1. A total of 46% of the voters in a certain city classify themselves as Independents, where as 30% classify themselves as Liberals and 24% say they are Conservatives. In s recent local election, 35% of the Independents, 62% of the Liberals and 58% of the Conservatives voted. A voter is chosen at random. Given that this person voted in the local election, the probability that he or she is an Independent is _________ [enter up to 3 decimal points]

Solution:

Given that

P(J) = 46%   Independents

P(L) = 30%   Liberals

P(C) = 24%   Conservatives

⇒ And also given that 35% of Independents are voted

P(V/J) = 35%

P(V/L) = 62%

P(V/C) = 58%

∴ We have to find P(J/V).

According to the Bays theorem,

$$P\left(\frac{J}{V}\right) = \frac{P(I \cap V)}{P(V)}$$

$$= \frac{P\left(\frac{V}{I}\right)P(I)}{P\left(\frac{V}{I}\right)P(I)+P\left(\frac{V}{L}\right)P(L)+P\left(\frac{V}{C}\right)P(C)}$$

$$= \frac{0.35\times0.46}{0.35\times0.45+0.62\times0.30+0.58\times0.24}$$

$$= 0.331$$

2. Mr. Elliot has just had a biopsy or a possibly cancerous tumor. Not wanting to spoil a weekend family event, he does not want to hear any bad news in the next few days. But if he tells the doctors to call only if the news is good, then if the doctor doesn’t call, Mr. Elliot can conclude that the news is bad.

Being a student of probability Mr. Elliot instruct the doctor to flip a coin. If it comes up head, the doctor is to call if the new is good and not call if the new is bad. If the coin comes up tails, the doctor is not to call whether the news is good or bad.

Solution:

Let α be the probability that the tumor is cancerous.

Let β be the probability that the tumor is cancerous given that the doctor doesn't call then $\beta =$ ________

(a) $\frac{\alpha}{1+\alpha}$

(b) $\frac{2\alpha}{1+3\alpha}$

(c) $\frac{1}{1+2\alpha}$

(d) $\frac{2\alpha}{1+\alpha}$

Solution for 3:

Let C be the event that tumor is cancerous.

Let N be the event that the doctor doesn't call.

So we need to find $P\left(\frac{C}{N}\right)$

$$\beta = P\left(\frac{C}{N}\right) = \frac{P(C \cap N)}{P(N)} = \frac{P\left(\frac{N}{C}\right)P(C)}{P\left(\frac{N}{C}\right)P(C) + P(\frac{N}{C'})P(C')}$$

$P\left(\frac{N}{C}\right)$:

It is given that he has cancer.

If coin turns to be head $\left[P(n) = \frac{1}{2}\right]$ he want call.

If coin turns to be tail $\left[P(t) = \frac{1}{2}\right]$ he want call.

$\therefore$ In both cases doctor won't call him $P\left(\frac{N}{C}\right) = 1$

$P\left(\frac{N}{C'}\right)$:

It is given that he don't have cancer.

$\therefore$ Doctor won't call him only when the coin turns to be tail $P(t) = \frac{1}{2}$

$\therefore P\left(\frac{N}{C'}\right) = \frac{1}{2}$

$$\therefore P\left(\frac{C}{N}\right) = \frac{P\left(\frac{N}{C}\right)P(C)}{P\left(\frac{N}{C}\right)P(C) + P\left(\frac{N}{C'}\right)P(C')}$$

$$= \frac{1\times\alpha}{1\times\alpha+\frac{1}{2}\times(1-\alpha)}$$

$$= \frac{\alpha}{\alpha+\frac{1}{2}(1-\alpha)}$$

$$= \frac{2\alpha}{2\alpha+(1-\alpha)}$$

$$= \frac{2\alpha}{1+\alpha}$$

$$\therefore\ \beta = \frac{2\alpha}{1+\alpha}$$

3. According to the question 2, which should be bigger α or β?

(a) $\alpha > \beta$

(b) $\alpha < \beta$

(c) $\alpha = \beta$

(d) Can’t say

<u>Solution 3</u>:

$$\frac{2\alpha}{1+\alpha} \geq \alpha\ [\textit{which is strictly inequal unless } \alpha = 1]$$

$$\beta \geq \alpha \Rightarrow \beta > \alpha$$

4. A family has children with probability P_i where $P_1 = 0.01, P_2 = 0.25, P_3 = 0.35, P_4 = 0.3$. A child from this family is randomly chosen. Given that this child is the eldest child in the family, the conditional probability that the family has 4 children ________.

