Glueballs and Hybrids (Gluons as Constituents)

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After a brief introduction to hybrid and glueball source operators, I summarize recent lattice results for these particles.

In addition to the familiar baryons made from three quarks and mesons made from a quark and an antiquark, the spectrum of QCD is expected to contain glueballs, hybrids and multiquark states. Since the earliest lattice QCD calculations it has been recognized that lattice QCD should be used to calculate the properties of these particles. Hopefully, lattice results will help to sort out which of the many known hadrons contain important glueball or hybrid components. The experimental situation for glueballs and hybrids with light quarks was recently reviewed in Ref. [1]. Evidence for an exotic 1^{-+} particle has been seen in several experiments[2], and is now substantial enough to merit inclusion in the Review of Particle Properties[3]. Unfortunately, the mass of this particle, 1400 MeV, is quite a bit lower than the lattice results so far, and its decays conflict with phenomenological flux-tube models.

Of course, we would like to know everything about these particles — their masses, their decays, their wavefunctions, their mixings However, the place to start, and where most of the calculations have stopped, is with a calculation of the masses. This works just like spectrum calculations for ordinary hadrons: find an operator with the desired quantum numbers $\mathcal{O}_i(\vec{x},t)$, and compute

$$\int d^3x \langle \mathcal{O}_i(0)\mathcal{O}_j(\vec{x},t)\rangle = A_i^{(0)} A_j^{(0)} e^{-m_0 t} + A_i^{(1)} A_j^{(1)} e^{-m_1 t} + \dots$$
 (1)

The ground state is the easiest to find; for excited states you must manipulate several \mathcal{O}_i and look for combinations with $A_i^{(0)} = 0$, or simultaneously fit to several masses. Even for the ground

state, it is still important to construct an operator minimizing overlap with excited states.

To make these operators, we can combine the following ingredients:

 q^a : quark, color 3, \bar{q}^a : antiquark, color $\bar{3}$, $\bar{q}^a\Gamma q^a$: color singlet quark bilinear $\bar{q}^a\Gamma q^b$: color octet quark bilinear B^{ab} : color magnetic field, color 8, $J^{PC}=1^{+-}$ E^{ab} : color electric field, color 8, $J^{PC}=1^{--}$ \bar{L} : orbital angular momentum For example, to make the simplest glueballs you might use:

$$\begin{array}{l} B_i^{ab}B_i^{ab} = 0^{++} \\ B_i^{ab}B_j^{ab} - \frac{1}{3}Trace() = 2^{++} \\ B_i^{ab}E_i^{ab} = 0^{-+} \end{array}$$

One way to understand the construction of the simplest hybrid operators is to replace one of the glue fields in these glueball operators with a $\bar{q}q$ bilinear. In particular, take a quark and in the 1S state, "break" the color, and insert the color magnetic field. In its simplest form, called "Basic" in the upper left of Fig. 1, the color magnetic field is just the difference between the product of links in a small loop traversed clockwise and counterclockwise connecting the quark spinor to the antiquark.

The very first thing you might try is to take a pion operator, $\bar{q}^a\gamma_5q^a$, and combine it with the color magnetic field to get $\bar{q}^a\gamma_5q^bB_i^{ab}$. The quantum numbers of this operator are found by combining the quantum numbers of the pion and the magnetic field — $0^{-+}\otimes 1^{+-}=1^{--}$ — the same quantum numbers as the ρ .

Similarly, if you take the quark and antiquark

to have spin one, $\bar{q}^a\bar{\gamma}q^a$, you can combine it with the color magnetic field to produce three different quantum numbers: $1^{--}\otimes 1^{+-} = 0^{-+}\oplus 1^{-+}\oplus 2^{-+}$ 0^{-+} : $\bar{q}^a\gamma_iq^bB_i^{ab}$: (q. num. of pion) 1^{-+} : $\epsilon_{ijk}\bar{q}^a\gamma_iq^bB_j^{ab}$: (**exotic**) 2^{-+} : $\bar{q}^a\gamma_iq^bB_j^{ab}+(i\leftrightarrow j)-Tr()$: (quantum numbers of π_2)

Thus, if hybrid spectroscopy works like conventional meson spectroscopy or glueball spectroscopy, we expect that the lowest multiplet of hybrid states contains 1^{--} , 0^{-+} , 1^{-+} and 2^{-+} particles, both isovector and isoscalar, which will be split by the color hyperfine interactions. The most interesting of these is the 1^{-+} , which has exotic quantum numbers.

