The Capital Asset Pricing Model

Understanding Risk

The total risk of a stock (or any asset) is measured by the variance or standard deviation of the stock's returns.

Total risk can be divided into 2 categories:

- Market-wide risk (Systematic risk)

- Firm-specific risk (Unique risk)

Which Risks Matter?

Scott has $5,000 to invest. He chooses to invest only in Merck.

What are the risks?

- Market:

  Interest rates may rise or fall.

  Unemployment may increase or decrease.

- Firm Specific:

  Other companies may develop more effective drugs.

  Merck may face patient lawsuits.

  US Food and Drug Administration may block new products.

Which Risks Matter?

Rose invests in 30 stocks across different industries. Her portfolio has the same level of market risk as Scott's investment in Merck.

What are the risks?

- Market:

  Interest rates may rise or fall.

  Unemployment may increase or decrease.

- Firm Specific:

  There is virtually no firm-specific risk.

Risk and Expected Returns

Scott invested only in Merck.

- He has market and firm-specific risk.

Rose invested in 30 stocks.

- She only has market risk.

Who should earn higher expected returns?

Can firm-specific risk be eliminated?

Yes. Invest in multiple stocks across different industries.

Should Scott be compensated for taking a risk that he could easily eliminate?

Can market risk be eliminated?

All stocks are exposed to market risk.

The extent to which stocks are exposed to market risk is likely to be a function of:

- Industry.
- Firm size.
- Firm leverage.

Since market risk affects all the firms in the economy, there is no opportunity to “diversify” its effect.

Market risk is the only risk that matters!

The Capital Asset Pricing Model (CAPM)

Sharpe (1964), Lintner (1965), and Black (1972) extend the work of Markowitz.

They show that the expected return on any stock reflects only risk that cannot be diversified away.

- Market risk.

The market risk of a stock is measured by the extent to which its return co-moves with the return on the market.

For any stock *i*, expected return $E(r_i)$ satisfies,

$$E(r_i) = r_f + \beta_i \left(E(r_m) - r_f \right)$$

$$\text{where } \beta_i = \frac{\operatorname{cov}(r_i, r_m)}{\operatorname{var}(r_m)} = \frac{\sigma_{im}}{\sigma_m^2},$$

The Capital Asset Pricing Model

β_i is the market risk of stock *i*.

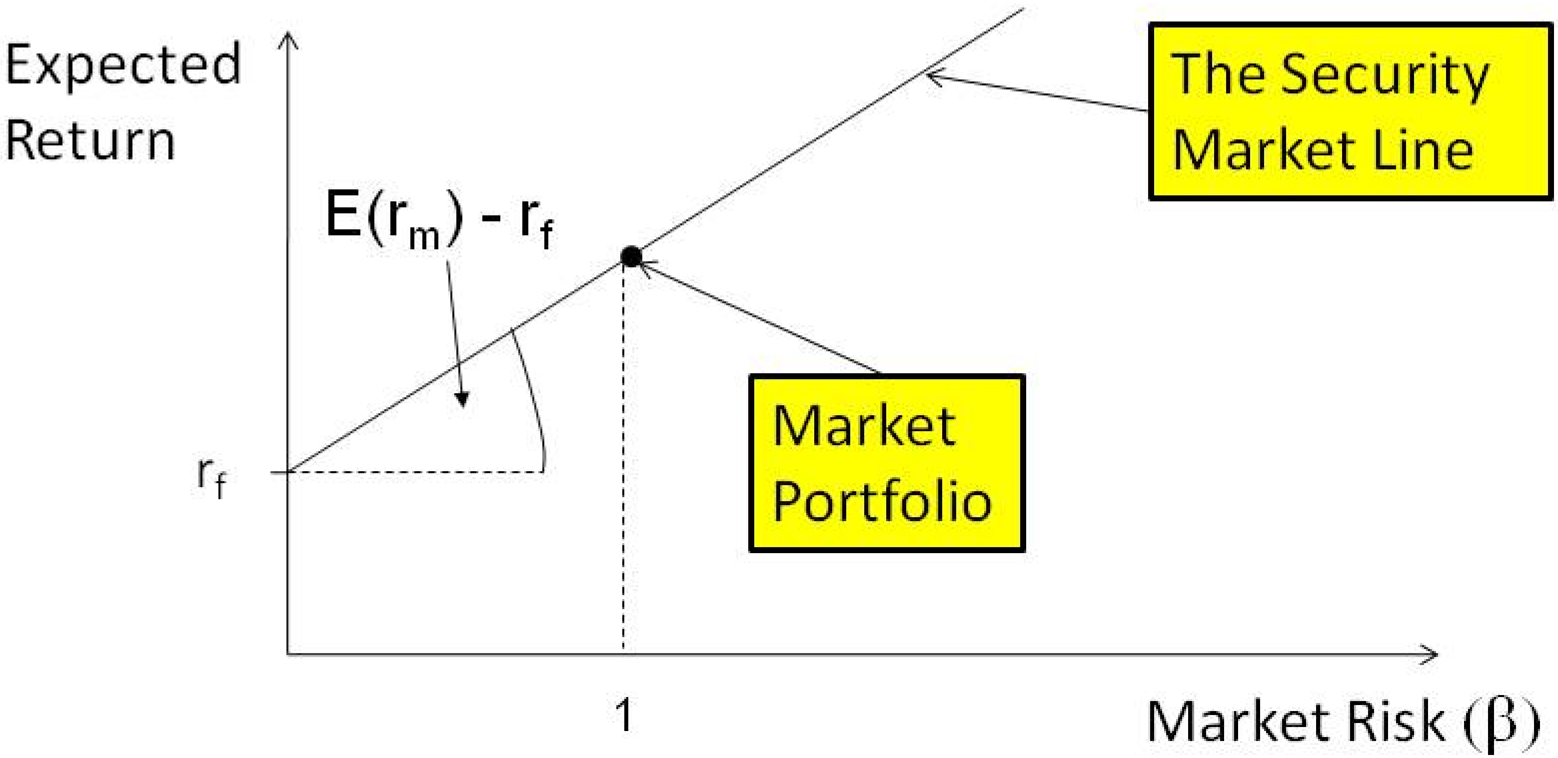
- If there is a 1% increase in the stock market, β_i tells us how much stock *i* is likely to increase by.

