Exercise 5: VB

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* 1. **VB update equations**

1. Univariate Gaussian model

As is a constant,

Joint distribution:

1. Find the joint distribution given prior
2. Find the optimal variational distribution over under the mean field approximation.

Regarding to the mean field approx.:

since the optimization is over integration terms excluding can be sorted into the constant term, hence:

The integrated term can be factorized as:

Move the term excluding outside the integral:

With the definition of expectation:

1. Reorder in (c) with and

Since is the only variable, terms and are constant, therefore:

1. Show that follows Gaussian distribution.

i.e., follows Gaussian distribution with

mean

And variance

1. Find the optimal variational distribution over under the mean field approximation.

As in (c), integration terms excluding can be sorted into the constant term, hence:

1. Rewrite with
2. Show that is given by a Gamma distribution

With the equation derived in (g):

Which follows distribution:

With

1. Formulate the negative free energy.

The negative free energy can be written as:

With the assumption of factorized

As we derived before:

(Grey means these terms are always constant)

The first integration becomes:

The second integration term:

Summing the two integrations up, we get the negative free energy:

* 1. **VB implementation**
  2. **The true posterior**