If we continue, and consider $\bar{q}q$ operators with the quark and antiquark in a P wave in the nonrelativistic limit, we can construct operators including exotics with quantum numbers 0^{+-} and 2^{+-} , and by using the color electric field you can make a 0^{--} exotic $(a_1 \otimes \vec{E} = 1^{++} \otimes 1^{--} = \mathbf{0}^{--} \dots)$.

The "Basic" cartoon of a hybrid operator in Fig. 1 is not really acceptable since it violates the cubic symmetries of the lattice — there is nothing special about the upper left plaquette. An obvious way to symmetrize it is to average over all the nearby plaquettes, producing the "clover" in the upper right of Fig. 1. Another way to proceed is to begin by moving the traversal of the plaquette in one of the two directions down to the lower left, as in the center of Fig. 1. This doesn't give the full cubic symmetry, but (with a few more loops out of the plane of the figure) does have rotational symmetry around one axis. Then one can imagine pulling the quark and antiquark apart along this axis, reaching the cartoon in the lower right of Fig. 1 by way of the intermediate stage in the lower left. In this way we make contact with a picture of a hybrid meson as an excited flux tube connecting the quark and antiquark.

For both glueballs and hybrids it is necessary to use "smeared" or "fuzzed" links for the gluon parts of the operator to get good overlap with the ground state particles. The details of how this is done vary, and more often than not the parameters are determined empirically.

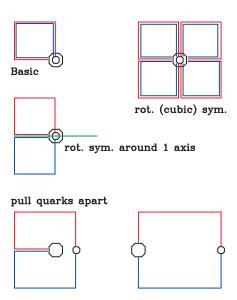


Figure 1. Construction of hybrid operators, illustrating the connection between the "local" operators and "excited potential" operators.

Along with the choice of the operator which creates the hybrid, we can choose how the quark and antiquark propagate. The ideal way is to use the full relativistic formulation. However, this is time consuming, and for heavy enough quarks it is much more sensible to use the nonrelativistic (NRQCD) propagators. In fact, one can go all the way to the infinite mass limit, and simply use static quarks. (This was the first method used to study hybrids on the lattice). In addition to its simplicity, the use of static quarks gives a very physical picture of the hybrid meson as a quark and antiquark moving in a potential that depends on the state of the flux tube connecting them (keywords: adiabatic potential, Born-Oppenheimer approximation), and within this approximation it is straightforward to compute the wavefunctions of the hybrids, as well as masses of excited states. (Since the quarks can't move in the static approximation, it is necessary to use the "flux tube" versions of the operators in Fig. 1 so that the quark and antiquark can be created at the desired separation.)

While the first lattice calculation of hybrids was

done in 1983, progress was slow until the last few years. Highlights of these calculations can be found in Refs. [4] through [19].

All of the lattice studies agree with our expectation that the 1^{-+} should be the lightest exotic hybrid, and this particle has received the most attention. To give a flavor for the state of the art, Table 1 shows a selection of results for the mass of the 1^{-+} hybrid or, in the case of heavy quarks, for the mass difference between the 1^{-+} hybrid and the $\bar{q}q$ S-wave mass. The tabulated results show a pleasing convergence with time, and for recent results (bold face) the agreement among different groups is quite satisfactory.

