Letter to the Editor

Theory of the inverse Faraday effect in metals

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Abstract

An analytic expression is given for the inverse Faraday effect, i.e., for the magnetization occuring in a transparent medium exposed to a circularly polarized high-frequency electromagnetic wave. Using a microscopic approach based on the Drude approximation of a free-electron gas, the magnetization of the medium due to the inverse Faraday effect is identified as the result of microscopic solenoidal currents generated by the electromagnetic wave. In contrast to the better known phenomenological derivation, this microscopic treatment provides important information on the frequency dependence of the inverse Faraday effect.
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1. Introduction

While the equations describing the transfer of energy and momentum to a medium by an electromagnetic wave are commonly known from undergraduate students' textbooks, the study of the transfer of angular momentum of an electromagnetic wave has received relatively little attention. The first prediction that circularly polarized light should cause an angular momentum flux is due to Sadovskii and dates back to 1897. The original work of Sadovskii is written in Russian and is difficult to find (see footnote 1 in Ref. [1]). The first detection of the Sadovskii effect, i.e., the mechanical measurement of a torque exerted by circularly polarized light on a half-wave plate, has been reported in 1936 by Beth [1] and Holbourn [2], independently. The interesting history of the study of angular momentum of electromagnetic waves is reported in several review papers [3,4]. Since magnetic moments have an angular momentum, the possibility to influence the magnetization in a magnetically ordered material by means of circularly polarized light appears to be a straightforward continuation of the old experiments by Beth and Holbourn. It is therefore remarkable that only very recently, about 70 years after these historic experiments, it has been demon-strated by Kimel et al. [5] that spins can indeed be manipulated with a circularly polarized laser beam. Apart from their importance for fundamental processes concerning the magnetization dynamics, these spectacular experiments are also interesting, because of their potential technical implications. These studies may pave the way towards an ultrafast, focussed laser-controlled magnetic writing process that could eventually replace the data storage technique being in use nowadays, which involves microscopic coils and write heads that may generate undesired fringing fields. The effect reported by Kimel et al. is attributed to the generation of a stationary magnetic field resulting from irradiation with circularly polarized light, which is known as the inverse Faraday effect (IFE).

The IFE has first been predicted by Pitaevskii more than 40 years ago [6], while the name (IFE) seems to have been coined only four years later by van der Ziel et al. [7,8]. The IFE was originally derived on a phenomenological basis and has received a certain publicity among physicists, because it was briefly treated in some older editions of a well-known textbook series on theoretical physics [9]. A more detailed description of the IFE has been elaborated in the late seventies and the early eighties by theoretical plasma physicists, mainly in the former Soviet Union [10–13]. This literature is presumably hardly known in the part of the scientific community that is currently investigating ultrafast magnetization processes. The scope of this

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paper is to provide a simple derivation of the IFE that involves microscopic currents, described in more detail by Karpman et al. [13], rather than a purely phenomenological approach. In the wake of the recent studies by Kimel et al., this short review of the theory of the IFE is likely to be of interest for researchers working in the domain of magnetism who may not happen to be experts in theoretical plasma physics.

2. Basic equations

When a material is irradiated with a high-frequency (HF) electromagnetic wave, the primary coupling between the wave and the material is given by the interaction between the electrons and the electric field of the wave. One may therefore neglect in a first approximation the wave's fluctuating magnetic field $\delta \boldsymbol{B}$.

For simplicity, any quantum mechanical effects are neglected and it is assumed that the band structure of the material is unimportant for the motion of the electrons. In the case of a metallic material the conducting electrons can be treated as a collisionless plasma in which the electrons can move freely, at least on the time scale given by the period of the HF field. Such an electron plasma can conveniently be represented as a fluid with density $n(\boldsymbol{r}, t)$ and velocity $\boldsymbol{v}(\boldsymbol{r}, t)$. The electron density n and the velocity $\boldsymbol{v}(\boldsymbol{r}, t)$ are related to each other by the continuity equation

$$\frac{\partial}{\partial t} n + \boldsymbol{\nabla}(n\boldsymbol{v}) = 0. \tag{1}$$

