Multifactor Models

Is the CAPM Dead?

- Fama and French (1992)
- - No evidence of a positive relation between β and average returns.
- - Firm size and the ratio of book value to market value are strongly associated with average returns:.

- Fama and French argue that size and book-to-market may proxy for unobserved risk factors.

The Fama-French 3 Factor Model

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i\left(r_{m,t} - r_{f,t}\right) + s_i r_{smb,t} + h_i r_{hml,t} + \varepsilon_t$$

- Rather than use the CAPM, Fama and French (1993) argue that the 3 factor model (above) is better able to explain stock returns.

- - SMB stands for **s**mall **m**inus **b**ig. The risk factor is constructed by calculating the returns of small stocks, and subtracting the returns of large stocks.

- - HML stands for **h**igh **m**inus **l**ow. The factor is constructed by calculating the returns of high book-to-market equity (value) stocks, and subtracting the returns of low book-to-market (growth) stocks.

The Fama-French 3 Factor Model

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i\left(r_{m,t} - r_{f,t}\right) + s_i r_{smb,t} + h_i r_{hml,t} + \varepsilon_t$$

- Fama and French (1993) show that their proposed model does have greater explanatory power than the CAPM in a time series test:
  - - Use 25 portfolios formed based on firm size and book-to-market equity.
  - - Many αs are significantly different from zero based on the CAPM .
  - - The αs for 25 portfolios are not significantly different from zero based on the 3 factor model.

The Fama-French 5 Factor Model

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i \left(r_{m,t} - r_{f,t} \right) + s_i r_{smb,t} + h_i r_{hml,t} + \gamma_i r_{RMW,t} + c_i r_{CMA,t} + \varepsilon_t$$

- Fama and French (2015) add profitability, and investment factors to their three factor model.

- RMW (Robust Minus Weak) is the average return of the robust operating profitability portfolios minus the average return of weak operating profitability portfolios.

- CMA (Conservative Minus Aggressive) is the average return of the conservative investment portfolios minus the average return of the aggressive investment portfolios.

Chinese 5 factors

- http://sf.cufe.edu.cn/info/1198/7027.htm
  数据来自中央财经大学中国资产管理研究中心

- **Description of Fama/French 5 Factors**
- http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/f-f_5_factors_2x3.html

Problems

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i\left(r_{m,t} - r_{f,t}\right) + s_i r_{smb,t} + h_i r_{hml,t} + \varepsilon_t$$

- SMB and HML are not likely candidates to proxy for macroeconomic risk factors.

- Fama and French justify their model on empirical grounds. It works across multiple time periods and countries.
  - - We do NOT know why!

The Carhart 4 Factor Model

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i \left(r_{m,t} - r_{f,t} \right) + s_i r_{smb,t} + h_i r_{hml,t} + m_i r_{mom,t} + \varepsilon_t$$

- Momentum
- MOM stands for momentum. The risk factor is constructed by calculating the returns of past winners, and subtracting the returns of past losers.
  - - Past winners: Stocks with high returns over last 6–12 months.
  - - Past losers: Stocks with low returns over last 6–12 months.

The Carhart 4 Factor Model

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i \left(r_{m,t} - r_{f,t} \right) + s_i r_{smb,t} + h_i r_{hml,t} + m_i r_{mom,t} + \varepsilon_t$$

- Carhart (1997) added a momentum factor to help explain returns on mutual funds.
  - - Commonly used when assessing the performance of stocks or funds.
- But:
- No economic justification for adding a momentum factor.

Fama-French 3 Factor Model

- Time Series Test
  - - Run a regression for each stock or portfolio of stocks:

$$\text{FF3F}: E(r_i) - r_f = \beta_{capm,i}\left(E(r_m) - r_f\right) + \beta_{SMB,i}E(r_{SMB}) + \beta_{HML,i}E(r_{HML})$$

$$\text{Regression}: r_{i,t} - r_{f,t} = \alpha_i + \beta_i\left(r_{m,t} - r_{f,t}\right) + \beta_{SMB,i}r_{SMB} + \beta_{HML,i}r_{HML} + \varepsilon_t$$

$$\text{Test}: \alpha_i = 0$$

Carhart 4 Factor Model

- Time Series Test
  - - Run a regression for each stock or portfolio of stocks:

$$\text{Carhart}: E(r_i) - r_f = \beta_{capm,i}\left(E(r_m) - r_f\right) + \beta_{SMB,i}E(r_{SMB}) + \beta_{HML,i}E(r_{HML}) + \beta_{MOM,i}E(r_{MOM})$$

