Classical Ruin Problem

Applications in Insurance Sector

Stochastic Processes

BSc. (Hons) Statistics

Semester V

By : Dhruv Aggarwal (2033102)

Overview

Every business involves risk and revenues and expenses may not always be easily predictable and maybe highly volatile therefore it makes sense to have tools to be able to manage the uncertainty. This is the premise of risk theory, the study of how results deviate from their expected values and how to prevent undesirable results.

More specifically, We attempt to model cash flows as a surplus process, a function describing how much money the firm has at a given time. Some types of cash flows are easily predictable and can therefore be modeled with great precision; others, though, are more random and are thus harder to predict. In the specific case of an insurer company, the most important cash flows are of course premiums and claims which together form the company's surplus.

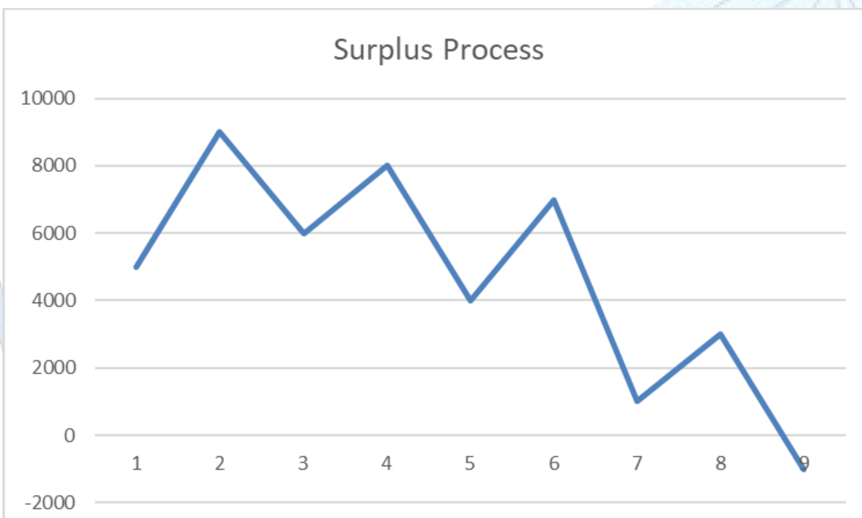
One of the worst events that could happen to an insurance company is called ruin defined as the company's surplus becoming negative, in other words, the collected premiums don't cover the customer's claim payments.

Basic Idea to Approach the Problem

Surplus Process

The surplus of an insurance company is the sum of three basic components: aggregate premiums, aggregate claims, and the initial surplus.Aggregate premiums can be represented as a simple linear model over time,c(t) while aggregate claims can be viewed as a stochastic process which we call S(t). The initial surplus is a constant,u. Altogether, we can then write the surplus process, represented by U(t), as

$$U(t) = u + ct - S(t)$$



As can be seen, the surplus undergoes periods of growth interrupted by sudden jumps downwards. The surplus drops below its initial level three times before finally becoming negative at 9, the time of ruin.

The time of ruin T, is the first time that the Aggregate Surplus becomes negative.

$$T := min(t: t \geq 0 \; and \; U(t) < 0)$$

The probability of ruin $\psi(u)$, is the probability that ruin will happen, i.e. that

$$\psi(u) = P(T < \infty)$$

Although using finite time would more accurately represent the reality of the business world

Aggregate Claims (S(t))

The only part of U(t) that requires further analysis is S(t), S(t) is a collection of individual claim-size random variables. But the no of claims itself is unknown i.e. itself a random variable representing how many claims will be made until the time t.

$$S(t) = X_1 + X_2 + ... + X_{N(t)}$$

Where the N(t) denotes the no of claims will be made until time t and the most logical and natural choice of the distribution is the Poisson Distribution

X_i denotes the amount of individual claim size and itself a random variable. The most natural and logical choice for the distribution of claim size is the distribution with thick tails normally lognormal , exponential , gamma and weibull.

And without any loss of generality it can be assumed that the no of claims in a time t does not depends upon the claim size i.e. N(t) is independent of the X_i.

So S(t) follows the Compound Poisson Process.

$$E(S(t)) = E(N(t))E(X_i)$$

And

$$V(S(t)) = E^2(X)V(N(t)) + E(N(t))V(X)$$

Our Case Study

An insurer is modeling its probability of ruin over time. The insurer’s surplus comprises three items:

- An initial surplus U = $5000 (in hundred thousand) that is invested in assets
- Premium income of $300 (in hundred thousand) per unit time, arriving at the end of each time period
- Claims on the policies issued, paid at the end of each time period.

We have simulated 1000 values in $(in hundred thousand) over the next ten time periods in the ‘Data’ tab of the spreadsheet. For the number of claims we have assumed a Poisson distribution with λ=30 and individual claim distribution area assumed to be a log-normal distribution with parameters μ=3 and $\sigma^2 = 1.1^2$.