$E(r_m)$-r_f is the expected market risk premium.

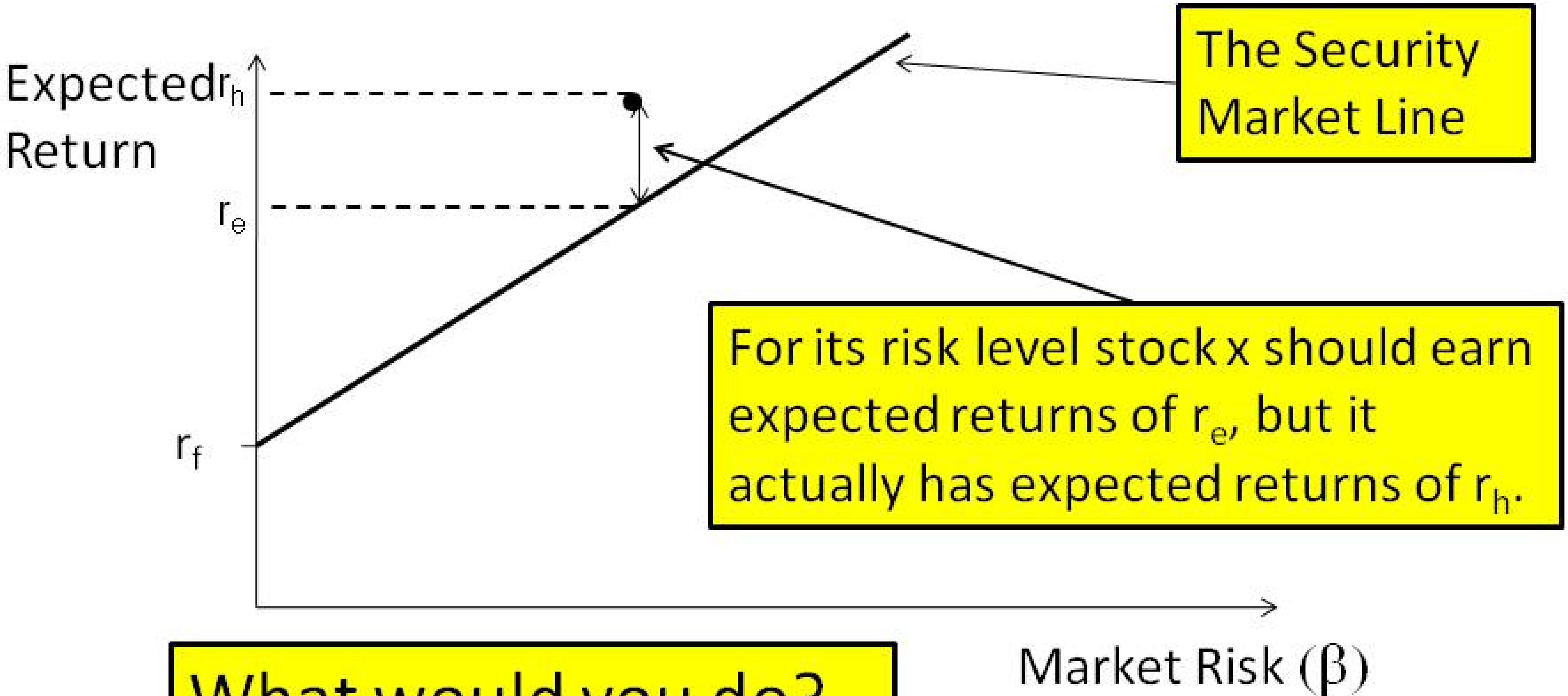
- The reward for bearing market risk.

The Security Market Line

The CAPM predicts a linear relationship between risk, as measured by β, and expected returns.



Equilibrium and the Security Market Line



What would you do?

In equilibrium all stocks should lie on the security market line.

