Exercise 3: DCM for EEG

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Group D: Clara Kümpel, Damola Agbelese, Yitong Li

* 1. **Dynamic equations**

With

derive:

According to *Leibniz’ rule:*

Again, plug term into *Leibniz’ rule:*

*Q.E.D.*

* 1. **Coupled harmonic oscillator**

1. Convert the 2nd order ODE of harmonic oscillator (HO) to a 1st order system.

We have

Let

Then we get a equation set

i.e.,

Q.E.D.

1. Coupled dynamic system: HO is driven by a second HO , i.e.,

Similarly, let:

Keeping notation in (a), we get:

i.e.,

Where:

1. Reconsider equation

Assume:

dynamics of a different population (also follows Eq. )

Transform Eq. to 1st order linear equation system by linearizing

Around , with Taylor expansion, we have:

Then:

Eq. becomes:

Meanwhile:

Let

* 1. **Inference on NN structure**

a) Integration of the system

with A and C as given in the exercise sheet.

Graphical user interface, text

Description automatically generatedThe solution for a) and b) is implemented in Matlab and uploaded in Moodle.

The declaration of the parameters and the integration with Euler methods (step size: 0.001) is shown in the code snippet.

To verify the integrated states (x) correspond to the data in x\_condition\_1, we calculated the absolute difference between the single values. The maximal difference

2.7756 \* 10-17.



Application

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Specifically, we verified x1 and x3:

The maximum errors between all integrated states and the given data were still negligibly small and for x3 even 0.

b) Parameter Grid search

For the grid search of the 4 parameters we used as a first range + / - the half of the standard value in a reasonable step size.

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By comparing the maximum

The grid search showed quick that the explained variance is the best for changing the k1 parameter. Listed below are the maximum sums and mean values of the explained variances for all grid search parameter estimations.

Table

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The estimations for x1 to x4 are shown in the table by applied grid search for k1.

Table

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The estimated k1 value with the best explained variance was 99.74 (index 60). For this value we got explained variances of 1.0, 0.999, 1.0 and 1.0 for x1 to x4. This means that for all variables the explained variance reached 99%. The implemented matrix A with the parameter estimate is shown below.

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When taking the absolute difference between the x\_condition\_2 and the predicted x values, the highest difference was 0.0014.

Table

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So our final model and the ensuing parameters that best explain the data are:

κ1 =  99.74, κ2 =  50

f1  =  50, f2  = 50

af = 3000

ab = 1000

c = 1

μ = 0.05

σ = 0.01