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SHORT COMMUNICATIONS

Electromagnetic Field of a Dipole in an Anisotropic Medium

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Abstract—The harmonically varying field of point electric and magnetic dipoles in an anisotropic medium with an anisotropic axis is found for the first time. © *2005 Pleiades Publishing, Inc.*

In this work, we for the first time give a complete and, more importantly, exact solution to the Maxwell equations for the radiation of point electric and magnetic dipoles in a homogeneous anisotropic medium where the conductivity and permittivity along the anisotropy axis differ from those in the transverse directions. The solution is found for any orientation of the dipole relative to the anisotropy axis. When finding the electromagnetic field of the dipole, which varies by a harmonic law, we apply two approaches, one of which is suggested in [1]. Here, the basic Maxwell equations, where displacement currents are ignored, are reduced to equations for vector potential that allow determination of these currents. The second approach [2–4] consists in converting the basic set of Maxwell equations (reduced at any point of the domain) to an equation for the electric field strength and applying the Fourier transformation to this equation. In this case, finding of the transforms of the electric field components is reduced to solving a set of linear algebraic equations and the field components themselves can be determined by applying the inverse Fourier transformation to their transforms. However, the creators of such an approach, also ignoring displacement currents, restricted the analysis to finding the transforms of only two electric field components and one magnetic field component [3], not explaining how it was obtained. In this work, we for the first time derive an exact solution to the Maxwell equation for electric and magnetic dipoles in an anisotropic medium, which allows for displacement currents, contrary to works [1–4]. To solve the problem, we chose the second approach. First, it is more general and does not require the presence of the anisotropy axis. Second, studying the electromagnetic field near the dipole is unnecessary in this approach.

Let a homogeneous anisotropic medium contain a point dipole in which the density of extraneous electric, $\mathbf{j}^E$, and magnetic, $\mathbf{j}^H$, currents varies by a harmonic law, $\exp(-i\omega t)$. We choose a rectangular coordinate system with the origin at the site of the dipole and assume that the conductivity and permittivity components in the tangential x and z directions are the same (γ_t and ε_t, respectively) and the respective components in the normal y direction are γ_n and ε_n. Note at once that the choice of direction where the conductivity differs is of no significance, since an otherwise stated problem is reduced to that under consideration merely by coordinate transformation. Let the electromagnetic field of the dipole also vary by a harmonic law, $\exp(-i\omega t)$. Then, according to the Maxwell equations, the amplitudes of the electric, $\mathbf{E}$, and magnetic, $\mathbf{H}$, fields of the dipole satisfy the equation [5]

$$\mathrm{curl}\mathbf{H} = \hat{\sigma}E + \mathbf{j}^E - i\omega\hat{\varepsilon}\mathbf{E}, \tag{1}$$

$$\mathrm{curl}\mathbf{E} = i\omega\mu\mathbf{H} - \mathbf{j}^H. \tag{2}$$

Here, $\hat{\sigma}$, $\hat{\varepsilon}$, and μ are the conductivity tensor, permittivity tensor, and permeability of the medium, respectively. Matrices $\hat{\sigma}$ and $\hat{\varepsilon}$ have the diagonal form

$$(\sigma)_{11} = (\sigma)_{33} = \gamma_t, \quad (\sigma)_{22} = \gamma_n,$$
$$(\sigma)_{kl} = 0 \quad (k \neq l),$$

$$(\varepsilon)_{11} = (\varepsilon)_{33} = \varepsilon_t, \quad (\varepsilon)_{22} = \varepsilon_n,$$
$$(\varepsilon)_{kl} = 0 \quad (k \neq l).$$

Applying the curl operator to both sides of Eq. (2), we obtain

$$\mathrm{curl}\,\mathrm{curl}\mathbf{E} - i\omega\mu\hat{\sigma}\mathbf{E} - \omega^2\mu\hat{\varepsilon}\mathbf{E} = i\omega\mu\mathbf{j}^E - \mathrm{curl}\mathbf{j}^H. \tag{3}$$

