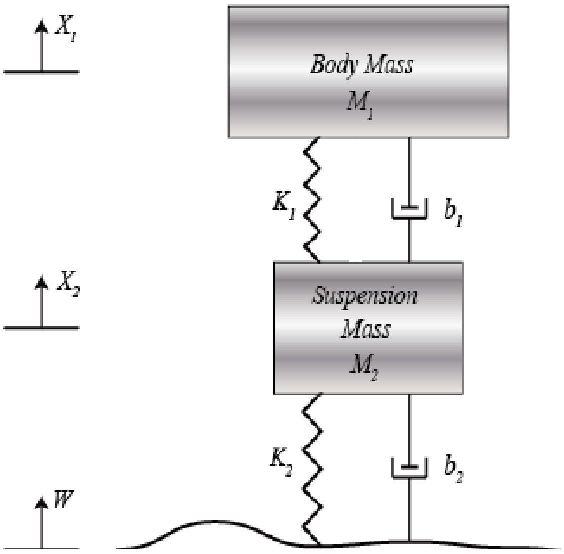
PASSIVE QUARTER CAR SUSPENSION SYSTEM

Dynamic Equations



$$m_1\ddot{x}_1 = -b_1 * (\dot{x}_1 - \dot{x}_2) - k_1 * (x_1 - x_2)$$

$$m_2\ddot{x}_2 = k_1 * (x_1 - x_2) + b_1 * (\dot{x}_1 - \dot{x}_2) - k_2 * (x_2 - W) - b_2 * (\dot{x}_2 - \dot{W})$$

Transfer Functions

$$(m_1 s^2 + b_1 s + k_1)x_1(s) = (b_1 s + k_2)x_2(s)$$

$$(m_2 s^2 + (b_1 + b_2)s + (k_1 + k_2))\,x_2(s) - (b_1 s + k_1)\,x_1(s) = (b_2 s + k_2)W(s)$$

Matlab code to find transfer functions $x_2(s)/W(s)$ & $x_1(s)/W(s)$

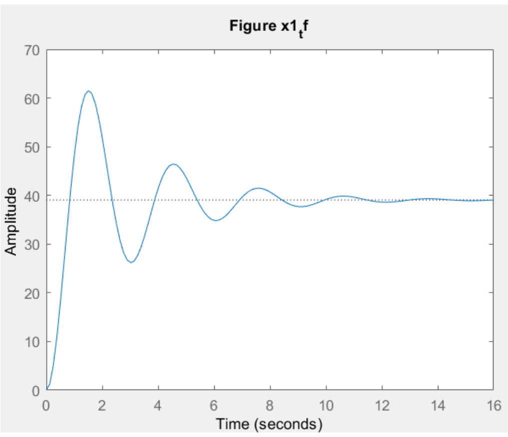
```
m1 = 2500;
m2 = 320;
k1 = 80000;
k2 = 500000;
b1 = 350;
b2 = 15020;

%transfer function
syms s x1s x2s
x1s = x2s*((b1*s+k2)/(m1*s^2 + b1*s + k1));
ws = ((m2*s^2+(b1+b2)*s+(k1+k2))*x2s-(b1*s+k1)*x1s)/(b2*s+k2);
x2_tf = x2s/ws;
pretty(collect(x2_tf,s))

syms s x1s x2s
```

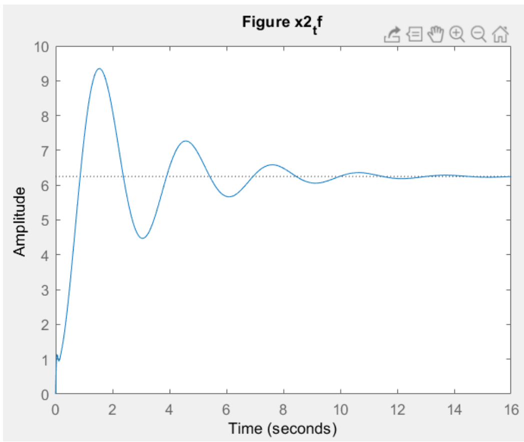
```
x2s = x1s/((b1*s+k2)/(m1*s^2 + b1*s + k1));
ws= ((m2*s^2+(b1+b2)*s+(k1+k2))*x2s-(b1*s+k1)*x1s)/(b2*s+k2);
x1_tf = x1s/ws;
pretty(collect(x1_tf,s))
```

```
    37550 s  + 1255257 s  + 1376600 s + 40000000
----------------------------------------------------
     4          3            2
800 s  + 38537 s  + 1480857 s  + 1229600 s + 6400000

                   2
          5257 s  + 7685000 s + 250000000
----------------------------------------------------
     4          3            2
800 s  + 38537 s  + 1480857 s  + 1229600 s + 6400000
```