Glueball mass calculations have a longer history than hybrid mass calculations. Most have been done in the quenched approximation, perhaps partly because glueballs are the actual excitations of quarkless QCD. The state of the art is represented by three accurate calculations in the quenched approximation: a UKQCD calculation[20], a series of works by the GF11 group[21] and a recent calculation by Morningstar and Peardon[22]. The first two of these are conventional calculations, using isotropic lattices with the one-plaquette action, while the third uses anisotropic lattices with an improved action in the spatial planes. All three use results at several lattice spacings to get good control of the $a \to 0$ limit for the lowest glueballs. In addition, the anisotropic calculation[22] gets convincing results for a few of the excited states. As emphasized by Teper[23], the UKQCD and GF11 calculations of the 0^{++} glueball mass in units of the lattice spacing are in excellent agreement, and the difference in their reported results for the mass in MeV comes from different methods of assigning a lattice spacing. Some of this ambiguity is inherent in quenched calculations, since we expect that quenched mass ratios in the $a \to 0$ limit will differ from real world mass ratios. To compare these calculations I follow contemporary practice and use r_0 to set the lattice spacing. Ref. [22] quotes masses in units of r_0 , and for the conventional action calculations I convert to physical units using the interpolating formula for r_0 of Guagnelli, Sommer and Witting[24] using $r_0 = 0.50$ fm. With this definition of lattice spacing, some of the results of these three calculations are: UKQCD

```
0++
          1645(50) \text{ MeV}
 2^{++}
          2337(100) MeV
GF11
 0++
          1686(24)(10) \text{ MeV}
 2++
          2380(67)(14) MeV
Morningstar and Peardon
 0++
          1659(43)(16) \text{ MeV}
 2^{++}
          2304(8)(24) \text{ MeV}
 0^{-+}
          2494(28)(24) \text{ MeV}
 0++*
          2561(173)(28) MeV
 2++*
          3499(43)(35) \text{ MeV}
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All three calculations are in reasonable agreement for the 0^{++} and 2^{++} glueball masses, and the Morningstar and Peardon results show the state of the art in extracting excited state masses.

There is also a growing body of work, much of it by the GF11 group[21], on the decay rates of glueballs, and on the mixing of glueballs with quarkonia to produce the physically observed hadrons. Here the idea is to take the quenched glueball masses and the quenched light quark isoscalar and $\bar{s}s$ quarkonia masses, together with either a measurement or a model for their overlap. and diagonalize a matrix to find the physical states. (Oddly, this is a calculation where we actually prefer a quenched mass spectrum to a full QCD lattice calculation.) The eigenstates are then identified with the experimental $f_0(1370)$, $f_0(1500)$ and $f_J(1710)$ (assuming J=0). Such an analysis has been done by the GF11 group using their own numerical results[21] and by Teper using numerical inputs from the literature [23].

Teper proposes:

```
f(1710) = 0.42g + 0.90\bar{s}s + 0.13\bar{u}u

f(1500) = 0.77g - 0.43\bar{s}s + 0.48\bar{u}u

f(1370) = -0.49q + 0.10\bar{s}s + 0.87\bar{u}u
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while the GF11 group suggests:

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f(1710) = 0.86(5)g + 0.30(5)\bar{s}s + 0.41(9)\bar{u}u
f(1500) = -0.13(5)g + 0.91(4)\bar{s}s - 0.40(11)\bar{u}u
f(1370) = -0.50(12)g + 0.29(9)\bar{s}s + 0.82(9)\bar{u}u
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where g is the glueball component, $\bar{s}s$ the strange onium component, and $\bar{u}u$ the (isoscalar) light quark component.

These results are quite different. The largest difference comes from differences in the unmixed masses, with Teper having $m_{\bar{u}u}=1.36$, $m_{\bar{s}s}=1.61$ and $m_g=1.48$ GeV and the GF11 group having $m_{\bar{u}u}=1.47$, $m_{\bar{s}s}=1.51$ and $m_g=1.63$ GeV. Note that in the first analysis the unmixed $\bar{s}s$ is heavier than the glueball, while in the second it is lighter. Not surprisingly, the heaviest physical particle is mostly $\bar{s}s$ or mostly glueball, respectively. Clearly it is important to know whether the quenched $\bar{s}s$ is heavier than the quenched glueball, and the GF11 group has attacked this question by calculating quarkonium and glueball masses on the same samples.