The electron density n and the velocity $\boldsymbol{v}$ give rise to an electric current density $\boldsymbol{j}$ according to

$$\boldsymbol{j} = en\,\boldsymbol{v}, \tag{2}$$

which, in the simplest approximation, is proportional to an electric field $\boldsymbol{E}$

$$\boldsymbol{j} = \sigma \boldsymbol{E}. \tag{3}$$

In the case of an oscillating electric field

$$\delta \boldsymbol{E}(\boldsymbol{x}, t) = \hat{\boldsymbol{E}} \exp(\mathrm{i}\boldsymbol{k}\boldsymbol{r} - \mathrm{i}\omega t) \tag{4}$$

of HF ω and wave vector $\boldsymbol{k}$, the conductivity σ of an isotropic, collisionless plasma is [9]

$$\sigma = \frac{\mathrm{i}\langle n \rangle e^2}{m\omega}, \tag{5}$$

where m is the electron mass. The brackets $\langle \rangle$ denote a time average of the electron density n over several periods of the HF field.

For the following, only the temporal oscillation of the local field at a given point $\boldsymbol{r}$ will be required, so that the spatial dependence of the electromagnetic wave $\delta \boldsymbol{E}$ can be omitted. Note that the amplitude $\hat{\boldsymbol{E}}$ is in general a complex quantity. The physically relevant real value of the field is given by

$$\Re\{\delta \boldsymbol{E}\} = \tfrac{1}{2} \hat{\boldsymbol{E}} \exp(-\mathrm{i}\omega t) + \text{c.c.}, \tag{6}$$

where c.c. denotes complex conjugation of the previous term.

3. Separation of time scales

To investigate the influence of an HF field on a plasma, it is useful to treat different time scales separately. The time scales are split by representing the electron density n, the velocity $\boldsymbol{v}$, and any other quantity a which is required for the description of the plasma as a sum of two parts

$$a = \langle a \rangle + \delta a, \tag{7}$$

one of which ($\langle a \rangle$) is the time-averaged value that is constant or slowly changing on the time scale given by the period of the HF field, and the other (δa) is an oscillating part with the same time dependence $\propto \exp(-\mathrm{i}\omega t)$ as the HF field [14].

One may safely assume that the magnitude of the oscillating part δn of the electron density, i.e., the size of the density fluctuations induced by the HF field is much smaller than the stationary value of n:

$$\delta n \ll \langle n \rangle. \tag{8}$$

Inserting the separation of stationary and oscillatory quantities into the continuity equation (1) yields

$$\frac{\partial}{\partial t}(\langle n \rangle + \delta n) + \boldsymbol{\nabla}(\langle n \rangle \langle \boldsymbol{v} \rangle + \langle n \rangle \delta \boldsymbol{v} + \delta n \langle \boldsymbol{v} \rangle + \delta n \delta \boldsymbol{v}) = 0. \tag{9}$$

Owing to $\langle \delta a \rangle = 0$, the time-averaged continuity equation is

$$\frac{\partial}{\partial t} \langle n \rangle + \boldsymbol{\nabla}(\langle n \rangle \langle \boldsymbol{v} \rangle + \langle \delta n \delta \boldsymbol{v} \rangle) = 0. \tag{10}$$

This is the so-called slow time scale of the continuity equation. Using the approximation of Eq. (8), the fast time scale reads

$$\frac{\partial}{\partial t} \delta n + \boldsymbol{\nabla}(\langle n \rangle \delta \boldsymbol{v}) = 0. \tag{11}$$

The same approximation yields the oscillating part $\delta \boldsymbol{j}$ of the current density $\boldsymbol{j}$ as

$$\delta \boldsymbol{j} = e\langle n \rangle \delta \boldsymbol{v}. \tag{12}$$

Hence, the fluctuating part of the velocity is

$$\delta \boldsymbol{v} = \frac{\sigma}{\langle n \rangle e} \delta \boldsymbol{E}. \tag{13}$$

According to Eqs. (11) and (13), the fluctuating part of the electron density n is given by

$$\delta n = -\frac{\mathrm{i}}{\omega e} \boldsymbol{\nabla}(\sigma \delta \boldsymbol{E}). \tag{14}$$

The slow time scale term of the current density $\langle \boldsymbol{j} \rangle$ is

$$\langle \boldsymbol{j} \rangle = e(\langle n \rangle \langle \boldsymbol{v} \rangle + \langle \delta n \delta \boldsymbol{v} \rangle) \tag{15}$$

or simply

$$\langle \boldsymbol{j} \rangle = e\langle \delta n \delta \boldsymbol{v} \rangle \tag{16}$$

if $\langle \boldsymbol{v} \rangle = 0$, which shall be assumed henceforth.