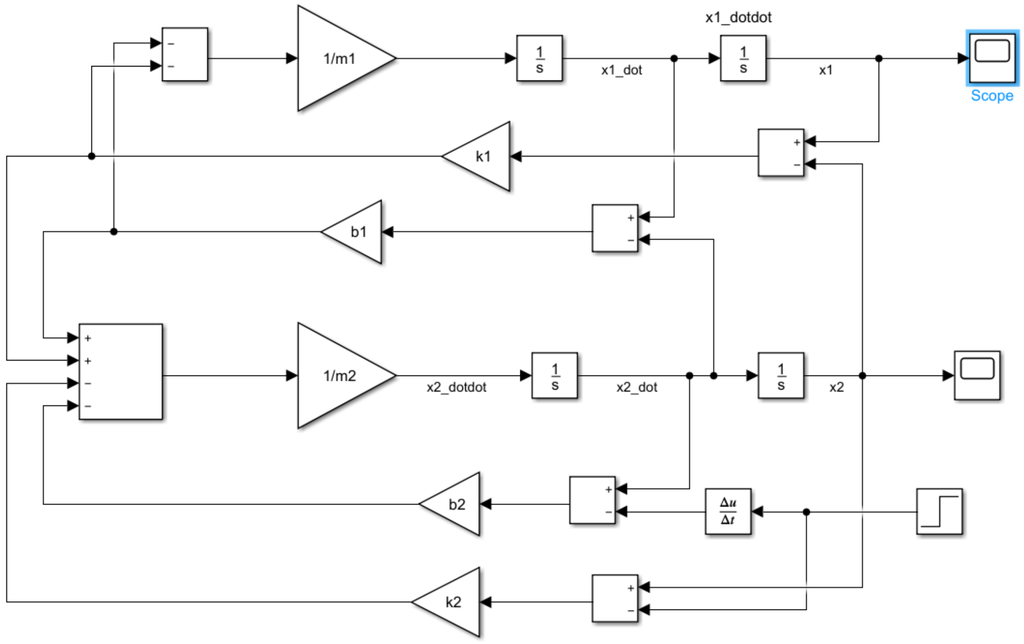
Plotting step response transfer functions:

```
% Plot step response
[num, den] = numden(x1_tf);
x1_tf = tf(sym2poly(num), sym2poly(den));
[num, den] = numden(x2_tf);
x2_tf = tf(sym2poly(num), sym2poly(den));

figure(1);
step(x1_tf);
title('Figure x1_tf');
figure(2);
step(x2_tf);
title('Figure x2_tf');
```

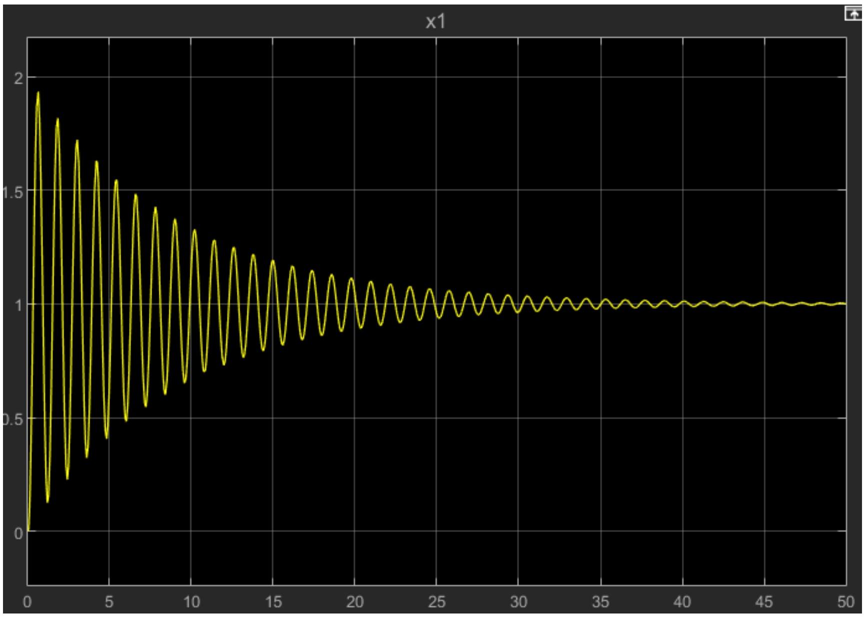




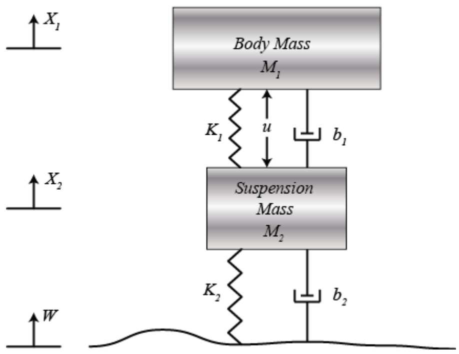
Modelling in Simulink



When we scope x1 it seems that the oscillation takes a long time to damped. PID control needs to be added.