The direct and inverse Fourier transformations of function $f(x, y, z)$ in coordinates x, y, and z look as follows:

$$f^+(\xi, \eta, m)$$
$$= \iiint_{-\infty}^{\infty} f(x, y, z)\exp(-i\xi x - i\eta y - imz)dxdydz,$$

$$f(x, y, z) = \frac{1}{(2\pi)^3}$$

$$\times \int\int\int_{-\infty}^{\infty} f^{+}(\xi, \eta, m)\exp(i\xi x + i\eta y + imz)d\xi d\eta dm.$$

Let us recast Eq. (3) componentwise by applying the Fourier transformation to both sides of the resulting set of equations. Eventually, we arrive at a set of linear algebraic equations for the electric field component transforms,

$$\begin{pmatrix} \eta^2 + m^2 + k_{\mathrm{t}}^2 & -\xi\eta & -\xi m \\ -\xi\eta & \xi^2 + m^2 + k_{\mathrm{n}}^2 & -\eta m \\ -\xi m & -\eta m & \xi^2 + \eta^2 + k_{\mathrm{t}}^2 \end{pmatrix} \begin{pmatrix} E_x^{+} \\ E_y^{+} \\ E_z^{+} \end{pmatrix} \quad (4)$$

$$= \begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix},$$

where

$$F_l = \int\int\int_{-\infty}^{\infty} (i\omega j_l^E - \mathrm{curl}_l \mathbf{j}^H)$$

$$\times \exp(-i\xi x - i\eta y - imz)dxdydz,$$

$$k_{\mathrm{t}}^2 = -i\omega\mu\gamma'_{\mathrm{t}}, \quad k_{\mathrm{n}}^2 = -i\omega\mu\gamma'_{\mathrm{n}},$$

$$\mathrm{Re}k_{\mathrm{t}} > 0, \quad \mathrm{Re}k_{\mathrm{n}} > 0,$$

$$\gamma'_{\mathrm{t}} = \gamma_{\mathrm{t}} - i\omega\varepsilon_{\mathrm{t}}, \quad \gamma'_{\mathrm{n}} = \gamma_{\mathrm{n}} - i\omega\varepsilon_{\mathrm{n}}.$$

Having found the electric field component transforms from a solution to set (4), we then find the desired field components by applying the inverse Fourier transformation to the transforms of these components. The magnetic field components can be derived from (2).

Let us write the electromagnetic field components when magnetic and electric dipoles are oriented along one of the coordinate axes. To do this, we introduce the following notation:

$$\lambda^2 = \gamma'_{\mathrm{t}}/\gamma'_{\mathrm{n}}, \quad r^2 = x^2 + z^2,$$

$$R^2 = r^2 + y^2, \quad \bar{R}^2 = r^2 + \lambda^2 y^2.$$

1. MAGNETIC DIPOLE

(i) Orientation along the *y* axis. In this case, only one component of the extraneous magnetic current is nonzero,

$$j_y^H = -i\omega\mu M\delta(x)\delta(y)\delta(z),$$

where M is the moment of the magnetic dipole.

The electromagnetic field components are given by

$$E_x = \frac{i\omega\mu Mz}{4\pi}\exp(-k_{\mathrm{t}}R)\left(\frac{k_{\mathrm{t}}}{R^2} + \frac{1}{R^3}\right),$$

$$E_y = 0,$$

$$E_z = -\frac{i\omega\mu Mx}{4\pi}\exp(-k_{\mathrm{t}}R)\left(\frac{k_{\mathrm{t}}}{R^2} + \frac{1}{R^3}\right),$$

$$H_x = \frac{Mxy}{4\pi}\exp(-k_{\mathrm{t}}R)\left(\frac{k_{\mathrm{t}}^2}{R^3} + \frac{3k_{\mathrm{t}}}{R^4} + \frac{3}{R^5}\right),$$