Summary

At the firm level the CAPM implies that there is linear relationship between the expected excess return of a stock and the stock's market risk, β .

$$E(r_i) = r_f + \beta_i \left(E(r_m) - r_f \right)$$

β measures the extent to which a stock moves with the market.

Investors are only rewarded for taking market risk.

- Investors are not be rewarded for firm-specific risk

The CAPM: A Review

- Investors are only rewarded for bearing market risk.

$$E(r_i) = r_f + \underbrace{\beta_i \big(\overbrace{E(r_m) - r_f}^{\text{Market Risk Premium}}\big)}_{\text{Stock Risk Premium}}$$

- β measures the extent to which a stock moves with the market, ceteris paribus.
- - Market risk.

How do we measure market risk?

- If the stock market has a return of 1%, what do we expect to happen to Merck?

- We can answer this question using regression analysis.
  - \- Regression analysis examines the relation of a dependent variable to specified independent variables.

Regression Analysis: A Review

$$Y = \alpha + \beta X + \varepsilon$$

Y is the dependent variable

- This is the variable we wish to explain/predict.

X is the independent variable

- This is the variable (or variables) we are using to explain Y.

Key Assumptions

– There is a linear relation between X and Y.
– The residual/error terms have constant variance.
– The error terms are independent.
– The error terms are normally distributed.

Estimating market risk for stock *i*

The Market Model

$$r_{i,t} = \alpha + \beta r_{market,t} + \varepsilon_t$$

In reality, no stock return will be perfectly explained by market returns. The error term captures all the firm-specific factors which affect stock returns

Interpretation

– β measures the expected effect of a 1% increase in the stock market on stock *i*.

Tests of the CAPM

- Test 1: Cross-sectional Test.
  - Given βs and average excess returns for stocks, the CAPM implies that there should be a positive relation between βs and average excess returns.
    - - The more market risk we take, the higher the returns we should earn.

$$\text{CAPM}: E(r_i) - r_f = \left(E(r_m) - r_f\right)\beta_i$$

Market Risk Premium should be positive.

Cross-sectional Test

- Run a regression of average excess returns (dependent variable) on β (independent variable).
  - \- If the CAPM holds what should we find?

$$\text{CAPM}: E(r_i) - r_f = \left(E(r_m) - r_f\right)\beta_i$$

Cross-sectional Test

- Run a regression of average excess returns (dependent variable) on β (independent variable).
  - - If the CAPM holds what should we find?

$$\text{CAPM}: E(r_i) - r_f = \left(E(r_m) - r_f\right)\beta_i$$

$$\text{Regression: } \overline{r_i - r_f} = \gamma_0 + \gamma_1 \beta_i + \varepsilon$$

$$\text{CAPM implies: } \gamma_0 = 0 \text{ and } \gamma_1 = E(r_m) - r_f$$

Hypothesis Testing

We want to test:

H0: Coefficient is not significantly different from 0.

H1: Coefficient is significantly different from 0.

Use two tailed T-test: $$t = \frac{\hat{\beta} - 0}{SE}$$

Statistical packages calculate the standard error (SE) for each coefficient.

For large samples (60 or more observations):

- Rough Benchmark: If absolute value of t > 1.96 then parameter is significantly different from 0 at a 95% confidence level.

Is the CAPM Dead?

- Fama and French (1992) conduct a cross-sectional test of the CAPM over the period 1940 – 1990.
  - They find no evidence of a positive relation between β and average returns.
  - Investors are not rewarded for taking market risk!
  - Firm size (mkt cap) is negatively associated with average returns.
  - Book-to-market equity is positively associated with average returns.

- Fama and French argue that size and book-to-market may proxy for some unobserved risk factors.

Is the CAPM Dead?

- There are considerable statistical difficulties implementing cross-sectional tests of the CAPM:
  - \- To test the CAPM estimated βs are regressed on average excess returns.

- Miller and Scholes (1972)
  - \- The major problem is the use of estimated βs.
  - \- Using estimated βs leads to a downward bias in the estimated risk premium (γ_1) and an upward bias in the intercept (γ_0).
    - \- This is consistent with Fama and French's (1992) findings.

Tests of the CAPM

- Test 2: A Time Series regression.
  - Run a regression for every stock or portfolio of stocks:

$$\text{CAPM}: E(r_i) - r_f = \beta_i \left(E(r_m) - r_f \right)$$

Tests of the CAPM

- Test 2: A Time Series regression.
  - Run a regression for every stock or portfolio of stocks:

$$\text{CAPM}: E(r_i) - r_f = \beta_i \left(E(r_m) - r_f \right)$$

$$\text{Regression}: r_{i,t} - r_{f,t} = \alpha_i + \beta_i \left(r_{m,t} - r_{f,t} \right) + \varepsilon_{i,t}$$

$$\text{CAPM implies: } \alpha_i = 0$$

Time Series Regressions

- Fama and French (1993) run the time series test of the CAPM for 25 portfolios over the period 1963 - 1990.
  - - The portfolios are formed based on firm size and book-to-market ratios.

