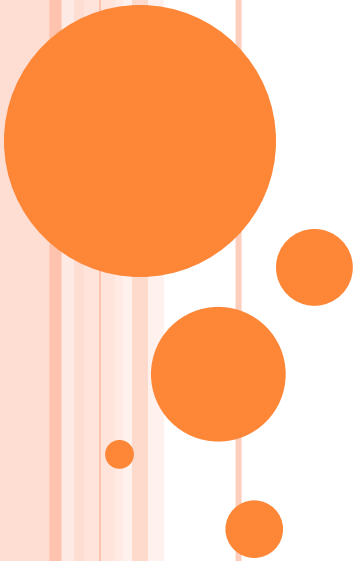

GRAPH THEORY

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VERTEX

- A vertex is usually conceptualized as a point. Abstractly, it is a member of the first of two sets that form a graph.
- Sometimes called as a node.

EDGE

- An edge is usually conceptualized as a line, either joining one vertex to another or joining a vertex to itself. Abstractly, it is a member of the second of two sets that form a graph.
- Sometimes called as a line.

PROPER EDGE

- A proper edge (or link) is an edge that joins one vertex to another.
- Most of the time when we talk about edges, it is a proper edge.

LOOP

- A loop (or self-loop) is an edge joining a vertex to itself.
- A graph is has no loop if it is told explicitly.

ENDPOINTS

- The endpoints of an edge are the vertices that it joins. A loop has only one endpoint.

GRAPH

- A graph is a set V of vertices and a set E of edges (both sets finite unless declared otherwise) such that all the endpoints of edges in E are contained in V .
- It is often denoted G = (V, E),or (V_G, E_G),or (V (G), E(G)).
- Sometimes, each edge is regarded as a pair of vertices.

NEIGHBOR

- A neighbor of a vertex is any vertex to which it is adjacent.

WEIGHTED GRAPH

- A weighted graph is a graph model in which each edge is assigned a number called the weight or the cost.

DIRECTED GRAPH

- A directed graph or digraph is a graph in which every edge is directed.

DEGREE

- The degree of vertex v, deg(v), is the number of proper edges plus twice the number of loops incident with v. Thus, in a drawing, it is the number of edge-endings at v.

SUBGRAPH

- A subgraph of a graph G = (V_G, E_G) is a graph H = (V_H , E_H) whose vertex set and edge set are subsets of V_G and E_G, respectively, such that for each edge e in E_H ,the endpoints of e (as they occur in G) are in V_H .

WALK

- A walk in a graph is an alternating sequence v_0, e_1, e_1, . . . , e_r , v_r of vertices and edges in which each edge e_i joins vertices v_{i-1} and v_i.

PATH

- A path is a trail in which all the vertices are different, except that the initial and final vertices may be the same.

CONNECTEDNESS

- A graph G is connected if for each pair of vertices in G, there is a path in G that contains them both.

CYCLE

- A path that the initial and final vertices are the same.

TREE

- A tree is a connected graph without any cycles as subgraphs.

FOREST

- A forest is a graph without any cycles as subgraphs.

DISTANCE

- The distance d(v, w) between two vertices v and w of a graph is the length of a shortest path between them, with d(v, v) = 0 and d(v, w) = ∞ if there is no path between v and w.

HAMILTON CYCLE

- A Hamilton cycle in a graph [digraph] is a cycle [directed cycle] that includes all vertices of the graph.

HAMILTONIAN GRAPH

- A graph or digraph is Hamiltonian if it contains a Hamilton cycle.

HAMILTON PATH

- A Hamilton path in a graph [digraph] is a path [directed path] that includes all vertices of the graph.

COMPLETE GRAPH

- The complete graph K_n is a simple graph with n vertices in which every pair of vertices is adjacent.

