GLUEBALL MASSES AND STRING TENSION IN LATTICE QCD

Ape Collaboration

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We study glueball masses in lattice QCD. We present the first numerical determination of the mass of the lowest spin 2 state in the scaling region, and find that it is close to the lowest spin 0 state. We present very precise results for the string tension and for the spin 0 state, obtained by analyzing a large set of operators. We find that finite size effects are significant.

Numerical simulations of lattice QCD allow a determination of glueball masses. These masses are determined by the large-time behaviour of the connected correlation function

$$G(t) \equiv \langle O(t) \cdot O(0) \rangle - \langle O(t) \rangle \cdot \langle O(0) \rangle \approx \exp(-mt) ,$$

where $O(t)$ is a suitable gauge invariant operator. The problem lies in the fact that $G(t)$ decreases rapidly, and is obtained as a difference of two fluctuating quantities of comparable magnitude. This difficulty, already challenging in the case of the spin 0 state, increases in the case of higher spin states. As a consequence the determination of the mass of the 2^{++}

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state is the object of a heated dispute. In this paper we try to solve this question, both by brute force (using our newly built Ape computer) and by methodological refinements – we propose a new method for choosing useful operators. The result is very satisfactory, and we succeed in measuring the mass of the 2^{++} state, that turns out to be comparable to that of the 0^{++}. We also give very precise estimates of the 0^{++} state mass and of the string tension, carefully studying the finite-size effects, and the convergence to the infinite-volume result. We study large lattices, but also small ones. We think this is the only way to define what a large lattice is.

We study a pure gauge lattice SU[3] theory, with a standard Wilson action. We use a Metropolis updating scheme; we sweep sequentially through the lattice, and we try 5 updates of each link (5 hits, in the usual jargon). We keep the single-hit acceptance close to 50%. We renormalize the SU(3) matrices and we measure all our observables once each 20 sweeps of the full lattice, except for the average energy, which is measured at every sweep.

The results of this paper have been obtained at $\beta \equiv 6/g^2 = 5.9$. We have chosen this value in order to achieve a good control on the finite size and finite-time effects, working as deep as we can in the scaling region. The subject of the scaling behaviour of our data will be studied in a forthcoming paper [1]. We work on lattices with periodic boundary conditions; we keep the time length of the lattices equal to 32 (the euclidian time label t will run in the following from 1 to 32), and we consider spatial volumes of 10^3, 12^3 and 16^3 sites. In this way we can study finite-size effects with no contamination from finite-time effects, which should be negligible. We start from a $\beta = \infty$ gauge field configuration, discarding at least 1500 configurations (4000 for the 10^3 lattice) for thermalization. We have then performed 64000 iterations on the 10^3 lattice, 56000 on the 12^3 and 24000 on the 16^3.

This computation has been performed on a 4 unit Ape computer [2], a general-purpose parallel processor, whose design has been optimized for the solution of homogeneous differential equations, and more specifically for dealing with lattice QCD. This 4 unit Ape is the first running prototype of the machine. It has a peak speed of 256 Mflops in complex arithmetics, and a 32 Mbyte memory. The whole Monte Carlo program is written in a high-level language which, especially designed for Ape, is cross-compiled on a Vax. The program updates a link in 50 μs (we estimate that with some programming effort we could improve the performance of our code by a significant factor). The results described in this paper have been obtained in about one month of continuous runs. The market cost of performing the same computation on a commercial supercomputer would have exceeded the entire cost of the Ape-INFN project, including the construction of two 4 unit Apes and a 16 unit computer, a one Gflops machine.

We measure the decay of correlations from a cold source [3] at $t=1$. This gives a stronger signal than in the case of operator–operator correlation functions, but also introduces a strong distortion at short distance from the source. We choose this method because it seems to be very effective on large lattices. Work done by using operator correlations (and the smearing technique we introduce below) is in progress [4]. We construct the source by setting to the identity all the $t=1$ links pointing in the $\hat{x}$ and in the $\hat{y}$ directions. With such a source we can monitor the behaviour of the 0^{++} and the 2^{++} gluonic states, and of the string tension. We estimate the energy of the system in absence of the source by averaging over time slices that are at least 8 lattice spacings away from the cold wall. We get results that are compatible for the 3 lattice sizes, that is 0.58187(6) at 10^3, 0.58182(4) at 12^3 and 0.58187(3) at 16^3. We have checked that these values are really asymptotic, in the sense that the mean value does not systematically change if we discard more time slices close to the source.