$$\text{Regression}: r_{i,t} - r_{f,t} = \alpha_i + \beta_i\left(r_{m,t} - r_{f,t}\right) + \beta_{SMB,i}r_{SMB} + \beta_{HML,i}r_{HML} + \beta_{MOM,i}r_{MOM} + \varepsilon_t$$

$$\text{Test}: \alpha_i = 0$$

Excel: LINEST

- LINEST(known_y's,known_x's,const,stats)
  - This function will run a regression.
  - - **known_y's**: the dependent variable.
  - - **known_x's**: the independent variable(s).
  - - **const** specifies whether you want a constant or not in the regression.
  - Leave it blank and a constant will be included.
  - - **stats**: If you only want the coefficient estimates type 0. If you also want standard errors type 1.
- LINEST stores the output in column vectors.
  - - There is 1 column for each independent variable and 1 column for the constant if a constant is specified.
  - - The parameters are listed in **reverse** order - the constant is always the last column.

Excel: INDEX

- INDEX(array,row_num,column_num)
  - **array**: the matrix/vector of interest
  - **row_num**: the row number of the cell you are interested in.
  - **col_num**: the column number of the cell you are interested in.

- To access components of the column vectors created by LINEST we can use the index function.
  - - Coefficient estimates are in the first row.
  - - Standard errors are stored in the second row.

Testing the 3 Factor Model

- Suppose we regress excess stock returns for stock A on excess stock market returns and the returns on SMB and HML.

- We want the value of the intercept (constant) and its standard error.

- =INDEX(LINEST(A1:A60,M1:M60,,1),1,4) returns the intercept.

- =INDEX(LINEST(A1:A60,M1:M60,,1),2,4) returns the standard error for the intercept.

Hypothesis Testing

We want to test:

H0: Intercept is not significantly different from 0.

H1: Intercept is significantly different from 0.

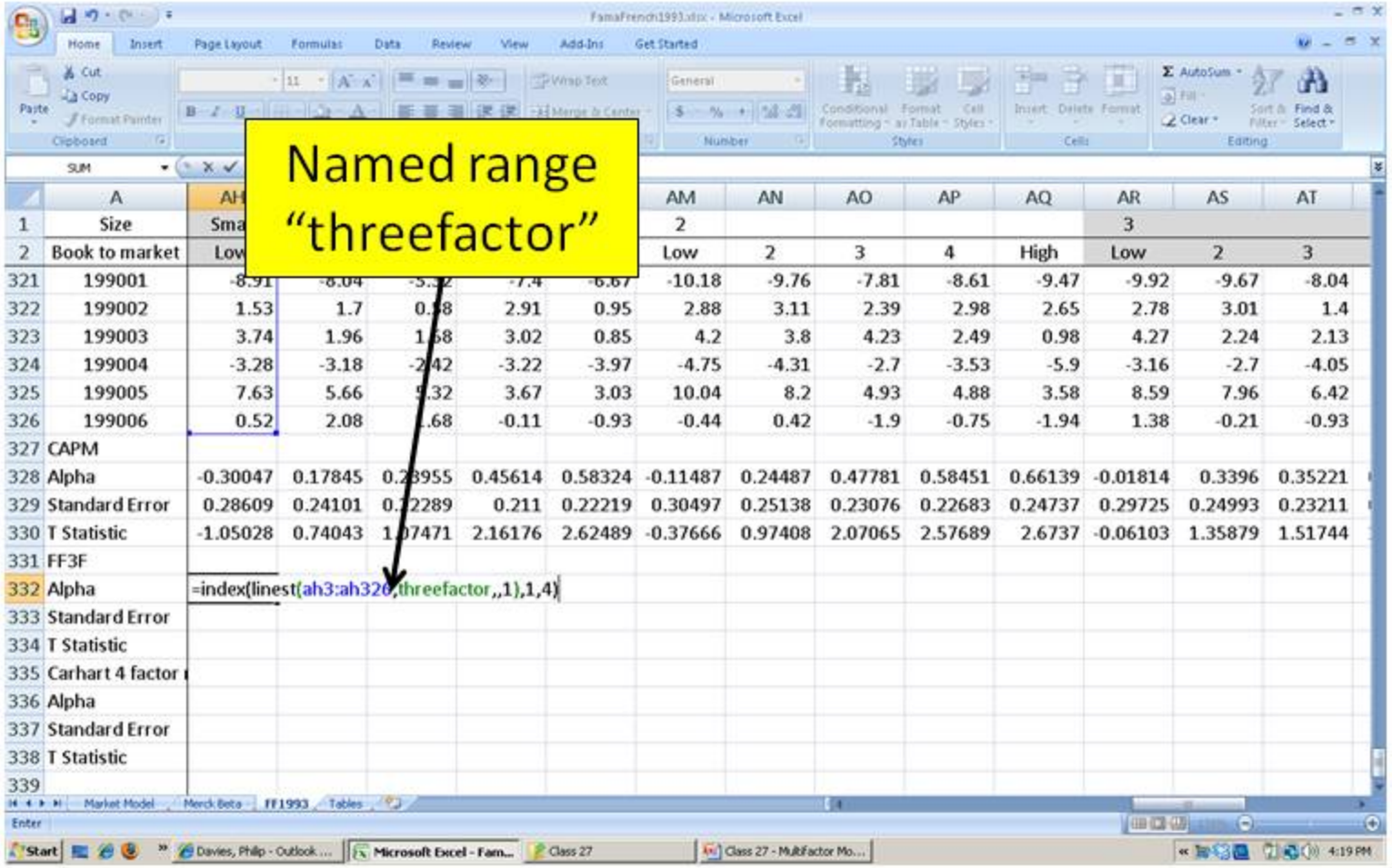
Use two tailed T-test: $$t = \frac{\hat{\alpha} - 0}{SE}$$