$$H_y = \frac{M}{4\pi}\exp(-k_{\mathrm{t}}R)\left[-\frac{k_{\mathrm{t}}^2}{R} - \frac{k_{\mathrm{t}}}{R^2} - \frac{1}{R^3}\right.$$

$$\left. + y^2\left(\frac{k_{\mathrm{t}}^2}{R^3} + \frac{3k_{\mathrm{t}}}{R^4} + \frac{3}{R^5}\right)\right],$$

$$H_z = \frac{Myz}{4\pi}\exp(-k_{\mathrm{t}}R)\left(\frac{k_{\mathrm{t}}^2}{R^3} + \frac{3k_{\mathrm{t}}}{R^4} + \frac{3}{R^5}\right).$$

(ii) Orientation along the *x* axis. Only one component of the extraneous magnetic current is nonzero,

$$j_x^H = -i\omega\mu M\delta(x)\delta(y)\delta(z).$$

The electromagnetic field components are given by

$$E_x = \frac{i\omega\mu Mxyz}{4\pi}\left\{\exp(-k_1 R)\left(\frac{k_{\mathrm{t}}}{r^2R^2} + \frac{1}{r^2R^3} + \frac{2}{r^4R}\right)\right.$$

$$\left. - \lambda\exp(-k_{\mathrm{n}}\bar{R})\left(\frac{k_{\mathrm{n}}}{r^2\bar{R}^2} + \frac{1}{r^2\bar{R}^3} + \frac{2}{r^4\bar{R}}\right)\right\},$$

$$E_y = \frac{i\omega\mu M\lambda z}{4\pi}\exp(-k_{\mathrm{n}}\bar{R})\left(\frac{k_{\mathrm{n}}}{\bar{R}^2} + \frac{1}{\bar{R}^3}\right),$$

$$E_z = -\frac{i\omega\mu My}{4\pi}\left\{\exp(-k_{\mathrm{t}}R)\left[\frac{k_{\mathrm{t}}}{R^2} + \frac{1}{r^2R} + \frac{1}{R^3}\right.\right.$$

$$\left.\left. - z^2\left(\frac{k_{\mathrm{t}}}{r^2R^2} + \frac{1}{r^2R^3} + \frac{2}{r^4R}\right)\right]\right\} - \frac{i\omega\mu My\lambda}{4\pi}$$

$$\times\left\{\exp(-k_{\mathrm{n}}\bar{R})\left[-\frac{1}{r^2\bar{R}} + z^2\left(\frac{k_{\mathrm{n}}}{r^2\bar{R}^2} + \frac{1}{r^2\bar{R}^3} + \frac{2}{r^4\bar{R}}\right)\right]\right\},$$

$$H_x = \frac{M}{4\pi}\left\{\exp(-k_t R)\left[\frac{k_t}{R^2} + \frac{1}{R^3} - \frac{k_t}{r^2}\right.\right.$$

$$\left.\left. + x^2\left(\frac{2k_t}{r^4} + \frac{k_t^2}{r^2 R} - \frac{k_t^2}{R^3} - \frac{3k_t}{R^4} - \frac{3}{R^5}\right)\right]\right\}$$

$$+ \frac{Mk_t}{4\pi}\left\{\exp(-k_n\bar{R})\left[\frac{k_n}{R} + \frac{1}{r^2} - x^2\left(\frac{k_n}{r^2\bar{R}} + \frac{2}{r^4}\right)\right]\right\},$$

$$H_y = -\frac{Mxy}{4\pi R^3}\exp(-k_t R)\left(k_t^2 + \frac{3k_t}{R} + \frac{3}{R^2}\right),$$

$$H_z = \frac{Mxz}{4\pi}\left\{\exp(-k_t R)\left(\frac{2k_t}{r^4} + \frac{k_t^2}{r^2 R} - \frac{k_t^2}{R^3} - \frac{3k_t}{R^4} - \frac{3}{R^5}\right)\right.$$

$$\left. - k_t\exp(-k_n\bar{R})\left(\frac{2}{r^4} + \frac{k_n}{r^2\bar{R}}\right)\right\}.$$