As we already mentioned we consider different operators, that we define by using a *smearing* procedure. The general point of view is the one proposed in refs. [3,5] that is we try to eliminate as much as possible the unphysical short-distance fluctuations. We use a sequence of operators $O^{(s)}$, such that $O^{(0)}$ is the elementary space-like plaquette (the four links that form it are pointing in 2 of the 3 space directions). The operator $O^{(s)}$ is recursively defined from the operator $O^{(s-1)}$ by applying a smearing procedure (see fig. 1)

$$U_i(n) \to U_i(n) + \epsilon \sum_{\eta=+1,-1} \sum_{j \neq i} P^{\eta}_{ij} , \qquad (1)$$

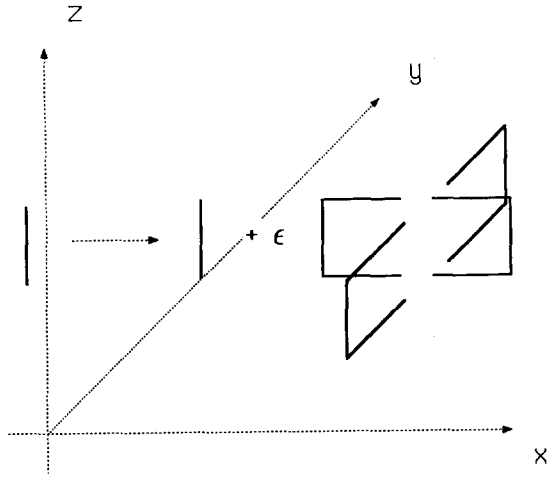

Fig. 1. The smearing procedure. We substitute a link with itself plus ϵ times the sum of the incomplete neighboring space like plaquettes, and we project it back into SU(3).

where the RHS of (1) is projected onto the SU(3) group, and $P^{\eta}_{i,j}$ is the oriented incomplete plaquette operator, product of 3 gauge matrices. It lies in the (i,j) plane, excluding the link $U_i(n)$. For $\eta=+1,-1$ $P^{\eta}_{i,j}$ has $U_i(n)$ as its left or right edge. In fig.1 we show this transformation. We use the analogous construction for the Polyakov loops.

This transformation averages the gauge fields with their neighbours, suppressing in this way the short-wavelength fluctuations in the gauge invariant sector. To clarify the meaning of this procedure we can consider the effect of a similar rule applied to a scalar field

$$\varphi^{(s)}(n)\to\varphi^{(s+1)}(n)=\varphi^{(s)}(n)+\epsilon\sum{}'\,\varphi^{(s)}(n')\,,$$

where the sum runs over the first space-like neighbours of n. For large s and small ϵ we can define a proper time for the smearing dynamics, $\tau=s\epsilon$, so that

$$\partial\varphi/\partial\tau=\nabla^2\varphi\,. \tag{2a}$$

In momentum space, the effect of the smearing operator S is the multiplication by $\exp(-\tau k^2)$. The same argument can be applied to our case. In the continuum limit, with proper time $\tau=s\epsilon$, eq. (1) is equivalent to

$$\partial A_i/\partial\tau=D_i\,F_{ik} \tag{2b}$$

and, for gauge invariant quantities, the smearing operator is easily seen to correspond, neglecting the non-linearities of eq. (2a), to multiplication by $\exp(-\tau k^2)$. If we assume that the wave function of the 0^{++} glueball is gaussian (i.e. $\propto\exp(-Wk^2)$), and we measure the expectation value of the smeared plaquette (that in the continuum limit corresponds to $F_{ij}^2\approx k^2$), we get, for $\tau\to\infty$,

$$\langle 0^{++}|O_{(k)}|0^{++}\rangle=\langle 0^{++}|S_{(k)}\,O_{(0)}|0^{++}\rangle$$

$$\approx(W+\tau)^{-5/2}\,. \tag{3}$$

One can see some analogies between this method and the Monte Carlo Renormalization Group ideas. We like in our method the idea that we do not have to choose a priori some operators (small in practice) on which to perform the matching. We consider instead a "continuous" set of operators that become very large after many smearing steps. If we get (as we do) results that are independent from the operator (at least for large smearing) we can be sure that these results are not influenced from a scale (the dimension of the matching operator) that we have chosen by hand.