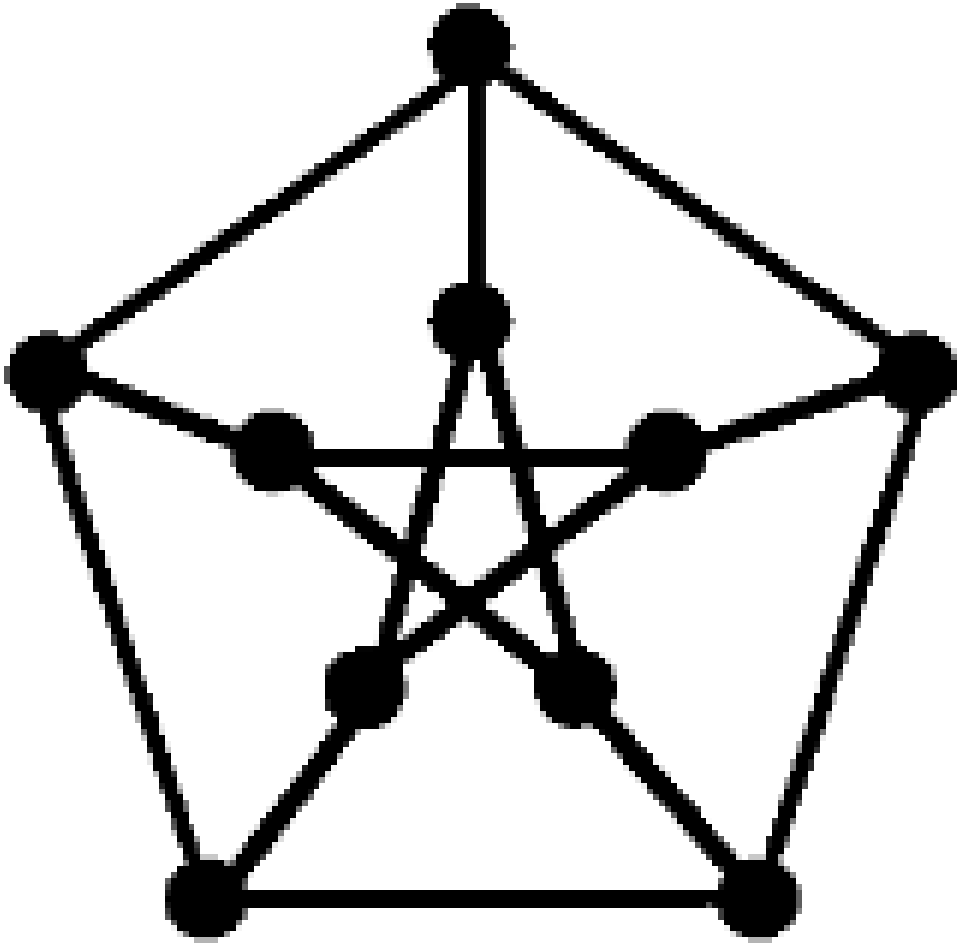
BIPARTITE GRAPH

- A bipartite graph is a graph whose vertices can be divided into two disjoint and independent sets U and V such that every edge connects a vertex in U to one in V. Vertex sets U and V are usually called the parts of the graph.

COMPLEMENT

- Let G = (V, E) be a graph. The complement $\bar{G}$ of G is a graph with the same vertex set as G and E($\bar{G}$) = {e ∉ E(G)}. i.e. G has edges exactly where there are no edges in G.

PETERSEN GRAPH



ENDPOINT TABLE

- An endpoint table for a graph is a tabular description of the incidence rule, that gives the endpoints of every edge. In a digraph or partially directed graph, the tail and head of each directed edge are distinguished, for instance, by marking the head, or by always giving the tail first.

ADJACENCY MATRİX

- The adjacency matrix of a loopless graph G with vertices $v_1, v_2, \ldots, v_n$ and edges $e_1, e_2, \ldots, e_m$ is the n × n matrix A_G with A_G [i, j] = the number of edges between v_i and v_j if i = j.
- If there are self-loops, then A_G[i, i] is usually defined to be the number of loops at v_i.
- It has some really good features.

QUESTIONS

- Euler’s theorem: In every graph, the sum of the degrees equals twice the number of edges.

- Let G be a disconnected graph. Prove that its complement $\bar{G}$ is connected.

- Let G be a connected graph. Prove that two paths which are both a longest path in the graph, contain at least one vertex in common.

