EE3980 Algorithms

Homework 10. Coin Set Design

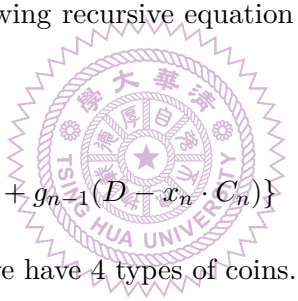
Due: May 24, 2020

In Taiwan, we have four types of coins: \$1, \$5, \$10, and \$50. Using these four types of coins, any dollar amount less than \$100 can represented. Let $(C_1, C_2, C_3, C_4) = (1, 5, 10, 50)$, and the numbers of each type of coin be (x_1, x_2, x_3, x_4), then the minimum number of coins for D dollars, $D \leq 99$, can be formulated as

$$\begin{aligned} \text{minimize} \quad & Ncoin = \sum_{i=1}^{4} x_i, \\ \text{subject to} \quad & D = \sum_{i=1}^{4} x_i C_i, \\ \text{and} \quad & x_i \in \mathbb{Z} \text{ and } x_i \geq 0. \end{aligned}$$

Let $g_n(D)$ be the function that returns the minimum number of coins, using n types of coins, $1 \leq n \leq 4$, then one can derive the following recursive equation of our minimum-coin problem, assuming $C_1 = 1$.

$$\begin{aligned} g_1(D) &= D, \\ g_n(D) &= \min_{x_n=0}^{\lfloor D/C_n \rfloor} \{x_n + g_{n-1}(D - x_n \cdot C_n)\} \qquad n > 1. \end{aligned}$$



And our our goal is to find $g_4(D)$ since we have 4 types of coins.

Your assignment is to write a function to calculate $g_n(D)$ using dynamic programming approach, and using this function to find

1. Given $\{C_1, C_2, C_3, C_4\} = \{1, 5, 10, 50\}$, find the average number of coins for $D = 1$ to 99.
2. Assuming C_4 is a variable find its value that minimizes the average for $D = 1$ to 99.
3. Assuming C_3 is a variable find its value that minimizes the average for $D = 1$ to 99.
4. Assuming both C_3 and C_4 are variables find their values that minimizes the average for $D = 1$ to 99.

Example of program execution is shown below.

```
$ a.out
For coin set {1, 5, 10, 50} the average is d.ddddd
Coin set {1, 5, 10, dd} has the minimum average of d.ddddd
Coin set {1, 5, dd, 50} has the minimum average of d.ddddd
```

```
Coin set {1, 5, dd, dd} has the minimum average of d.ddddd
```

Notes.

1. One executable and error-free `C` source file should be turned in. This source file should be named as `hw10.c`.
2. A `pdf` file is also needed. This report file should be named as `hw10a.pdf`.
3. Submit your `hw10.c` and `hw10a.pdf` on EE workstations using the following command:

   ```
   $ ~ee3980/bin/submit hw10 hw10.c hw10a.pdf
   ```

   where `hw10` indicates homework 10.
4. Your report should be clearly written such that I can understand it. The writing, including English grammar, is part of the grading criteria.