We compute the values of the glueball masses (of quantum numbers $J^{P,C}$) $m(0^{++})$ and $m(2^{++})$. We also compute the value of the string tension, by monitoring the behaviour of the Polyakov loops at large separation from the source. The 0^{++} Green function is obtained by summing

$$G_0^{(s)}(t)\equiv O^{(s)}(x,y)+O^{(s)}(y,z)+O^{(s)}(x,z)\,, \tag{4}$$

where the plaquettes O are evaluated at time t, and the s indicate the different operators corresponding to different smearing. The source is at $t=1$ in the xy plane. For the 2^{++} state we have

$$G_2^{(s)}(t)\equiv -2O^{(s)}(x,y)+O^{(s)}(y,z)+O^{(s)}(x,z)\,. \tag{5}$$

The two gluonic channels are contaminated by a state of mass $2L\sigma$, where σ is the string tension and L is the spatial extent of the lattice (see ref. [6] and references therein for discussions about this point). From what is known in the current literature [7–9] it is clear a piori that for $\beta=5.9$ the situation is very dangerous on a 10^3 lattice. Indeed in this case $20\sigma\approx m(0^{++})$. The situation is analogous, if not worse, in the 2^{++} channels. In such conditions when we look at the ground state in the glueball channel

we do not really know what we are observing.

Our data analysis tries to use as few prejudices as possible. The effective masses are a very usefull tool. They are defined as the log of the ratio of the correlation function at time t with the one at $t+1$, but for the correction of the hyperbolic cosine effect coming from the boundary (in most of our data this correction is negligible). We obtain a reliable estimate when the effective mass does not depend on the time at which it is estimated, and on the operator used in the measurement. We stress that we are working with a very large set of operators, and that all the results we give here do not depend on the operator we consider.

In figs. 2a, 2b we give a typical example of what we mean by stability of the result. For the 12^3 lattice we plot the effective string tensions at different times versus the operator we consider. The operator is plotted as a function of the inverse smearing number. The points at $(\text{smearing})^{-1}=1$ are the ones one would get by considering the usual Polyakov loops. The results from $G(1)/G(2)$ are strongly dependent on the choice of operator. Already at time 2 over 3 the result is less dependent on the operator, and at this scale the results of $G(3)/G(4)$, $G(4)/G(5)$, $G(5)/G(6)$ and $G(6)/G(7)$ are identical. In fig. 2b we just show the largest operators (higher smearing), by magnifying fig. 2a, to evidentiate the residual 10% difference between times $G(3)/G(4)$ and $G(4)/G(5)$. $G(5)/G(6)$ and $G(6)/G(7)$ coincide with $G(4)/G(5)$, but with a larger statistical uncertainty. The crossing of two different curves for operators with $s\approx 10$ can be interpreted as a change of sign of the coefficient of the first excited state. This interpretation is confirmed by a two-state fit of the t dependence at different smearing, and offers a possible way to estimate the mass of the lowest state by using the point where two curves at different euclidian time cross (effective mass versus the smearing number). That will be a point in which the effect of the first excited state is very suppressed. We have verified that such an estimate, given for example by using the crossing of $G(2)/G(3)$ versus $G(3)/G(4)$, which can be determined accurately, is systematically lower by 5% than the infinite-time plateau.

We also use standard fitting routines. Our data are good enough to allow precise fits, which use the information coming from very large euclidian times