- Fama and French show that significantly positive αs exist for portfolios of small firms with high book-to-market ratios.
  - - This is NOT consistent with the CAPM.

Is the CAPM Dead?

- Fama and French (1993) estimate the model using over 30 years of monthly data.
  - β is likely to change over time for a stock or portfolio.

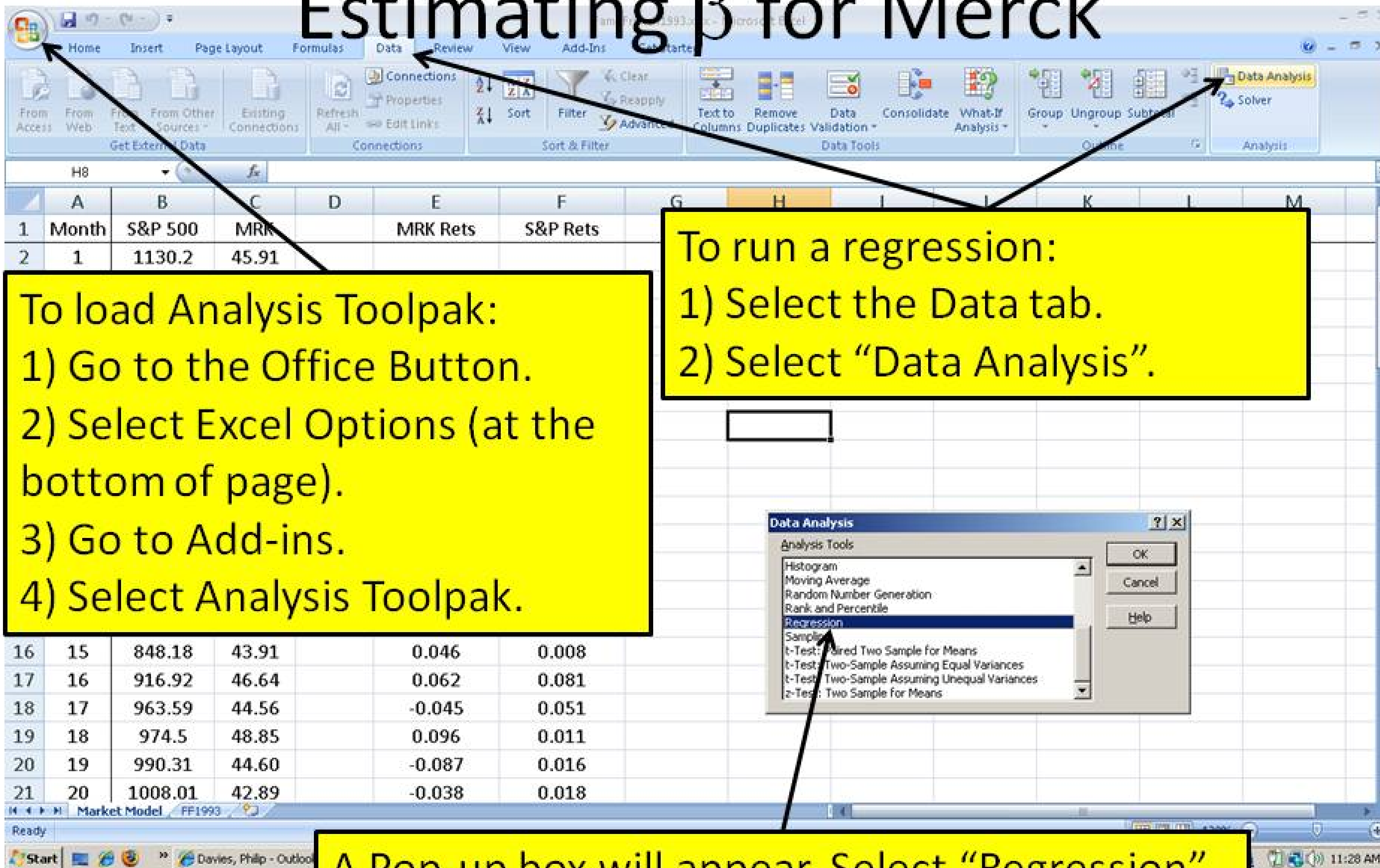
- Not allowing β to change leads to an upward bias in estimated αs.

- Ang and Chen (2006) show that when β is allowed to vary over time, there is little evidence that αs are significantly greater than zero.

Is the CAPM dead?

- Despite being an elegant theory the CAPM is very difficult to test.
- The evidence is mixed. Over the last 3 decades evidence has mounted against the CAPM.
- Recent evidence suggests that when the statistical difficulties are overcome, the data may be more consistent with the CAPM.
- If the CAPM does not hold:
  - - What should it be replaced with?
  - - How should we think about risk and return?

Estimating β for Merck



To load Analysis Toolpak:

1) Go to the Office Button.

2) Select Excel Options (at the bottom of page).

3) Go to Add-ins.

4) Select Analysis Toolpak.