Statistical packages calculate the standard error (SE) for each coefficient.

For large samples (60 or more observations)

- Rough Benchmark: If absolute value of t > 2 then parameter is significantly different from 0 at a 95% confidence level.

Time Series Test of 3 Factor Model



	A	AH					AM	AN	AO	AP	AQ	AR	AS	AT
1	Size	Sma					2					3		
2	Book to market	Low					Low	2	3	4	High	Low	2	3
321	199001	-8.91	-8.04	-5.32	-7.4	-6.67	-10.18	-9.76	-7.81	-8.61	-9.47	-9.92	-9.67	-8.04
322	199002	1.53	1.7	0.38	2.91	0.95	2.88	3.11	2.39	2.98	2.65	2.78	3.01	1.4
323	199003	3.74	1.96	1.68	3.02	0.85	4.2	3.8	4.23	2.49	0.98	4.27	2.24	2.13
324	199004	-3.28	-3.18	-2.42	-3.22	-3.97	-4.75	-4.31	-2.7	-3.53	-5.9	-3.16	-2.7	-4.05
325	199005	7.63	5.66	5.32	3.67	3.03	10.04	8.2	4.93	4.88	3.58	8.59	7.96	6.42
326	199006	0.52	2.08	1.68	-0.11	-0.93	-0.44	0.42	-1.9	-0.75	-1.94	1.38	-0.21	-0.93
327	CAPM													
328	Alpha	-0.30047	0.17845	0.23955	0.45614	0.58324	-0.11487	0.24487	0.47781	0.58451	0.66139	-0.01814	0.3396	0.35221
329	Standard Error	0.28609	0.24101	0.22289	0.211	0.22219	0.30497	0.25138	0.23076	0.22683	0.24737	0.29725	0.24993	0.23211
330	T Statistic	-1.05028	0.74043	1.07471	2.16176	2.62489	-0.37666	0.97408	2.07065	2.57689	2.6737	-0.06103	1.35879	1.51744
331	FF3F													
332	Alpha	=index(linest(ah3:ah326,threefactor,,1),1,4)												
333	Standard Error													
334	T Statistic													
335	Carhart 4 factor													
336	Alpha													
337	Standard Error													
338	T Statistic													
339														

Testing the 3 Factor Model

Alphas

		Book to market				
		Low	2	3	4	High
Size	Small	-0.380	-0.065	-0.084	0.083	0.088
	2	-0.113	0.023	0.152	0.153	0.101
	3	-0.047	0.102	-0.006	0.121	0.029
	4	0.101	-0.135	0.047	0.035	0.032
	Large	0.205	0.019	-0.029	-0.061	-0.173

T Stats

		Book to market				
		Low	2	3	4	High
Size	Small	-3.543	-0.820	-1.263	1.322	1.312
	2	-1.277	0.313	2.252	2.336	1.410
	3	-0.590	1.332	-0.081	1.714	0.342
	4	1.275	-1.531	0.566	0.410	0.299
	Large	2.856	0.247	-0.312	-0.779	-1.538

Testing the 4 Factor Model

Alphas

		Book to market				
		Low	2	3	4	High
Size	Small	-0.417	-0.027	-0.066	0.091	0.081
	2	-0.131	0.106	0.179	0.150	0.089
	3	-0.009	0.097	0.052	0.143	0.006
	4	0.114	-0.048	0.075	0.086	0.067
	Large	0.230	0.013	-0.098	-0.030	-0.190