(a) 0.24

(b) 0.18

(c) 0.26

(d) 0.16

Solution:

Let E be the event the child selected is the eldest.

Let F_i be the event that the family has j children.

We need to find

$$P\left(\frac{F_4}{E}\right) = \frac{P(F_4\cap E)}{P(E)}$$

$$P(F_4 \cap E) = P\left(\frac{E}{F_4}\right)P(F_4)$$

$$= \frac{1}{4}\times 0.3$$

$$= \frac{0.3}{4}$$

$$= 0.075$$

$$P(E) = \Sigma_i P(F_i) P\left(\frac{E}{F_i}\right) = P(F_1)P\left(\frac{E}{F_1}\right) + P(F_2)P\left(\frac{E}{F_2}\right) + P(F_3)P\left(\frac{E}{F_3}\right) + P(F_4)P\left(\frac{E}{F_4}\right)$$

$$P(E) = 0.1\times 1 + 0.25\times\frac{1}{2} + 0.35\times\frac{1}{3} + 0.3\times\frac{1}{4}$$

$$= 0.4155$$

$$\therefore P\left(\frac{F_4}{E}\right) = \frac{0.075}{0.4155} = 0.18$$

5. A gambler has in his pocket a fair coin and a two-head coin. He selects one of the coins at random; when he flips it, it shows head. What is the probability that it is the fair coin?

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) $\frac{1}{5}$

Solution:

It is given that, the win shows head

We need to find

$$P\left(\frac{fair}{h}\right) = \frac{P(fair \cap h)}{P(h)}$$

$$= \frac{P\left(\frac{h}{fair}\right)P(fair)}{P\left(\frac{h}{fair}\right)P(fair) + P\left(\frac{h}{fair'}\right)P(fair')}$$

$$= \frac{\frac{1}{2}\times\frac{1}{2}}{\frac{1}{2}\times\frac{1}{2} + 1\times\frac{1}{2}}$$

$$= \frac{\frac{1}{4}}{\frac{1}{4}+\frac{1}{2}}$$

$$= \frac{\frac{1}{4}}{\frac{3}{4}}$$

$$= \frac{1}{3}$$

6. A gambler has a fair coin and a two headed coin. He selects one of the coins at random flipped it twice and got two heads. Now the probability that it is the fair coin is _______.

(a) $\frac{1}{2}$

(b) $\frac{1}{3}$

(c) $\frac{1}{4}$

(d) $\frac{1}{5}$

Solution:

It is given that he got two heads

∴ we have to find,

$$P\left(\frac{fair}{hh}\right) = \frac{P(fair \cap hh)}{P(hh)}$$

$$= \frac{P\left(\frac{hh}{fair}\right)P(fair)}{P\left(\frac{hh}{fair}\right)P(fair)+P\left(\frac{hh}{not}fair\right)P(no\ fair)}$$

$$P\left(\frac{hh}{fair}\right) = \frac{1}{4}[(hh),(hT),(Th),(TT)]$$

$$P\left(\frac{hh}{notfair}\right) = 1$$

$$\therefore P\left(\frac{fair}{hh}\right) = \frac{\frac{1}{4}\times\frac{1}{2}}{\frac{1}{4}\times\frac{1}{2}+1\times\frac{1}{2}}$$

$$= \frac{\frac{1}{8}}{\frac{1}{8}+\frac{1}{2}}$$

$$= \frac{1}{5}$$

7. A gambler has a fair coin and a two headed coin. He selects one of the coins at random. He flipped the same coin thrice. In the first time it showed head, in the second time also head and in the third time it showed tails. The probability that the coin selected is fair coin is ________.