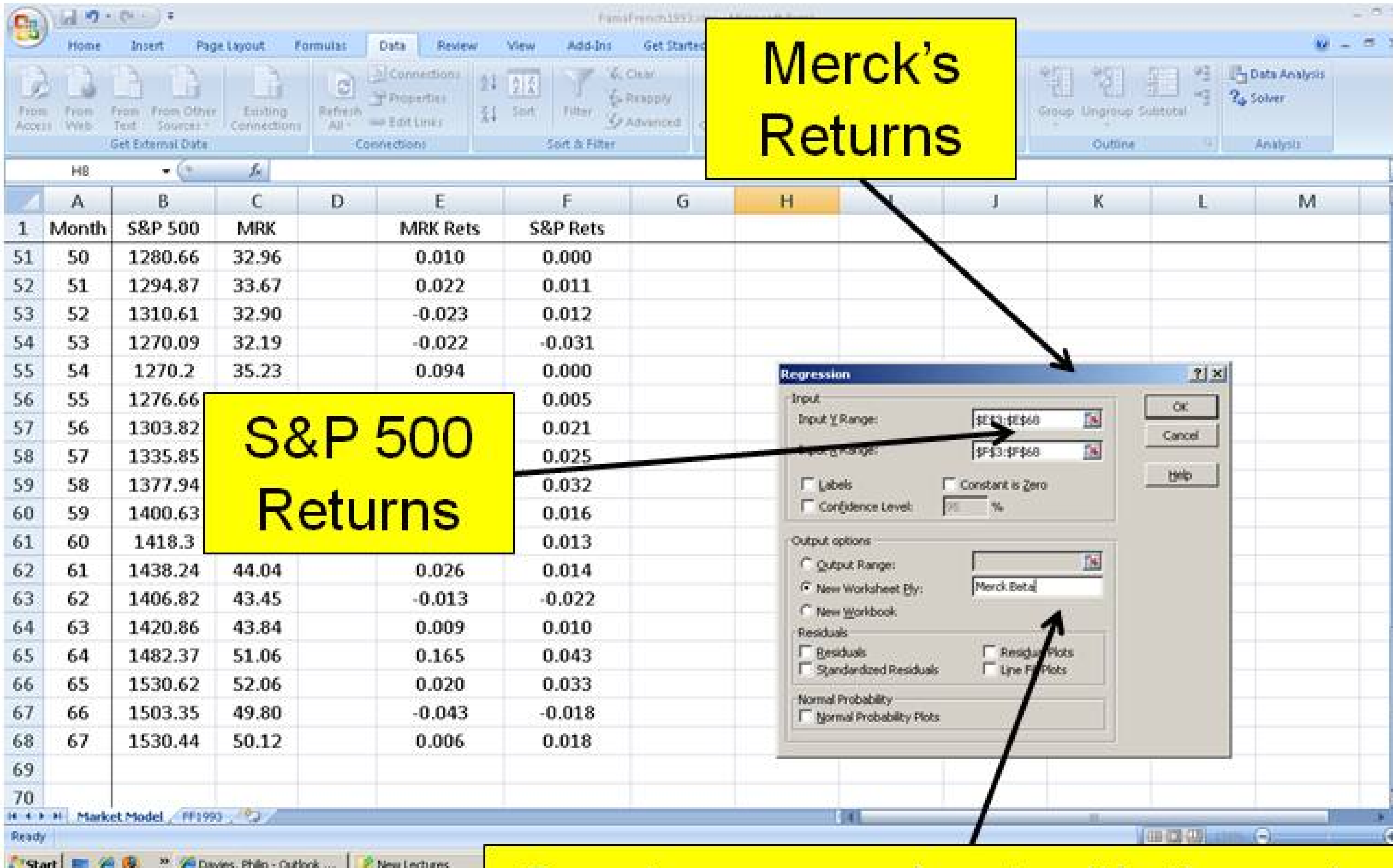
To run a regression:

1) Select the Data tab.

2) Select "Data Analysis".

A Pop-up box will appear. Select "Regression"