ACTIVE QUARTER CAR SUSPENSION SYSTEM



$$m_1 \ddot{x_1} = -b_1 * (\dot{x_1} - \dot{x_2}) - k_1 * (x_1 - x_2) + U$$

$$m_2 \ddot{x_2} = b_1 * (\dot{x_1} - \dot{x}_2) - b_2 * (\dot{x}_2 - \dot{W}) + k_1 * (x_1 - x_2) - k_2 * (x_2 - W) - U$$

State Space Equations

$z_1 = x_1 \qquad \dot{z}_1 = z_2$

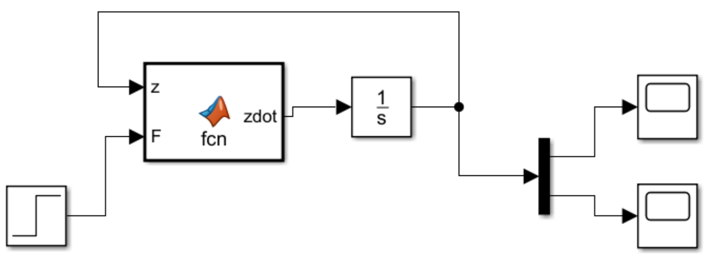
$z_2 = \dot{x}_1 \qquad \dot{z}_2 = \left(\frac{-b_1}{m_1}\right) z_2 + \left(\frac{b_1}{m_1}\right) z_4 + \left(\frac{-k_1}{m_1}\right) z_1 + \left(\frac{k_1}{m_1}\right) z_3 + \left(\frac{1}{m_1}\right) U$

$z_3 = x_2 \qquad \dot{z}_3 = z_4$

$z_4 = \dot{x}_2 \qquad \dot{z}_4 = \left(\frac{b_1}{m_2}\right) z_2 + \left(\frac{-b_1}{m_2}\right) z_4 + \left(\frac{-b_2}{m_2}\right) z_4 + \left(\frac{k_1}{m_2}\right) z_1 + \left(\frac{-k_1}{m_2}\right) z_3 + \left(\frac{-k_2}{m_2}\right) z_3 + \left(\frac{b_2}{m_2}\right) \dot{W} + \left(\frac{k_2}{m_2}\right) W$

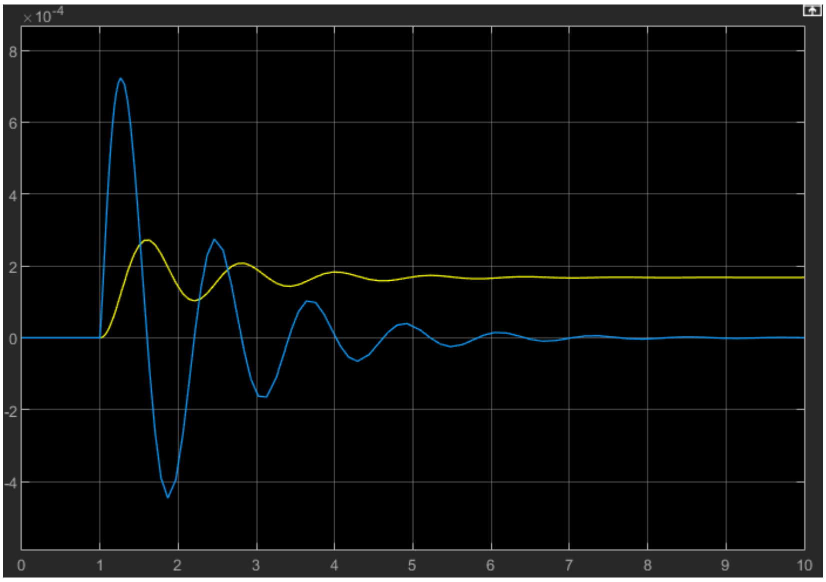
$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-k_1}{m_1} & \frac{-b_1}{m_1} & \frac{k_1}{m_1} & \frac{b_1}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k_1}{m_2} & \frac{b_1}{m_2} & \frac{-k_1-k_2}{m_2} & \frac{-b_1-b_2}{m_2} \end{bmatrix} * \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m_1} \\ 0 \\ \frac{1}{m_2} \end{bmatrix} * F$$

Modelling in Simulink



Matlab function block:

```
function zdot = fcn(z,F)
m1 = 300;
m2 = 60;
k1 = 16000;
k2 = 19000;
b1 = 1000;
b2 = 1000;


A=[0 1 0 0;
 -k1/m1 -b1/m1 k1/m1 b1/m1;
 0 0 0 1;
 k1/m2 b1/m2 (-k1-k2)/m2 (-b1-b2)/m2];

B=[0;
  1/m1;
  0;
  1/m2];

zdot = A*z+ B*F;
end
```



Plotting x1 and x2