Solution:

The given sequence of outcomes is HHT.

It is given that there is a tail in the outcomes which means it is not a two-head coin we can say it is fair coin.

∴ Answer = 1

8. English and American spellings are “rigour” and “rigor” respectively. A man staying at a Parisian hotel writes this word, and a letter taken at random from his spelling is found to be a vowel.

If the 40% of the English speaking men at the hotel are English and 60% are Americans. What is the probability that the writer is an Englishman?

(a) 5/13 (b) 4/9

(c) 5/11 (d) 2/5

Solution:

We have to find

$$P\left(\frac{E}{V}\right) = \frac{P\left(\frac{V}{E}\right)P(E)}{P\left(\frac{V}{E}\right)P(E) + P\left(\frac{V}{A}\right)P(A)}$$

$$= \frac{\frac{3}{6}\times\frac{40}{100}}{\frac{3}{6}\times\frac{40}{100}+\frac{2}{5}\times\frac{60}{100}}$$

$$= \frac{\frac{1}{2}\times\frac{2}{5}}{\frac{1}{2}\times\frac{2}{5}+\frac{2}{5}\times\frac{3}{5}}$$

$$= \frac{5}{11}$$

9. One probability class (Moon) of 30 students 15 that are good, 10 that are fair, and 5 that are of poor quality. A second probability class (Sun) also of 30 students, contains 5 that are good 10 that are fair, and 15 that are poor. You are aware of these numbers, but you have no idea which is class is which if you examine one student selected at random from each class and find that the student from class A is a fair student whereas the student from class B is a poor student. What is the probability that class A is the moon?

(a) ¼ (b) ¾

(c) ½ (d) none of the above

Solution:

We have to find,

$$P\left({}^{A = moon}/_{A\ fair, B\ poor}\right) = \frac{P(A = moon \cap A\ fair, B\ poor)}{P(A\ fair, B\ poor)}$$

$$=\frac{P\left({}^{A\ fair, B\ poor}/_{A=moon}\right)P(A=moon)}{P\left({}^{A\ fair, B\ poor}/_{A=moon}\right)P(A=moon)+P\left({}^{A\ fair, B\ poor}/_{A=sun}\right)P(A=sun)}$$

$$=\frac{\frac{10}{30}\times\frac{5}{30}\times\frac{1}{2}}{\frac{10}{30}\times\frac{5}{30}\times\frac{1}{2}+\frac{10}{30}\times\frac{15}{30}\times\frac{1}{2}}$$

$$=\frac{50}{50+150}$$

$$=\frac{50}{200}$$

$$=\frac{1}{4}$$

10. You randomly choose a treasure chest to open and then randomly choose a coin from that treasure chest. If the coin you choose is gold then what is the probability that you choose chest A?

Treasure 1 = 100 gold coins

Treasure 2 = 50 gold coins + 50 silver coins

(a) 2/3 (b) 1/3

(c) ½ (d) ¼

Solution:

Let G be the event of selecting a Gold coin.

⇒ Let A be the event selecting Treasure 1.

⇒ Let B be the event selecting Treasure 2.

$\therefore P\left(\frac{G}{A}\right)=1$

$P\left(\frac{G}{B}\right)=\frac{1}{2}$

We need to find

$$P\left(\frac{A}{G}\right)=\frac{P(A\cap G)}{P(G)}$$

$$=\frac{P(A)\cdot P\left(\frac{G}{A}\right)}{P(A)\cdot P\left(\frac{G}{A}\right)+P(B)\cdot P\left(\frac{G}{B}\right)}$$

$$=\frac{\frac{1}{2}\cdot 1}{\frac{1}{2}\cdot 1+\frac{1}{2}\cdot\frac{1}{2}}$$

$$= \frac{2}{3}$$