Exercise 5: VB

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* 1. **VB update equations**

## Univariate Gaussian model

As is a constant,

Joint distribution:

## Find the joint distribution given prior

## Find the optimal variational distribution over under the mean field approximation.

Regarding to the mean field approx.:

since the optimization is over integration terms excluding can be sorted into the constant term, hence:

The integrated term can be factorized as:

Move the term excluding outside the integral:

With the definition of expectation:

## Reorder in (c) with and

Since is the only variable, terms and are constant, therefore:

## Show that follows Gaussian distribution.

i.e., follows Gaussian distribution with

mean

And variance

## Find the optimal variational distribution over under the mean field approximation.

As in (c), integration terms excluding can be sorted into the constant term, hence:

## Rewrite with

## Show that is given by a Gamma distribution

With the equation derived in (g):

Which follows distribution:

With

## Formulate the negative free energy.

The negative free energy can be written as:

With the assumption of factorized

As we derived before:

(Grey means these terms are always constant)

The first integration becomes:

The second integration term:

Summing the two integrations up, we get the negative free energy:

# VB implementation

# Code is provided in an executable Matlab file.

1. Generated observations from univariate Gaussian modelText

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example generated y data: 

1. Update equations for μ and τ

A picture containing calendar

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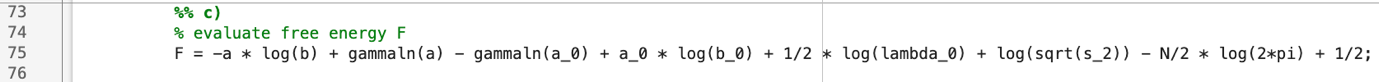
Text

Description automatically generated

Text, letter

Description automatically generated

1. Evaluate Free energy



1. Function to loop over the update equations and the free energyTable

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Monitored free energy:

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The program stopped after 3 iterations.

1. Run with generated data and prior parameters

Starting values as given in exercise script:

m = μ0

s2 = b0 / (a0 \* λ0)

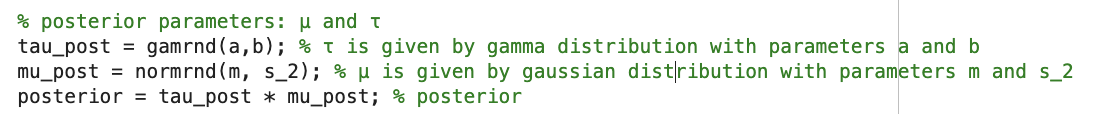
a = a0

b = b0

Then run the function vb() with prior parameters and generated y data:



Calculate posterior parameters:



Calculate random starting values:

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Results after one run with prior:

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | m | s2 | a | b | *F* | τ | μ | posterior |



10 runs with random values:

**Table

Description automatically generated with medium confidence**

If you can report the results from one run only, which one would you choose?