T Stats

		Book to market				
		Low	2	3	4	High
Size	Small	-3.750	-0.331	-0.950	1.400	1.156
	2	-1.420	1.420	2.547	2.200	1.198
	3	-0.111	1.220	0.632	1.946	0.062
	4	1.392	-0.533	0.865	0.970	0.610
	Large	3.090	0.168	-1.008	-0.371	-1.626

Size and Value in China

- Journal of Financial Economics, 2019
- Liu, Jianan, Robert F. Stambaugh, and Yu Yuan

- We construct size and value factors in China. The size factor excludes the smallest 30% of firms, which are companies valued significantly as potential shells in reverse mergers that circumvent tight IPO constraints. The value factor is based on the earnings-price ratio, which subsumes the book-to-market ratio in capturing all Chinese value effects. Our three-factor model strongly dominates a model formed by just replicating the Fama and French (1993) procedure in China.

Size factor

- In China, the IPO market is strictly regulated, and a growing demand for public listing confronts the low processing capacity of the regulatory bureau to approve IPOs.

- As a consequence, private firms seek an alternative approach, a reverse merger, to become public in a timely manner. In a reverse merger, a private firm targets a publicly traded company, a so-called shell, and gains control rights by acquiring its shares. The shell then buys the private firm's assets in exchange for newly issued shares.

- The smallest listed firms are the most likely shells. In fact, 83% of the reverse mergers in China involve shells coming from the smallest 30% of stocks.

Value Factor

- The value effect in China is best captured by the earnings-price (EP) ratio, versus other valuation ratios.

- we run a horse race among all candidate valuation ratios, including EP, book-to-market (BM), asset-to-market, and cash-flow-to-price ratios.

- Relying on our result for China, we use EP to construct our value factor.

The three-factor model, CH-3

- For comparison, we construct an alternative three-factor model, FF-3, by simply replicating the Fama and French (1993) procedure. We find that CH-3 strongly dominates FF-3.

- We also investigate the ability of CH-3 to explain previously reported return anomalies in China. A total of ten anomalies are unexplained by the CAPM; CH-3 explains eight of them, while FF-3 explains three.

- Hou, Xue, and Zhang (2015) and Fama and French (2015) add two factors based on investment and profitability measures in their recently proposed factor models, Q-4 and FF- 5. Investment does not produce a significant CAPM alpha in China, and profitability is fully explained by CH-3. We find that a replication of FF-5 in China is dominated by CH-3.

The horse race test

Quantity	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Intercept	0.0149	0.0581	0.0571	0.0659	0.0629	0.0690	0.0564	0.0716	0.0728
	(1.94)	(3.32)	(3.19)	(3.90)	(3.74)	(4.03)	(3.19)	(4.40)	(4.39)
β	−0.0002		−0.0010	−0.0018	−0.0017	0.0002	−0.0010	−0.0002	−0.0004
	(−0.09)		(−0.37)	(−0.71)	(−0.67)	(0.07)	(−0.37)	(−0.06)	(−0.15)
log*ME*		−0.0049	−0.0046	−0.0046	−0.0048	−0.0068	−0.0047	−0.0066	−0.0064
		(−2.91)	(−2.69)	(−2.73)	(−3.00)	(−4.34)	(−2.80)	(−4.49)	(−4.40)
log*BM*				0.0057				0.0022	0.0035
				(3.21)				(1.31)	(1.76)
log*AM*					0.0045			0.0014	
					(3.03)			(0.99)	
EP^+						0.9503		0.7825	0.7960
						(4.88)		(4.38)	(5.06)
$D(EP<0)$						0.0006		−0.0005	−0.0001
						(0.31)		(−0.29)	(−0.04)
CP^+							0.0546	0.0181	
							(3.41)	(1.35)	
$D(CP<0)$							0.0019	0.0016	
							(3.11)	(2.37)	
R^2	0.0196	0.0277	0.0441	0.0652	0.0677	0.0615	0.0454	0.0832	0.0776

Abilities of models CH-3 and FF-3 to explain each other's size and value factors

Our size and value factors, denoted as *SMB (small-minus-big) and VMG (value-minus- growth), combine the returns on these six portfolios as follows:*

$$SMB = \frac{1}{3}(S/V + S/M + S/G) - \frac{1}{3}(B/V + B/M + B/G),$$

$$VMG = \frac{1}{2}(S/V + B/V) - \frac{1}{2}(S/G + B/G).$$

	Alphas with respect to:	
Factors	CH-3	FF-3
Panel A: Alpha (t-statistic)		
FFSMB	–0.04	–
	(–0.66)	–
FFHML	0.34	–
	(0.97)	–
SMB	–	0.47
	–	(7.03)
VMG	–	1.39
	–	(7.93)