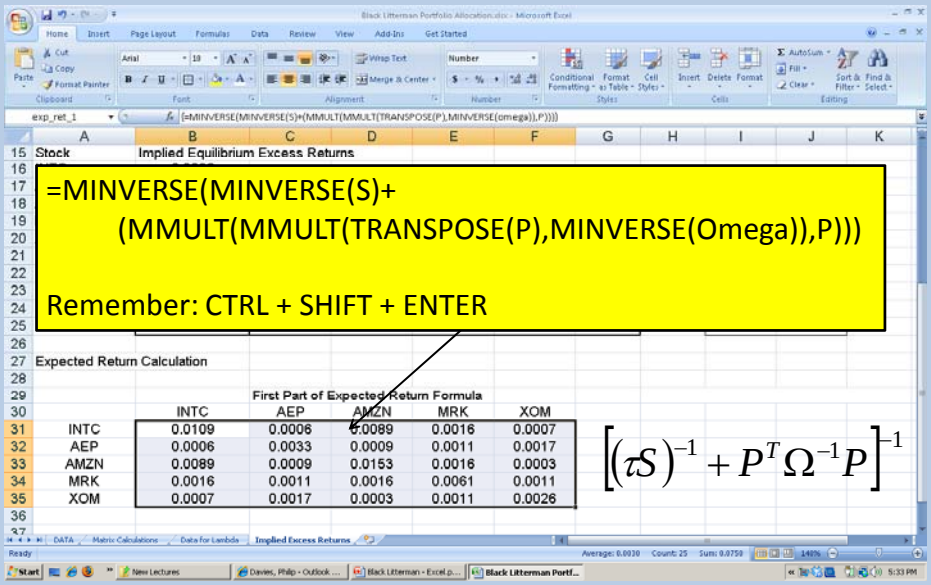
Link Matrix, P

	INTC	AEP	AMZN	MRK	XOM
1	0	1	0	0	-1
2	1	0	-1	0	0

Omega

0.0051	-0.0012
-0.0012	0.0168

Calculating Expected Excess Returns



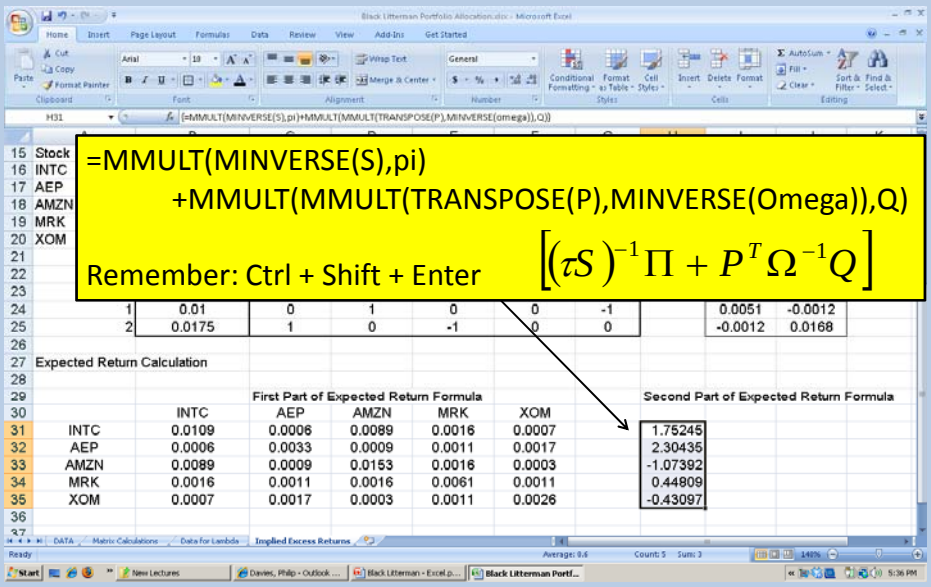
Expected Return Calculation

First Part of Expected Return Formula

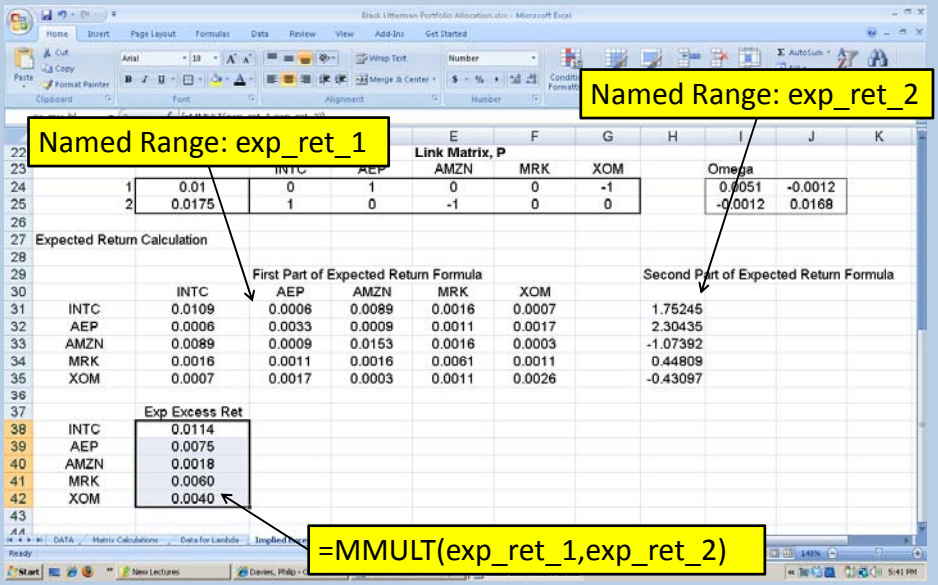
	INTC	AEP	AMZN	MRK	XOM
INTC	0.0109	0.0006	0.0089	0.0016	0.0007
AEP	0.0006	0.0033	0.0009	0.0011	0.0017
AMZN	0.0089	0.0009	0.0153	0.0016	0.0003
MRK	0.0016	0.0011	0.0016	0.0061	0.0011
XOM	0.0007	0.0017	0.0003	0.0011	0.0026