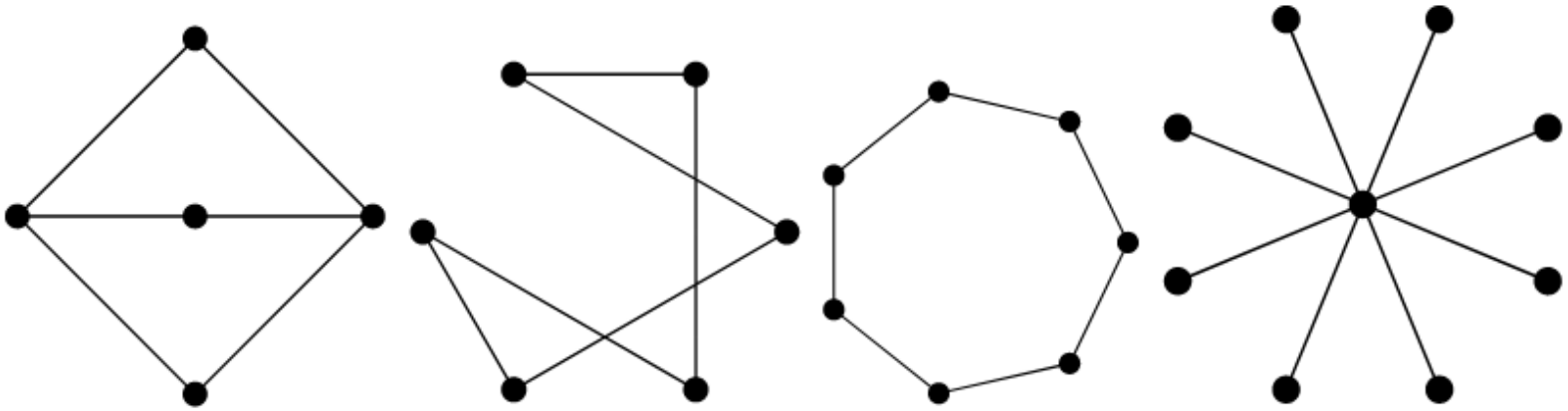
- Let n be a positive odd integer. There are n computers and exactly one cable joining each pair of computers. You are to color the computers and cables such that no two computers have the same color, no two cables joined to a common computer have the same color, and no computer is assigned the same color as any cable joined to it. Prove that this can be done using n colors.

- (Characterization of Trees) Let G be a connected graph with n vertices. The following statements are equivalent:

1. G does not contain any cycles.
2. G contains exactly $n-1$ edges.
3. For any two vertices, there exists exactly one path joining the two vertices.
4. The removal of any edge disconnects the graph

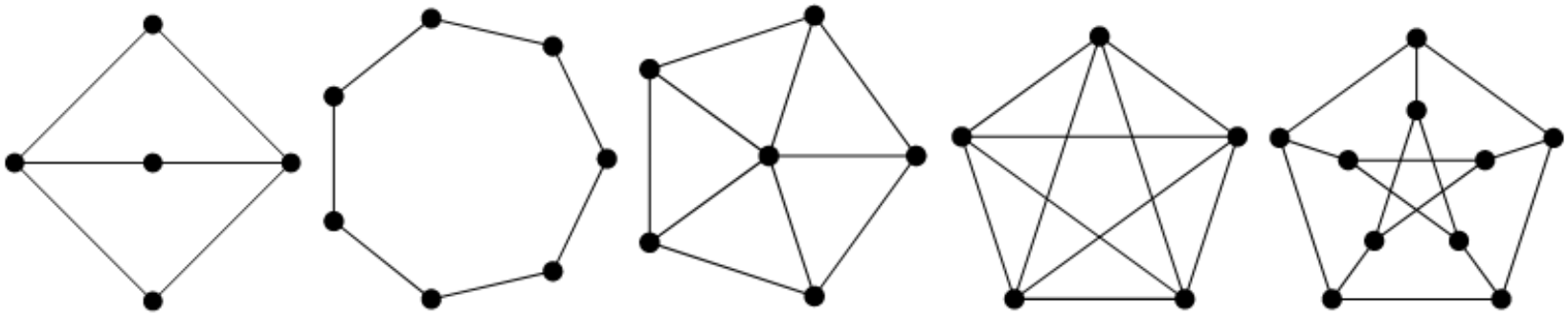
- Among a group of 5 people, is it possible for everyone to be friends with exactly 2 of the people in the group? What about 3 of the people in the group?

- Which of the graphs below are bipartite?



- There are n pieces of candy in a pile. One is allowed to separate a pile into two piles, and add the product of the sizes of the two new piles to a running total. The process terminates when each piece of candy is in its own pile. Show that the final sum is independent of the order of the operations performed.

- What is the smallest number of colors you need to properly color the vertices of these?



- Space communication has been introduced between the nine planets of the solar system. Rockets fly along the following routes: Earth - Mercury, Pluto - Venus, Earth - Pluto, Pluto - Mercury, Mercury - Venus, Uranus - Neptune, Neptune - Saturn, Saturn - Jupiter, Jupiter - Mars and Mars - Uranus. Is it possible to get from Earth to Mars?

- The king has 19 vassal barons. Could it be that each vassal barony has one, five or nine neighboring baronies?

- There are 100 circles on a plane that make up a coherent figure.
  Prove that this figure can be drawn without lifting the pencil from the paper or drawing the same line twice.
- (A little hard)

- Each of the edges of a complete graph with 6 vertices is colored in one of two colors.
  Prove that there are three vertices, all the edges between them are the same color.

- Prove that it is possible to arrange arrows on the edges of a connected graph in such a way that from some vertex one can get to any other vertex using arrows.