4. Inverse Faraday effect

Eq. (16) describes the occurrence of a stationary current density $\langle \boldsymbol{j} \rangle$ that is solely due to the HF oscillations of the electron density and velocity induced by the HF field. This term will eventually lead to an expression for the IFE, i.e., for the magnetization of the plasma in the field of a circularly polarized electromagnetic wave.

To further evaluate the right-hand side of Eq. (16) it should be beared in mind that the term $\delta n \cdot \delta \boldsymbol{j}$ describes the product of two *real* quantities. Therefore,

$$\delta n \cdot \delta \boldsymbol{v} = \Re\{\hat{n} \exp(-i\omega t)\} \cdot \Re\{\hat{\boldsymbol{j}} \exp(-i\omega t)\}$$
$$= \frac{1}{4} [\hat{n}\hat{\boldsymbol{v}}^* + \hat{n}^*\hat{\boldsymbol{v}} + \hat{n}\hat{\boldsymbol{j}} \exp(-2i\omega t)$$
$$+ \hat{n}^*\hat{\boldsymbol{v}}^* \exp(2i\omega t)] \tag{17}$$

and hence

$$\langle \delta n \delta \boldsymbol{v} \rangle = \frac{1}{4}(\hat{n}\hat{\boldsymbol{v}}^* + \hat{n}^*\hat{\boldsymbol{v}})$$
$$= -\frac{i}{4e^2\langle n \rangle \omega}[\sigma^*\hat{\boldsymbol{E}}^* \cdot \boldsymbol{\nabla}(\sigma\hat{\boldsymbol{E}}) - \text{c.c.}]. \tag{18}$$

The stationary current density $\langle \boldsymbol{j} \rangle$ can therefore be written as

$$\langle \boldsymbol{j} \rangle = -\frac{i}{4e\langle n \rangle \omega}\boldsymbol{\nabla} \times [\sigma^*\hat{\boldsymbol{E}}^* \times \sigma\hat{\boldsymbol{E}}] + \boldsymbol{\Gamma} \tag{19}$$

with

$$\boldsymbol{\Gamma} = \frac{1}{4e\langle n \rangle \omega}[(i\sigma^*\hat{\boldsymbol{E}}^*\boldsymbol{\nabla})(\sigma\hat{\boldsymbol{E}}) + \text{c.c.}]. \tag{20}$$

The term $\boldsymbol{\Gamma}$ describes the so-called ponderomotive force acting in the plasma. The ponderomotive force leads to a special type of stationary currents generated by a HF field. These currents result from spatial inhomogeneities of the plasma or of the field of the wave. This term $\boldsymbol{\Gamma}$ is discussed in detail, e.g., in Ref. [13]. The current densities connected with the ponderomotive force do not describe the IFE. It is the first term on the right-hand side of Eq. (19) that has the form of a solenoidal magnetization current $\boldsymbol{j}_{\text{m}}$:

$$\boldsymbol{j}_{\text{m}} = \boldsymbol{\nabla} \times \boldsymbol{M}. \tag{21}$$

Hence, the magnetization $\boldsymbol{M}$ generated in the plasma by the HF field is

$$\boldsymbol{M} = -\frac{i\varepsilon_0 m\omega_{\text{p}}^2}{4e^3\langle n \rangle^2 \omega}[\sigma^*\hat{\boldsymbol{E}}^* \times \sigma\hat{\boldsymbol{E}}] \tag{22}$$

$$= \frac{i\varepsilon_0 e\omega_{\text{p}}^2}{4m\omega^3}[\hat{\boldsymbol{E}} \times \hat{\boldsymbol{E}}^*] \tag{23}$$

where $\omega_{\text{p}} = (\langle n \rangle e^2/m\varepsilon_0)^{1/2}$ is the plasma frequency.