Assets are assumed to increase in value at a rate of 2% p.a. compounded annually. Once the surplus falls below zero, it is set to zero and remains there indefinitely (i.e. the insurer becomes insolvent).

Calculate the surplus process for the insurer in each simulation and calculate the insurer’s probability of ruin after each year.

SOLUTION:

The capital at the end of each year from t=1 to 10 shows us the returns from the investment of the assets. We have calculated aggregate claims at the end of each year.

As mentioned above,

$$U(t) = u + ct - S(t)$$

Here as per our situation,

$$U(t) = u(1 + i)^t + cS_{\overline{n|}} - S(t)$$

Data Values

	Initial Cap	5000		**premium**	300					
	interest	0.02								
Sno	**Year 1**	**Year 2**	**Year 3**	**Year 4**	**Year 5**	**Year 6**	**Year 7**	**Year 8**	**Year 9**	**Year 10**
Capital	5400	5808	6224.16	6648.643	7081.616	7523.248	7973.713	8433.188	8901.851	9379.888
1	1170.035	2155.264	3030.625	4175.395	5122.148	6215.062	8081.068	9173.394	10245.19	11727.82
2	1218.8154	2497.577	3227.981	4032.847	5102.232	6258.613	6801.887	7697.093	8479.633	9299.674
3	800.8546	1490.529	2324.666	3054.739	5054.122	5992.277	6591.265	7265.265	8163.979	9515.828
4	1191.8947	1847.469	3159.42	5086.452	6224.711	7290.057	7980.698	9627.628	10734.75	11881.69
5	970.04954	1939.861	2875.034	3506.885	5424.756	6614.372	7707.631	8618.728	9561.103	10854.05
6	1352.5392	2122.88	4358.688	5337.485	6275.005	7565.833	8389.527	9393.724	10863.86	11902.23
7	748.71873	2647.323	3274.879	4221.467	5667.941	6967.608	8149.707	9655.158	10069.77	10843.71
8	883.9926	1514.092	2697.303	3546.265	4173.78	4784.167	5674.998	6670.622	7988.223	8501.037
9	1359.8516	2277.279	4023.195	4951.572	6028.209	7042.623	8065.365	8812.669	9465.429	10187.63
10	742.28654	1843.254	3194.282	3920.776	5197.251	6308.617	6510.461	8227.626	10872.35	11563.19
11	813.40547	1957.982	3453.738	4273.247	4763.007	5484.783	6170.374	7076.2	8097.54	8886.397
12	1522.3052	3006.872	3600.706	4571.861	5078.609	6106.709	6884.739	7624.404	8508.201	10058.58
13	931.88577	1909.712	3270.838	4201.159	4918.24	5957.032	7039.97	7958.963	8483.943	9267.647
14	834.71007	1727.141	2534.642	3694.656	5022.425	6106.162	7788.923	8584.88	10024.74	11012.51
15	808.61675	1648.516	2752.916	4163.385	5038.145	5754.46	7647.857	9789.15	10713.09	11575.41
16	461.6451	1414.839	2330.278	3360.187	4154.943	5008.125	6010.614	7031.726	8052.426	8938.116
17	999.14158	3050.442	4021.338	5096.24	6081.423	6897.242	7708.329	8733.234	9566.48	10216.15
18	716.65872	1328.877	2555.84	3505.987	4327.828	5190.859	6492.511	7872.683	8533.753	10352.23
19	1379.2087	2888.361	4015.161	4844.266	6344.847	7450.841	8442.255	9329.325	11248	12154.8
20	1464.5964	2133.316	3327.082	4807.904	5871.317	6910.135	7979.502	8643.403	10234.57	11173.6
21	1289.5663	1833.911	3217.855	4490.781	5265.236	6360.627	7377.569	8144.268	9114.027	9715.86
22	713.7075	1647.081	2692.236	4161.206	5779.477	7177.014	8323.757	9089.11	9855.508	11309.31
23	1407.1301	2923.342	3890.897	4455.01	5508.553	6191.443	7735.026	8592.406	9580.935	10582.47
24	645.66701	2231.816	3428.887	4193.631	5070.253	6020.435	6731.022	7909.727	8911.886	9823.755
25	789.45687	1297.97	1777.331	2869.652	3696.45	4600.202	5821.161	6459.904	7246.81	8430.462
26	728.17654	1204.309	2262.636	3309.315	4612.165	5261.181	6026.044	6677.823	8474.853	9255.765
27	1492.5876	2864.048	3804.503	4921.361	6278.806	7381.494	8007.656	9204.311	10476.16	13243.94
28	943.42487	1729.135	3568.41	5027.784	6745.86	7178.811	8675.564	10596.6	11766.48	13511.13
29	548.88382	1196.453	2399.984	3960	5012.382	6876.528	7655.965	9163.245	10247.82	11504.16
30	1380.7701	2215.405	3521.143	4185.535	5576.416	6367.278	7262.422	8500.924	9296.397	10044.27
31	876.82386	2186.279	3221.373	3958.099	5010.805	5975.021	6714.527	7442.957	8145.995	8927.624
32	1221.4308	3045.005	4759.991	5883.378	6738.051	7353.969	8501.025	9341.46	10380.3	11779.34
33	892.68515	2262.473	3196.371	4098.482	5089.626	6123.755	6937.526	7620.845	8884.618	9970.244
34	1405.7378	2038.622	2731.468	3585.497	5072.657	5684.724	6712.324	7931.941	9003.67	10201.93
35	1234.7566	2137.814	3462.58	4061.273	5035.456	6310.637	7969.187	8973.785	9830.232	10581.97