It is also interesting to test the effects of dynamical quarks on the glueball spectrum. In principle this is tricky, since a full QCD spectrum calculation with a glueball source operator will produce the masses of the physical states, which are mixtures of glueballs and quarkonia. In practice, the calculations that have been done so far have used quark masses large enough that the quarkonium mass exceeds the glueball mass, and it is reasonable to simply say thay they have measured the glueball mass. The largest calculation is from the $T_{\chi}L/SESAM$ collaborations[25], and another preliminary result from UKQCD was presented at this conference [26]. The $T\chi L/SESAM$ results for the 0⁺⁺ glueball are within errors of the quenched results, although they do see hints of a larger dependence on the lattice size. However, the preliminary UKQCD results have a much smaller mass, despite being done at approximately the same sea quark mass, as measured by m_{π}/m_{ρ} . The UKQCD results are done on much coarser lattices, albeit with the clover action rather than the Wilson quark action. (UKQCD used $\beta = 5.2$ while $T\chi L/SESAM$ used 5.6.) The 0^{++} glueball mass has long been known to be small on coarse lattices[27], but the preliminary UKQCD results are even smaller than we would expect from our experience with the quenched theory.

Since the exotic 1⁻⁺ signal found in experiments at 1400 MeV is much lower than expected from lattice calculations (and most other theoretical approaches), it is tempting to ask whether it could be something else, most likely a 4-quark

Table 1

Some results for 1^{-+} hybrid masses				
Date	Ref.	Method	$\Delta M \; (\text{GeV})$	
$ar{b}bq - ar{b}b$:				
1990	[9]	St.	1.11(3)(?)	
1993	[10]	NR.	0.8(?)(?)	
1997	[13]	NR.	1.68(10)	
			1.40(14)	(1)
1997	[15]	NR.	1.14(21)	
1997	[14]	St.	1.3	
1998	[18]	NR.(An.)	1.542(8)	
1999	[19]	St+NR(An.)	1.49(2)(5)	
$\bar{c}cg - \bar{c}c$:				
1990	[9]	St.	0.94(3)	
1996	[12]	Rel.(Wil.)	1.34(8)(20)	
1998	[17]	Rel.(Clo.)	1.22(15)(?)	
1999	[18]	NR.(An.)	1.323(13)	
$ar{s}sg$			M (GeV.)	
1996	[11]	Rel.(Clo.)	2.00(20)	(2)
1996	[12]	Rel.(Wil.)	2.17(8)(20)	(3)
$\bar{u}dg$				
1996	[11]	Rel.(Clo.)	1.88(20)	(4)
1996	[12]	Rel.(Wil.)	1.97(9)(30)	(5)
1998	[16]	Rel.(Wil.)	1.90(20)	(6)
1998	[17]	Rel.(Clo.)	2.11(10)(?)	(7)
Abbreviations: $St. = Static$, $NR. = NRQCD$, Rel .				
$= {\it Relativistic, An.} = {\it anisotropic, Wil.} = {\it Wilson,}$				
Clo. = clover.				

Notes: (1): value with a determined differently, (2): a = 0.095 fm, (3): a = 0.075 fm, (4): Model to extrapolate to $m_q = 0$, 120 MeV below $\bar{s}s$ mass, a = 0.095 fm, (5): Extrapolation from several am_q values, a = 0.075 fm, (6): $N_f = 2$ dynamical quarks, extrapolate from several am_q values, a = 0.086 fm, (7): Same as (5).

 $(\bar{q}\bar{q}qq)$ state. In principle, this question is answerable with lattice methods, but it is a difficult subject. Nonetheless, there is a small but growing body of work on lattice 4-quark states [28,29], beginning with the simplest case where all four quarks are static, and moving into the case of two static and two moving quarks. An amusing limit has recently been studied by Michael and Pennanen [29], where the two quarks are very heavy, and the two antiquarks light. The two quarks have an attractive interaction in the $\bar{3}$ color combination, and since they are very heavy they can bind into

a small object which is effectively an antiquark. Then the three antiquarks may be regarded as an antibaryon, with one constituent very heavy.

I expect that lattice calculations will continue to be important in sorting out the rich structure of the QCD spectrum. It is important to extend our work to calculations of decay rates and mixings among particles.

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