$$\left[(\tau S)^{-1} + P^T \Omega^{-1} P\right]^{-1}$$

Calculating Expected Excess Returns



Calculating Expected Excess Returns

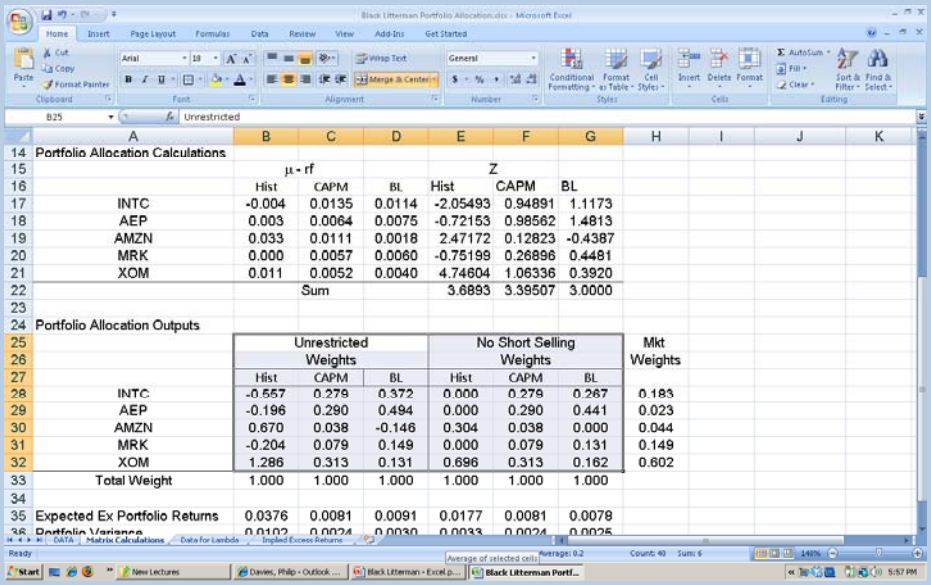


Do the Expected Returns Make Sense?

	Imp Ex Returns	Exp Ex Returns
INTC	0.0092	0.0114
AEP	0.0032	0.0075
AMZN	0.0074	0.0018
MRK	0.0059	0.0060
XOM	0.0064	0.0040

1) We think AEP will outperform XOM by 1% per month.
   - The expected excess return for AEP has gone up from 0.32% to 0.75%, while XOM returns have fallen to 0.40% per month.
2) We think INTC will outperform AMZN by 1.75% per month.
   - INTCs expected excess returns have increased to 1.14%, while AMZN expected excess returns have decreased to 0.18%.

Portfolio Weights



	A	B	C	D	E	F	G	H	I	J	K
14	Portfolio Allocation Calculations										
15		μ - rf			Z						
16		Hist	CAPM	BL	Hist	CAPM	BL				
17	INTC	-0.004	0.0135	0.0114	-2.05493	0.94891	1.1173				
18	AEP	0.003	0.0064	0.0075	-0.72153	0.98562	1.4813				
19	AMZN	0.033	0.0111	0.0018	2.47172	0.12823	-0.4387				
20	MRK	0.000	0.0057	0.0060	-0.75199	0.26896	0.4481				
21	XOM	0.011	0.0052	0.0040	4.74604	1.06336	0.3920				
22			Sum		3.6893	3.39507	3.0000				
23											
24	Portfolio Allocation Outputs										
25		Unrestricted Weights			No Short Selling Weights			Mkt Weights			
26											
27		Hist	CAPM	BL	Hist	CAPM	BL				
28	INTC	-0.557	0.279	0.372	0.000	0.279	0.267	0.183			
29	AEP	-0.196	0.290	0.494	0.000	0.290	0.441	0.023			
30	AMZN	0.670	0.038	-0.146	0.304	0.038	0.000	0.044			
31	MRK	-0.204	0.079	0.149	0.000	0.079	0.131	0.149			
32	XOM	1.286	0.313	0.131	0.696	0.313	0.162	0.602			
33	Total Weight	1.000	1.000	1.000	1.000	1.000	1.000				
34											
35	Expected Ex Portfolio Returns	0.0376	0.0081	0.0091	0.0177	0.0081	0.0078				
36	Portfolio Variance	0.0102	0.0024	0.0030	0.0033	0.0024	0.0025				

Do the results make sense?

	Market Weights	New Portfolio Weights
INTC	0.18	0.372
AEP	0.02	0.494
AMZN	0.04	-0.146
MRK	0.15	0.149
XOM	0.60	0.131
Total Weight	1.00	1.00

1) We think AEP will outperform XOM by 1% per month.
   - There is a large increase in our investment in AEP, at the expense of investment in XOM.
2) We think INTC will outperform AMZN by 1.75% per month.
   - There is a large increase in our holdings of INTC, while we actually sell AMZN short.