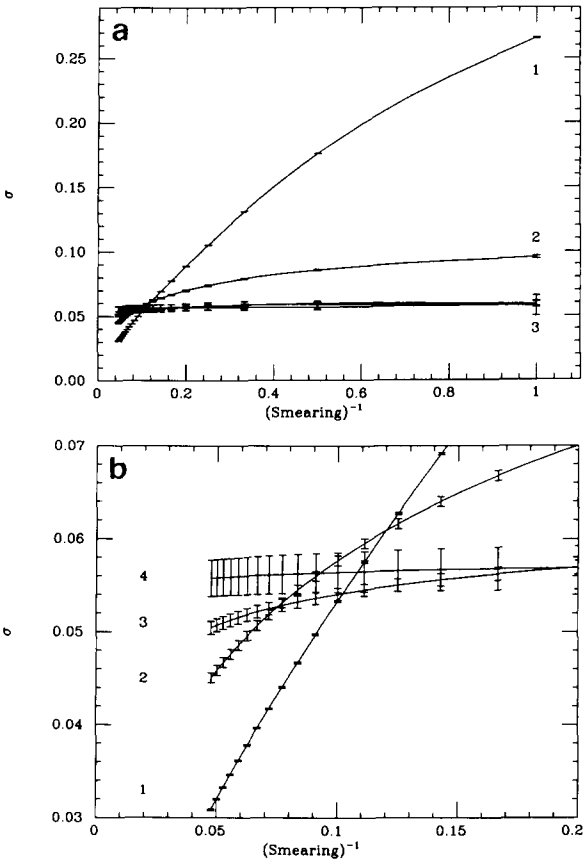

Fig. 2. The effective string tension as a function of the smearing. (a) At $(\text{smearing})^{-1}=1$, starting from the top, we have the curve (1) derived from $G(1)/G(2)$, next curve (2) is the one from $G(2)/G(3)$. The curves (3) for $G(3)/G(4)$, $G(4)/G(5)$ and $G(5)/G(6)$ are undistinguishable on this scale. (b) On a large scale one can see the effect obtained when going from $G(3)/G(4)$ to $G(4)/G(5)$ (respectively curves 3 and 4). The effective masses at higher times would coincide with $G(4)/G(5)$.

(up to a distance of order 10). We have found that the best approach was to use the simplest fit ($G\approx A\exp(-mt)$, with a single mass), discarding enough points close to the origin. This approach works, and we obtain fitted results which are independent from the operator and from the lowest time used in the fit. We have also performed two-mass fits including all points but one. The value of the low mass obtained in these fits agrees with that obtained in the simpler fits. The value of the high mass fluctuates too much when we divide our data in independent subsamples, and cannot be considered meaningful.

The last note about the analysis is that one can try to use results such as the one obtained in eq. (3) in order to get, for example, informations about the shape of the wave function. The results of the global fits are compatible with those from the fits described above.

Quoting our numerical results we will always quote what we judge to be true errors, evaluated over uncorrelated measurements. This is true both for effective masses and mass ratios, and for the results of fits done by using standard minimization programs. In the case of fitted results, due to the high non-linearity of the observables, we never trust the standard error given by the fitting routine, but we always repeat the fit over n independent subsamples, and take as standard error the dispersion over the results of different fits. As central value we quote the result of the single fit performed over the whole statistical sample. For the 10^3 lattice we consider five subsamples of 12000 consecutive iterations each, for 12^3 seven groups of 8000 iterations, and for 16^3 six groups of 4000 iterations.

We now present our results. We start from the 0^{++} gluonic state. For the 10^3 lattice we estimate from the effective masses

$$m(0^{++}, L=10)=0.65(3)\,. \tag{6}$$

A one-mass fit with $t_{\min}=4$ gives a slightly higher result, independent from the operator, but with a larger statistical error. On the 12^2 lattice from the effective masses we get

$$m(0^{++}, L=12)=0.76(4)\,. \tag{7}$$

A one-mass fit with $t_{\min}=4$ gives a very accurate result (with a mass that turns out to be independent from the operator) of 0.75(5). In fig. 3 we plot the result of the fit versus the operator we have used in the fit. In this case, like in all the other ones, a two-mass fit is not very useful.

For the 16^3 lattice we get from the effective masses

$$m(0^{++}, L=16)=0.82(5)\,. \tag{8}$$

The same fit we used for 12^3 gives a very nice, operator-independent result, with a slightly higher error, 0.83(7). The results seem to exhibit a possible residual finite-size effect of an order of 5%, between $L=12$ and $L=16$. We will discuss later the physical meaning of such results.

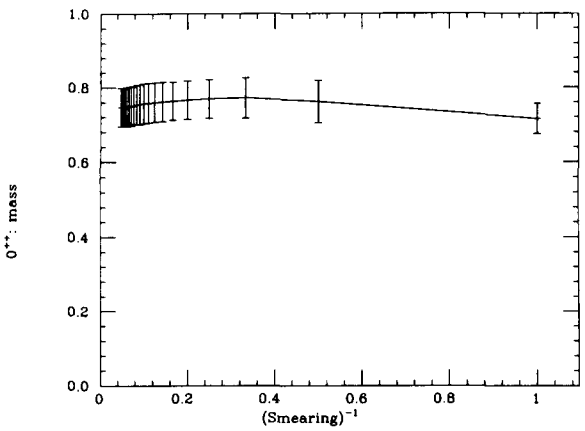

Fig. 3. Result of a fit for the 0^{++} mass. 12^3 lattice, $t_{\min}=4$.

The amplitudes one gets from the one-mass fits are very reasonable, and qualitatively consistent with the behaviour predicted in eq. (3), and with the global fits, in which we were using as inputs all the operators.