Estimating β for Merck



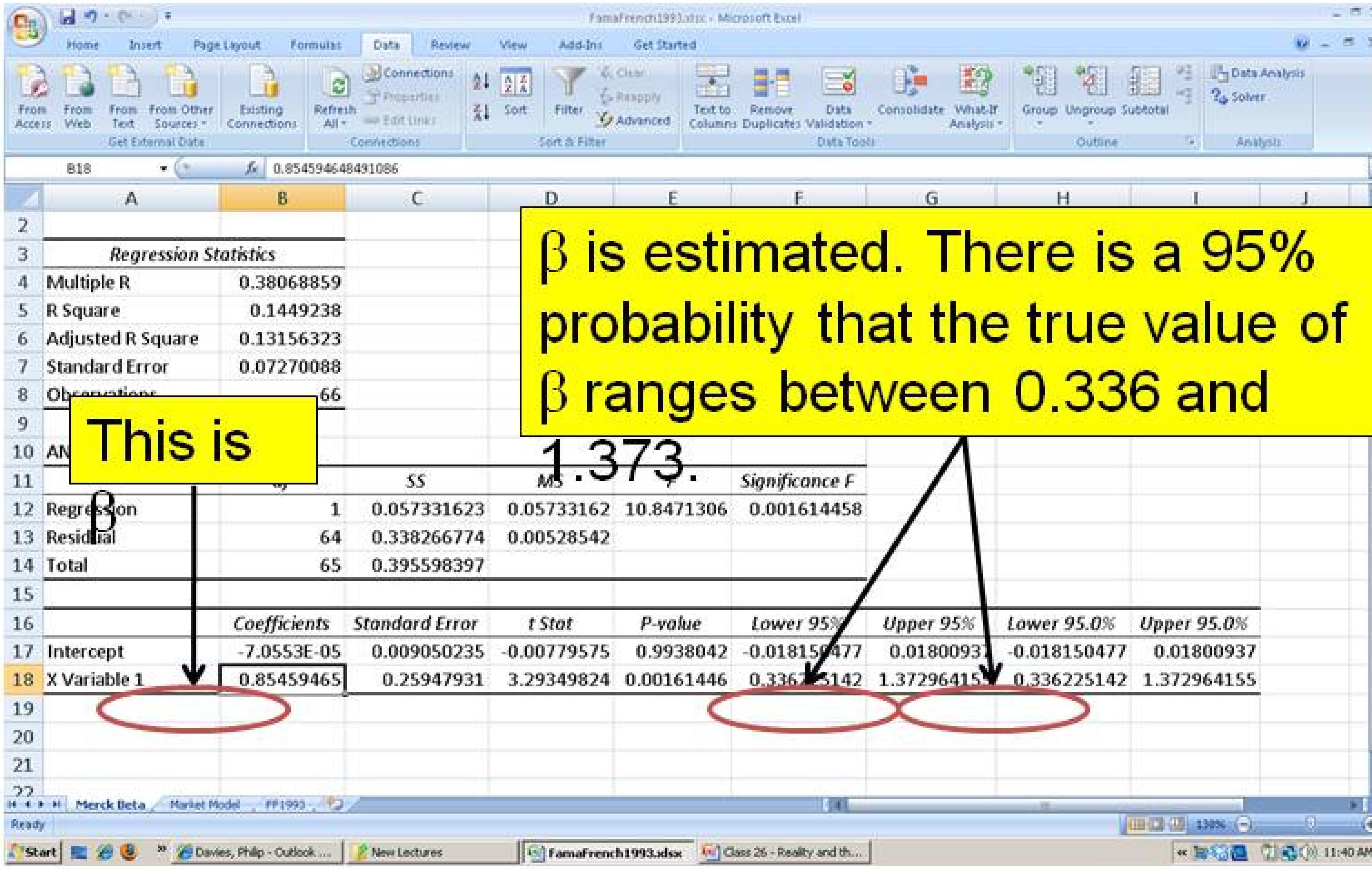
	Month	S&P 500	MRK		MRK Rets	S&P Rets
51	50	1280.66	32.96		0.010	0.000
52	51	1294.87	33.67		0.022	0.011
53	52	1310.61	32.90		-0.023	0.012
54	53	1270.09	32.19		-0.022	-0.031
55	54	1270.2	35.23		0.094	0.000
56	55	1276.66				0.005
57	56	1303.82				0.021
58	57	1335.85				0.025
59	58	1377.94				0.032
60	59	1400.63				0.016
61	60	1418.3				0.013
62	61	1438.24	44.04		0.026	0.014
63	62	1406.82	43.45		-0.013	-0.022
64	63	1420.86	43.84		0.009	0.010
65	64	1482.37	51.06		0.165	0.043
66	65	1530.62	52.06		0.020	0.033
67	66	1503.35	49.80		-0.043	-0.018
68	67	1530.44	50.12		0.006	0.018

Merck's Returns

S&P 500 Returns

Create a new sheet with the output

Estimating β for Merck



B18 f_x 0.854594648491086

Regression Statistics	
Multiple R	0.38068859
R Square	0.1449238
Adjusted R Square	0.13156323
Standard Error	0.07270088
Observations	66

ANOVA

	df	SS	MS	F	Significance F
Regression	1	0.057331623	0.05733162	10.8471306	0.001614458
Residual	64	0.338266774	0.00528542		
Total	65	0.395598397			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%	Lower 95.0%	Upper 95.0%
Intercept	-7.0553E-05	0.009050235	-0.00779575	0.9938042	-0.018150477	0.01800937	-0.018150477	0.01800937
X Variable 1	0.85459465	0.25947931	3.29349824	0.00161446	0.336225142	1.372964155	0.336225142	1.372964155

This is β

β is estimated. There is a 95% probability that the true value of β ranges between 0.336 and 1.373.

There is a lot of uncertainty about the true value of β for Merck.

Calculating Expected Returns

$$E(r_i) = r_f + \beta_i\left(E(r_m) - r_f\right)$$

– Assumptions:
  - - Annual Risk Free Rate is 4%.
  - - Annual Expected Market Risk Premium is 8%.

– The expected return for Merck is:
  - - $E(r_{Merck}) = 4 + 0.85 \times 8 = 10.8\%$ per year

– But the 95% confidence interval for Merck is a β ranging from 0.33 – 1.35.
  - - Merck's expected return could range from 6.6% per year to 15% per year!