(iii) Orientation along the *z* axis. Only one component of the extraneous magnetic current is nonzero,

$$j_z^H = -i\omega\mu M\delta(x)\delta(y)\delta(z).$$

All the electromagnetic field components are found from the previous case by substituting z for x, x for z, and $-\mu$ for μ.

2. ELECTRIC DIPOLE

(i) Orientation along the *y* axis. In this case, only one component of the extraneous electric current is nonzero,

$$j_y^E = I\delta(x)\delta(y)\delta(z),$$

where I is the moment of the electric dipole.

The electromagnetic field components are given by

$$E_x = -\frac{i\omega\mu I\lambda xy}{4\pi k_n^2\bar{R}^3}\exp(-k_n\bar{R})\left(k_n^2 + \frac{3k_n}{\bar{R}} + \frac{3}{\bar{R}^2}\right),$$

$$E_y = -\frac{i\omega\mu I\lambda}{4\pi k_n^2 R^2}$$

$$\times\exp(-k_n\bar{R})\left\{2k_n + \frac{2}{\bar{R}} - r^2\left(\frac{k_n^2}{\bar{R}} + \frac{3k_n}{\bar{R}^2} + \frac{3}{\bar{R}^3}\right)\right\},$$

$$E_z = -\frac{i\omega\mu I\lambda yz}{4\pi k_n^2\bar{R}^3}\exp(-k_n\bar{R})\left(k_n^2 + \frac{3k_n}{\bar{R}} + \frac{3}{\bar{R}^2}\right),$$

$$H_x = \frac{I\lambda z}{4\pi\bar{R}^2}\exp(-k_n\bar{R})\left(k_n + \frac{1}{\bar{R}}\right),$$

$$H_0 = 0,$$

$$H_z = -\frac{I\lambda x}{4\pi\bar{R}^2}\exp(-k_n\bar{R})\left(k_n + \frac{1}{\bar{R}}\right).$$

(ii) Orientation along the *x* axis. Only one component of the extraneous electric current is nonzero,

$$j_x^E = I\delta(x)\delta(y)\delta(z).$$

The electromagnetic field components are given by

$$E_x = \frac{i\omega\mu I}{4\pi k_t}\left\{\exp(-k_t R)\left[\frac{k_t}{R} + \frac{1}{r^2} - x^2\left(\frac{k_t}{r^2 R} + \frac{2}{r^4}\right)\right]\right\}$$

$$+ \frac{i\omega\mu I\lambda}{4\pi k_t^2}\left\{\exp(-k_n\bar{R})\left[\frac{k_n}{\bar{R}^2} + \frac{1}{\bar{R}^3} - \frac{k_n}{r^2}\right.\right.$$

$$\left.\left. + x^2\left(\frac{k_n^2}{r^2\bar{R}} - \frac{k_n^2}{\bar{R}^3} + \frac{2k_n}{r^4} - \frac{3k_n}{\bar{R}^4} - \frac{3}{\bar{R}^5}\right)\right]\right\},$$

$$E_y = -\frac{i\omega\mu I\lambda xy}{4\pi k_n^2\bar{R}^3}\exp(-k_n\bar{R})\left(k_n^2 + \frac{3k_n}{\bar{R}} + \frac{3}{\bar{R}^2}\right),$$

$$E_z = -\frac{i\omega\mu Ixz}{4\pi k_t^2}\left\{\exp(-k_t R)\left(\frac{k_t^2}{r^2 R} + \frac{2k_t}{r^4}\right)\right.$$