For the 2^{++} state we find useful results up to $t=4$. The signal on the fifth time slice is already too noisy to be really usefull. The results obtained on the 10^3 lattice have a shape that is different from the other two lattice sizes. This is not a surprise as we know from the larger lattices that $m(2^{++})$ is close to $m(0^{++})$. In this circumstance we expect a large contamination from the string tension, which on a 16^3 lattice corresponds to a state of $2L\sigma \approx 1.0$. If we want to read a mass from the 10^3 lattice we have to quote, from the crossing of $G(2)/G(3)$ and $G(3)/G(4)$ that $m(2^{++}, L=10)=0.6(3)$. For the 12^3 and 16^3 lattice we get a nicer signal from $G(3)/G(4)$. The result is still dependent on the operator we choose, but from the crossing of $G(2)/G(3)$ and $G(3)/G(4)$ we get

$$m(2^{++}, L=12)=0.80(20),$$

$$m(2^{++}, L=16)=1.00(15)\,. \tag{9}$$

We estimate that the systematic error of these results is by far smaller than the statistical error (previous considerations on the 0^{++} glueball may suggest a systematic positive shift of 0.05). If we average the Green functions over the 12^3 and the 16^3 lattices we find

$$m(2^{++}, L=12 \text{ and } 16)=0.85(12)\,. \tag{10}$$

In this case we also have a signal for $G(4)/G(5)$

Table 1
The observables we have considered versus the lattice size. $m(0^{++})/\sqrt{\sigma(L)}$ is computed directly from an effective ratio. $\sigma(\infty) \equiv \sigma(L) + \pi/(3L^2)$.

L	$m(0^{++})$	$m(2^{++})$		$\sigma(\infty)$	$\frac{m(0^{++})}{\sqrt{\sigma(L)}}$
10	0.65(3)	0.6(3)		0.058(3)	3.0(2)
12	0.76(4)	0.8(2)		0.062(1)	3.2(2)
16	0.82(5)	1.00(15)	0.85(12)	0.058(2)	3.4(4)

for large operators, consistent with the previous estimate. That makes our claim about the systematic error stronger.

The string tension is measured very precisely. The results are given in table 1, together with the ones for the glueball states. In this case a very clear operator and time independence gives a result compatible with the one coming from the fit, and with that coming from the crossing point.

Our data are in reasonable agreement with a $\pi/(3L^2)$ [10,8] contribution to the string tension, coming from the gaussian string. The difference in the value of the string tension we would expect between 12^3 and 16^3, due to the $\pi/3$, would be 0.003. We have a difference of 0.001 in the wrong direction, but with an error of 0.002. Instead the deviation of the σ measured on the 10^3 lattice from the asymptotic value can be explained by such an effect (we do agree with the numerical result obtained in ref. [8] on the 10^3 lattice at the same β). If we include the $\pi/3$ we get (in the following we assume $\sqrt{\sigma} = 420$ MeV)

$$a^{-1} = 1700(50)\ \text{MeV}\,, \tag{11}$$

that would be raised by 100 MeV neglecting the $\pi/3$ term. This is equivalent to

$$\Lambda_L/\sqrt{\sigma} = 10.7(2) \times 10^{-3} \tag{12}$$

a value that is 10% higher than that obtained in ref. [7] from the asymptotic behaviour of large Wilson loops at $\beta = 6.2$.

We have also measured directly $m(0^{++})/\sqrt{\sigma}$. We give these results in table 1. At the 10% level we are not able to exhibit any statistically significant lattice dependence of this ratio. We thus agree with the result obtained with a different action in ref. [11], and our 0^{++} glueball has a mass of 1400(100) MeV.

The main result of this paper is that the 2^{++} state has a low mass, with a ratio $m(2^{++})/m(0^{++})$ of order one (with an error of order 20%). This is a reasonable result, in general agreement with what we usually learn about spin forces for quarks. With our statistics we cannot say which state is the lighter one (we have indications, if any, for the 0^{++} being lighter then the 2^{++}), but the point of view assumed in ref. [4]. seems to lead to a roughly correct picture. This finding is in contrast with the claims of ref. [12], asserting asymptotic scaling of the 2^{++} state with $m(2^{++})/m(0^{++}) \approx 3$.

Our result for the 0^{++} mass on the 10^3 lattice agrees with the one given in ref. [9]. On the contrary the results on larger lattices are very different from the result of extrapolating from the 10^3 lattice by means of the small volume results [13]; the dimensionless three-glueball coupling estimated in (7) was already very high (i.e. $O(10^2)$). The same procedure applied to our data would give a three-glueball coupling of the order of 10^3, but for such large couplings the inclusion of multiple exchanges cannot be avoided. We rather believe that the finite-volume effects on the glueball masses originate from the mixing with states of mass $2L\sigma$ [14].

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