Tests of the CAPM

- Test 2: A Time Series regression.
  - Run a regression for every stock or portfolio of stocks:

$$\text{CAPM} : E(r_i) - r_f = \beta_i \left(E(r_m) - r_f \right)$$

$$\text{Regression} : r_{i,t} - r_{f,t} = \alpha_i + \beta_i \left(r_{m,t} - r_{f,t} \right) + \varepsilon_t$$

$$\text{CAPM implies} : \alpha_i = 0$$

Excel: LINEST

- LINEST(known_y's,known_x's,const,stats)
  - This function will run a regression.
  - \- **known_y's**: the dependent variable.
  - \- **known_x's**: the independent variable(s).
  - \- **const** specifies whether you want a constant or not in the regression.
    - \- Leave it blank and a constant will be included.
  - \- **stats**: If you only want the coefficient estimates type 0. If you also want standard errors type 1.

- LINEST stores the output in column vectors.
  - \- There is 1 column for each independent variable and 1 column for the constant if a constant is specified.
  - \- The parameters are listed in **reverse** order - the constant is always the last column.

Excel: INDEX

- INDEX(array,row_num,column_num)
  - **array**: the matrix/vector of interest
  - **row_num**: the row number of the cell you are interested in.
  - **col_num**: the column number of the cell you are interested in.

- To access components of the column vectors created by LINEST we can use the index function.
  - \- Coefficient estimates are in the first row.
  - \- Standard errors are stored in the second row.

Excel: Example

- Suppose we regress excess stock returns for stock A on excess stock market returns.

- We want the value of the intercept (constant) and its standard error.

- =INDEX(LINEST(A1:A60,M1:M60,,1),1,2) returns the intercept.

- =INDEX(LINEST(A1:A60,M1:M60,,1),2,2) returns the standard error for the intercept.

Hypothesis Testing

We want to test:

H0: Intercept is not significantly different from 0.

H1: Intercept is significantly different from 0.

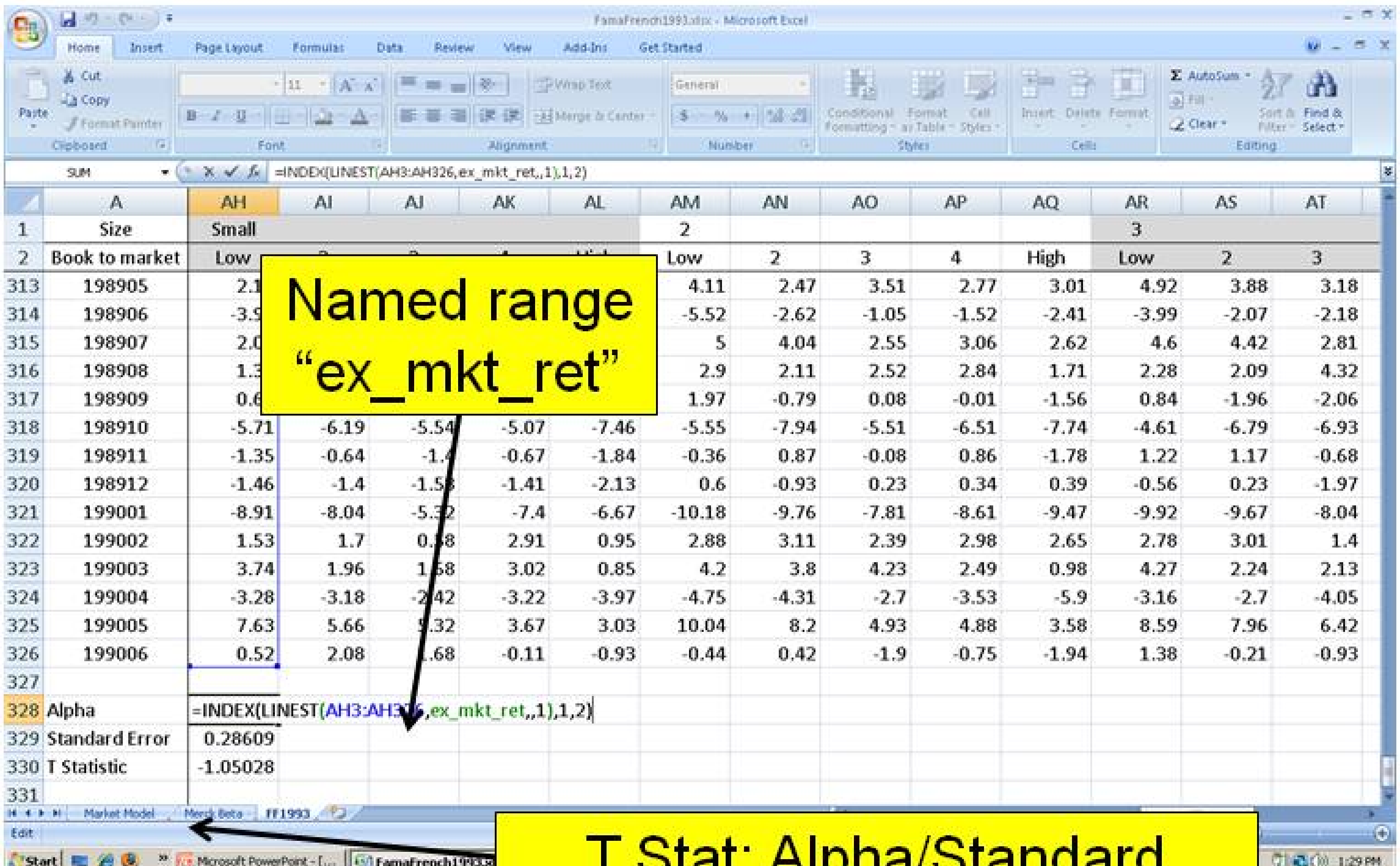
Use two tailed T-test: $$t = \frac{\hat{\alpha} - 0}{SE}$$