5. Discussion

It should be noted that Eq. (23) has been derived without making any assumption concerning the polarization of the electromagnetic wave. In the case of a circularly polarized wave propagating in the z direction, the last term on the right-hand side can be written as

$$\hat{\boldsymbol{E}} \times \hat{\boldsymbol{E}}^* = \pm i|\boldsymbol{E}|^2 \cdot \boldsymbol{e}_z, \tag{24}$$

while it is equal to zero in the case of linear polarization. The plus and the minus sign in Eq. (24) refers to left and right circular polarization, respectively. Therefore, $\boldsymbol{M}$ is a real-valued, stationary magnetization that is induced in the medium by a circularly polarized electromagnetic wave. The magnetization is parallel to the axis of propagation of the wave and its sign depends on the chirality of the wave.

If the IFE is applied to change the magnetization in magnetically ordered materials, the magnetization induced by the HF field according to Eq. (23) should reorient the magnetic moments in the sample, leading to a permanent magnetization that persists after the HF field is switched off. The precise mechanism that converts the field-induced magnetization currents into a permanent magnetization is not always clear. The simplest explanation would be the magnetostatic alignment of the magnetic moments in the field generated by the IFE. It has been reported that ultrashort circularly polarized laser pulses can generate a magnetic field that persists for only about 100 fs. In a ferro- or ferrimagnetic material, such a short field pulse can cause a precessional rotation of the magnetization into a new direction [15]. Other mechanisms of spin orientation driven by circularly polarized light have been discussed by Edelstein [16] and Smirnov [17].

In the derivation of the IFE according to Eq. (23) it has been assumed that the electromagnetic wave is not absorbed by the medium. Since the value of the HF conductivity is purely imaginary, cf. Eq. (5), the induced current density $\delta \boldsymbol{j}$ is phase-shifted by $\pi/2$ with respect to the electromagnetic wave. Hence, the power density transferred to the plasma

$$\langle \delta \boldsymbol{j} \delta \boldsymbol{E} \rangle = \frac{ie^2 n}{2m\omega}|\boldsymbol{E}|^2 \tag{25}$$

is also purely imaginary. However, it should not be concluded from this absorption-free derivation of the IFE that the absorption of a circularly polarized electromagnetic wave does *not* lead to a magnetization of the medium.

Interestingly, none of the historical arguments on the transfer of angular momentum mentioned in the Introduction has been used for the derivation of the IFE. In fact, the investigation of the angular momentum transported by a circularly polarized electromagnetic plane wave leads to several complications. It has been reported that an infinitely extended, circularly polarized plane wave, surprisingly, does not carry any angular momentum [4]. This counterintuitive result is not in contradiction to the experimental proof of the angular momentum demonstrated by Beth [1] and Holbourn [2], since it has also been shown that a real, collimated beam of circularly polarized light *does* have an angular momentum flux [3]. Using higher order terms in the so-called geometric optics approximation [18], an analytic expression for the spin

flux density of a collimated circularly polarized wave in a plasma has been reported [19]. However, angular momentum considerations are not necessary and do not seem to be helpful for the derivation of the IFE in metals.

6. Conclusion

The ongoing research efforts to find faster and more precise methods to store information in increasingly miniaturized magnetic particles has led to several successful collaborations between scientists specialized in different domains. This is especially true for modern experimental investigations, e.g., on ultrafast magnetization process in nanostructures, where usually several different techniques are involved. The recently demonstrated manipulation of spins by means of a circularly polarized laser beam is a further advance in this field, which requires an expansion of the knowledge about magnetization processes driven by electromagnetic waves. Similar to the positive synergy effects that have been obtained in experiments, the theory can also benefit significantly by combining the theory of magnetism with theoretical plasma physics, in which the interaction of electrons with radiation is of central importance.

Compared to the relatively well-known phenomenological treatment of the IFE, the microscopic derivation of the IFE presented in this paper gives a clearer insight into the processes leading to the magnetization of the sample. A further important aspect is that Eq. (23) is free of material constants and thus allows for quantitative predictions of the IFE. In particular, a strong dependence of the magnetization generated by the IFE on the frequency of the applied field $M \propto \omega^{-3}$ is predicted. An experimental investigation on the frequency dependence of the IFE would be desirable to validate this equation.

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