And similar 965 simulations more.

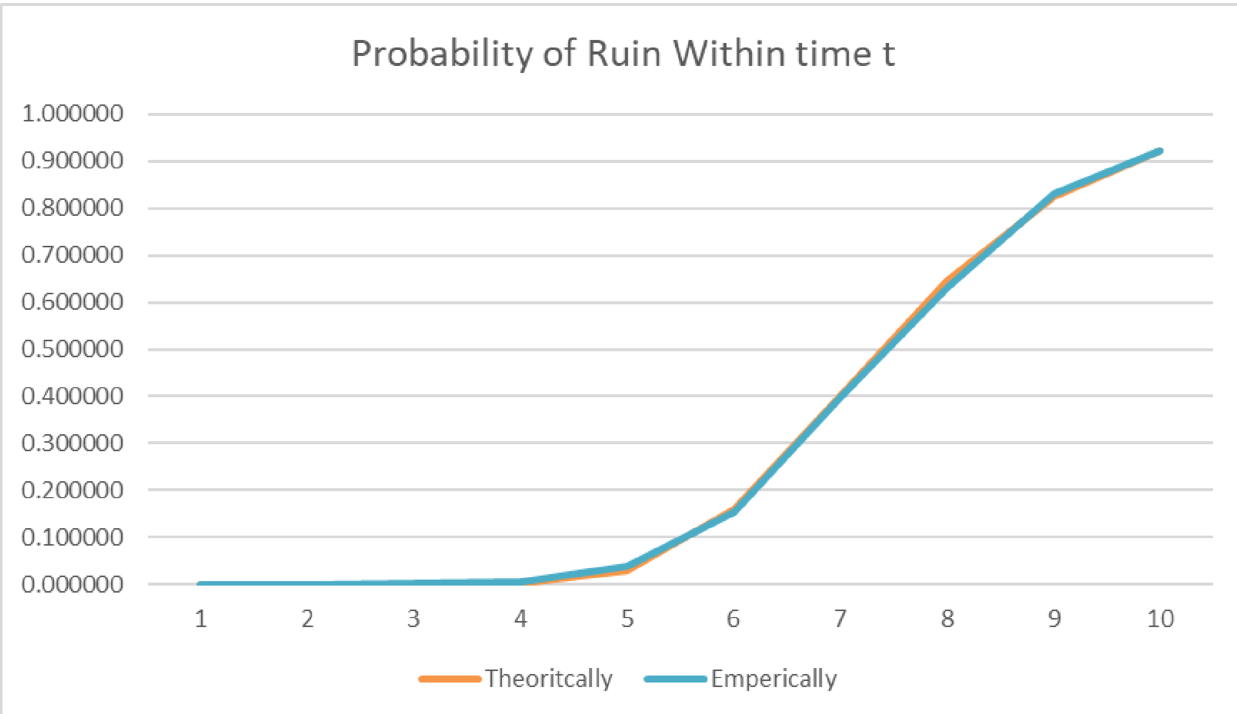
Generated Result

	Initial Cap	5000		**premium**	300					
	interest	0.02								
Sno	**Year 1**	**Year 2**	**Year 3**	**Year 4**	**Year 5**	**Year 6**	**Year 7**	**Year 8**	**Year 9**	**Year 10**
Capital	5400	5808	6224.16	6648.643	7081.616	7523.248	7973.713	8433.188	8901.851	9379.888
1	4229.965	3652.736	3193.535	2473.248	1959.468	1308.187	0	0	0	0
2	4181.185	3310.423	2996.179	2615.796	1979.384	1264.635	1171.827	736.0945	422.2187	80.21395
3	4599.145	4317.471	3899.494	3593.904	2027.494	1530.971	1382.448	1167.922	737.8724	0
4	4208.105	3960.531	3064.74	1562.191	856.9048	233.1911	0	0	0	0
5	4429.95	3868.139	3349.126	3141.758	1656.86	908.8761	266.082	0	0	0
6	4047.461	3685.12	1865.472	1311.158	806.6109	0	0	0	0	0
7	4651.281	3160.677	2949.281	2427.176	1413.675	555.6402	0	0	0	0
8	4516.007	4293.908	3526.857	3102.378	2907.836	2739.082	2298.716	1762.566	913.6286	878.8511
9	4040.148	3530.721	2200.965	1697.071	1053.407	480.6258	0	0	0	0
10	4657.713	3964.746	3029.878	2727.867	1884.366	1214.631	1463.252	205.5616	0	0
11	4586.595	3850.018	2770.422	2375.397	2318.609	2038.465	1803.34	1356.987	804.3117	493.4916
12	3877.695	2801.128	2623.454	2076.783	2003.007	1416.54	1088.974	808.7837	393.6505	0
13	4468.114	3898.288	2953.322	2447.484	2163.377	1566.216	933.7437	474.2247	417.9086	112.2415
14	4565.29	4080.859	3689.518	2953.987	2059.191	1417.087	184.7907	0	0	0
15	4591.383	4159.484	3471.244	2485.258	2043.471	1768.788	325.8561	0	0	0
16	4938.355	4393.161	3893.882	3288.456	2926.673	2515.123	1963.099	1401.461	849.425	441.7721
17	4400.858	2757.558	2202.822	1552.403	1000.194	626.0062	265.3847	0	0	0
18	4683.341	4479.123	3668.32	3142.657	2753.788	2332.39	1481.202	560.5043	368.0981	0
19	4020.791	2919.639	2208.999	1804.377	736.7693	72.40734	0	0	0	0
20	3935.404	3674.684	2897.078	1840.739	1210.299	613.1133	0	0	0	0
21	4110.434	3974.089	3006.305	2157.862	1816.38	1162.622	596.144	288.9197	0	0
22	4686.292	4160.919	3531.924	2487.437	1302.139	346.2347	0	0	0	0
23	3992.87	2884.658	2333.263	2193.633	1573.063	1331.805	238.6878	0	0	0
24	4754.333	3576.184	2795.273	2455.012	2011.363	1502.813	1242.692	523.4603	0	0