Statistical packages calculate the standard error (SE) for each coefficient.

For large samples (60 or more observations)

- Rough Benchmark: If absolute value of t > 1.96 then parameter is significantly different from 0 at a 95% confidence level.

Time Series Test of CAPM



=INDEX(LINEST(AH3:AH326,ex_mkt_ret,,1),1,2)

	A	AH	AI	AJ	AK	AL	AM	AN	AO	AP	AQ	AR	AS	AT
1	Size	Small					2					3		
2	Book to market	Low	[illegible]	[illegible]	[illegible]	[illegible]	Low	2	3	4	High	Low	2	3
313	198905	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	4.11	2.47	3.51	2.77	3.01	4.92	3.88	3.18
314	198906	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	-5.52	-2.62	-1.05	-1.52	-2.41	-3.99	-2.07	-2.18
315	198907	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	5	4.04	2.55	3.06	2.62	4.6	4.42	2.81
316	198908	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2.9	2.11	2.52	2.84	1.71	2.28	2.09	4.32
317	198909	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.97	-0.79	0.08	-0.01	-1.56	0.84	-1.96	-2.06
318	198910	-5.71	-6.19	-5.54	-5.07	-7.46	-5.55	-7.94	-5.51	-6.51	-7.74	-4.61	-6.79	-6.93
319	198911	-1.35	-0.64	[illegible]	-0.67	-1.84	-0.36	0.87	-0.08	0.86	-1.78	1.22	1.17	-0.68
320	198912	-1.46	-1.4	[illegible]	-1.41	-2.13	0.6	-0.93	0.23	0.34	0.39	-0.56	0.23	-1.97
321	199001	-8.91	-8.04	[illegible]	-7.4	-6.67	-10.18	-9.76	-7.81	-8.61	-9.47	-9.92	-9.67	-8.04
322	199002	1.53	1.7	[illegible]	2.91	0.95	2.88	3.11	2.39	2.98	2.65	2.78	3.01	1.4
323	199003	3.74	1.96	[illegible]	3.02	0.85	4.2	3.8	4.23	2.49	0.98	4.27	2.24	2.13
324	199004	-3.28	-3.18	-2.42	-3.22	-3.97	-4.75	-4.31	-2.7	-3.53	-5.9	-3.16	-2.7	-4.05
325	199005	7.63	5.66	[illegible]	3.67	3.03	10.04	8.2	4.93	4.88	3.58	8.59	7.96	6.42
326	199006	0.52	2.08	[illegible]	-0.11	-0.93	-0.44	0.42	-1.9	-0.75	-1.94	1.38	-0.21	-0.93
327														
328	Alpha	=INDEX(LINEST(AH3:AH326,ex_mkt_ret,,1),1,2)												
329	Standard Error	0.28609												
330	T Statistic	-1.05028												
331														

Named range “ex_mkt_ret”

T Stat: Alpha/Standard Error.

Testing the CAPM

Alphas

		Book to market				
		Low	2	3	4	High
Size	Small	-0.258	0.231	0.284	0.504	0.649
	2	-0.173	0.216	0.453	0.542	0.639
	3	-0.131	0.252	0.274	0.477	0.541
	4	-0.076	-0.074	0.259	0.383	0.500
	Large	-0.073	-0.035	0.021	0.196	0.226

T Stats

		Book to market				
		Low	2	3	4	High
Size	Small	-1.095	1.115	1.504	2.732	3.112
	2	-0.944	1.378	3.078	3.879	3.676
	3	-0.930	2.104	2.238	4.058	3.322
	4	-0.713	-0.787	2.639	3.228	3.234
	Large	-0.749	-0.445	0.209	1.762	1.508

You think that it is possible to outperform the CAPM by forming a portfolio which goes long stocks with high information risk and sells short stocks with low information risk. Information risk relates to the transparency of the company and its public disclosures.

Discuss: How would you test your theory?

You run a time-series test of the CAPM to assess whether the information risk strategy earns alpha. The results are reported below with standard errors in parentheses (1% = 0.01):

Coefficient	Results
Intercept	0.007 (0.003)
Excess Market Ret	0.400 (0.138)

The information risk strategy earns significant alpha.

A. True

B. False