$$\left. + \lambda\exp(-k_n\bar{R})\left(-\frac{k_n^2}{r^2\bar{R}} + \frac{k_n^2}{\bar{R}^3} - \frac{2k_n}{r^4} + \frac{3k_n}{\bar{R}^4} + \frac{3}{\bar{R}^5}\right)\right\},$$

$$H_x = \frac{Ixyz}{4\pi r^2}\left\{\exp(-k_t R)\left(\frac{k_t}{R^2} + \frac{2}{r^2 R} + \frac{1}{R^3}\right)\right.$$

$$\left. - \lambda\exp(-k_n\bar{R})\left(\frac{k_n}{\bar{R}^2} + \frac{2}{r^2\bar{R}} + \frac{1}{\bar{R}^3}\right)\right\},$$

$$H_y = -\frac{Iz}{4\pi R^2}\exp(-k_t R)\left(k_t + \frac{1}{R}\right),$$

$$H_z = -\frac{Iy}{4\pi}\left\{\exp(-k_t R)\left[\frac{1}{r^2 R}\right.\right.$$

$$\left.\left. - z^2\left(\frac{k_t}{r^2 R^2} + \frac{2}{r^4 R} + \frac{1}{r^2 R^3}\right)\right]\right\} + \frac{Iy\lambda}{4\pi}\{\exp(-k_n\bar{R})$$

$$\times\left[\frac{k_n}{\bar{R}^2} + \frac{1}{r^2\bar{R}} + \frac{1}{\bar{R}^3} - z^2\left(\frac{k_n}{r^2\bar{R}^2} + \frac{2}{r^4\bar{R}} + \frac{1}{r^2\bar{R}^3}\right)\right]\}.$$

(iii) Orientation along the z axis. Only one component of the extraneous electric current is nonzero,

$$j_z^E = I\delta(x)\delta(y)\delta(z).$$

All the electromagnetic field components are found from the previous case by substituting z for x, x for z, $-\mu$ for μ, and $-I$ for I.

Below, magnetic and electric dipoles were oriented along one of the coordinate axes. The field of an arbitrary oriented dipole is found as a superposition of the field projections onto the coordinate axes.

APPENDIX

It should be noted that not all the field components can be found by direct integration of the transforms; specifically, we omitted from consideration the orientation along the axis $r = 0$. The field components along this axis are found by passing to the limit $r \longrightarrow 0$. Below, we give only those field components along the axis $r = 0$ which appear nontrivial.

Magnetic dipole oriented along the x axis, $r = 0$.

$$E_x = 0,$$

$$E_z = -\frac{i\omega\mu My}{4\pi}\exp(-k_t|y|)\left\{\frac{k_t}{2y^2}\left(1+\frac{1}{\lambda^2}\right)+\frac{1}{|y|^3}\right\},$$

$$H_z = \frac{M}{4\pi}\exp(-k_t|y|)\left\{\frac{k_t^2}{2|y|}\left(1+\frac{1}{\lambda^2}\right)+\frac{k_t}{y^2}+\frac{1}{|y|^3}\right\},$$

$$H_z = 0.$$

The components of the magnetic dipole oriented along the z axis are found by making an appropriate change of variables (see Sect. 1(iii)).

Electric dipole oriented along the x axis, $r = 0$.

$$E_x = \frac{i\omega\mu I}{4\pi k_t^2}\exp(-k_t|y|)\left(\frac{k_t^2}{2|y|}+\frac{k_t}{\lambda^2 y^2}+\frac{1}{\lambda^2|y|^3}+\frac{k_t^2}{2\lambda^2|y|}\right),$$

$$E_z = 0,$$

$$H_x = 0,$$

$$H_z = \frac{Iy}{4\pi}\exp(-k_t|y|)\left(\frac{k_t}{2y^2}+\frac{k_t}{2\lambda^2 y^2}+\frac{1}{\lambda^2|y|^3}\right).$$

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