25	4610.543	4510.03	4446.829	3778.992	3385.166	2923.047	2152.552	1973.284	1655.041	949.4261
26	4671.823	4603.691	3961.524	3339.329	2469.451	2262.068	1947.67	1755.365	426.9986	124.1238
27	3907.412	2943.952	2419.657	1727.283	802.8096	141.7543	0	0	0	0
28	4456.575	4078.865	2655.75	1620.859	335.7556	344.4372	0	0	0	0
29	4851.116	4611.547	3824.176	2688.643	2069.234	646.7204	317.7486	0	0	0
30	4019.23	3592.595	2703.017	2463.108	1505.2	1155.97	711.2913	0	0	0

And similar 970 simulations more.

Ruin at time t	Frequency
3	3
4	3
5	34
6	113
7	243
8	234
9	202
10	91
No Ruin	77

Capital	t	u	sigma^2	sigma	P(Insurer will ruin within time t)	
					Theoritcally	Emperically
5400	1	1103.451	136107.1	368.9270	0.000000	0
5808	2	2206.902	272214.2	521.7415	0.000000	0
6224.16	3	3310.353	408321.3	639.0002	0.000003	0.003
6648.643	4	4413.804	544428.4	737.8539	0.001227	0.006
7081.616	5	5517.255	680535.5	824.9458	0.028959	0.04
7523.248	6	6620.706	816642.6	903.6828	0.158961	0.153
7973.713	7	7724.157	952749.7	976.0890	0.399103	0.396
8433.188	8	8827.608	1088856.8	1043.4830	0.647279	0.63
8901.851	9	9931.059	1224963.9	1106.7809	0.823791	0.832
9379.888	10	11034.51	1361071	1166.6495	0.921944	0.923



From the graph and table mentioned above, we can observe that the empirical values are close or approximately equal to the theoretical values.

	Theoretically	Emperically
Expected Time in which Company will get ruined	**7.018733**	**7.017**

Measures to Control Losses

- The insurance company can raise funds from other investors, market to increase the surplus.
- The insurance company can take reinsurance to control their losses. They can take proportional reinsurance or a retention limit reinsurance.
- The insurance company should invest their funds at a higher rate of return as 2% is very less.
- They can introduce a loading factor θ to cover up their expenses such as transaction costs and legal formalities. This is a very complicated factor to decide, generally, it lies between 0 to 1. The insurance company can't set this factor too high as it will drive the customer away and it can't be set too low.


Thank you!