Computational Physics: Project 3 - Penning Trap

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Implementing a model of the Penning Trap and investigating systems of one and two particles using the Euler-Cromer and Runge Kutta 4th order methods. Observing some suspicious results which casts some doubts on the implementation of the numerical methods, especially the Runge Kutta method. Concluding there are probably some serious flaws in the code and numerical methods..

I. GITHUB

https://github.com/daglyd/Computational_Physics/tree/main/project_3

II. INTRODUCTION

In this project we will model a Penning trap using and external electric and magnetic field, this to trap particles. The forces acting on the particles will be modeled using external field aswell as the electric field set up by the particles as point charges. Describing the forces acting on the particles as the Lorentz force the movement of the particles will be governed by Newtons second law. We attempt to solve the equations of motions using the Euler-Cromer method aswell as the Runge Kutta 4th order numerical methods. Using the numerical methods we will be modeling systems of one and two particles looking at its path in space aswell as its phase plots.

III. METHODS

The Penning trap will be modeled using a external electric and magnetic field.
The electric field is given by

$$E = -\nabla V$$

Where V is given by

$$V(x, y, z) = \frac{V_0}{2d^2}(2z^2 - x^2 - y^2)$$

The external magnetic field is given by:

$$B = B_0\hat{e}_z = (0, 0, B_0)$$

There will be an electric field given by the particles in the trap modeled as point charges with charges $\{q_1, ..., q_n\}$ given by

$$E = k_e \sum_{j=1}^{n} q_j \frac{\vec{r} - \vec{r_j}}{|\vec{r} - \vec{r_j}|^3}$$

Total force on a particle is given by the Lorentz force:

$$F = qE + q\vec{v} \times B$$

The algorithm

For the numerical intergration of the equation of motions I will be using the Euler-Cromer method. Given an differenctial equation

$$\frac{d^2x}{dt^2} = \ddot{x} = f(t, v)$$

With initial conditions $x(t_0) = x_0, v(t_0) = v_0$

Algorithm 1 Euler-Cromer

$x(t_0) = x_0$
$v(t_0) = v_0$
for $i = 1, 2, \ldots, n$ **do**
  $a = f(t_i, v_i)$ ▷ Acceleration given by NSL
  $v(t_{i+1}) = v(t_i) + a * dt$
  $x(t_{i+1}) = x(t_i) + v(t_{i+1}) * dt$

IV. RESULTS AND DISCUSSION

[h!] Beginning with the implementation of the Euler-Cromer method we see in figure 1 the particle oscillating on its path. As well as circling the center of the system. The local oscillations grow as time progresses and seemingly becomes more unstable. Looking at figure 2 we see that after some time in a stable position in the z-dimension it falls as the oscillations becomes bigger. The travel path in the z-direction seems very unrealistic as the scale of the z-positions grows very large.

Looking at the implementation of the Runge Kutta method in figure 3 the path is much smoother ending in the center. Here we also see a drop for the particle in z-dimension after some time.
I am somewhat unsure why there is a drop after some time, it was an unexpected result. Thinking maybe it could be due to loss of energy because of flaws in the numerical methods.
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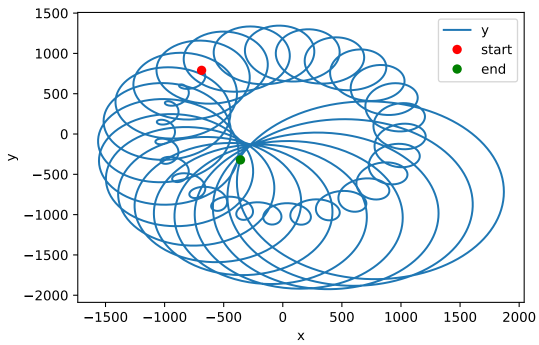

FIG. 1. The xy plot of a particle in the Penning trap.

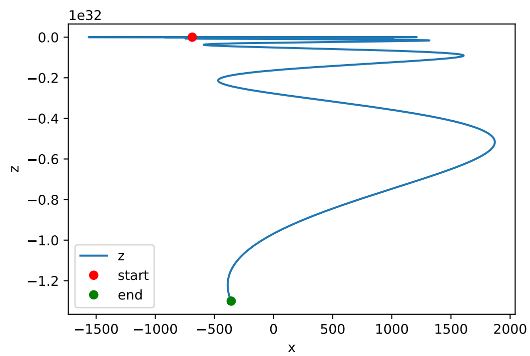

FIG. 2. The xz-plot of the single particle modeled with Euler-Cromer.x‘

z-dimension after some time.
I am somewhat unsure why there is a drop after some time, it was an unexpected result. Thinking maybe it could be due to loss of energy because of flaws in the numerical methods. I am somewhat suspicious about the results from the RungeKutta method as the plot for a single particle seems less realistic than the one for the Euler-Cromer method. And as the phase plot for RK4 produced nonsense I have decided to mostly use the Euler-Cromer method.

With the introduction of a second particle we get much the same results as the first experiment with the oscillation around the center, with ever increasing oscillations. We also see a stable persistence in the z-dimension for some time before the particle travels off. Looking at the xz plots we see the two particles both starting of about the same place on the z-axis but going of in the opposite directions. This would be expected as the charges are both positive.
The phase plots seems to make sense as the particles have the highest velocities around the center of the systems.
Looking at the phase plot for the z-v_z dimensions in 6 there seems to be a linear increase in velocity as the

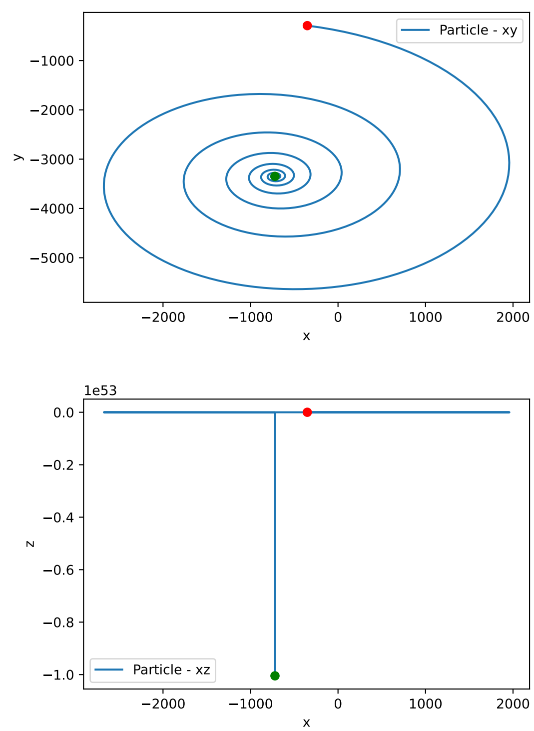

FIG. 3. xy and xz plot of a single particle modelled with Runge Kutta 4th order method.

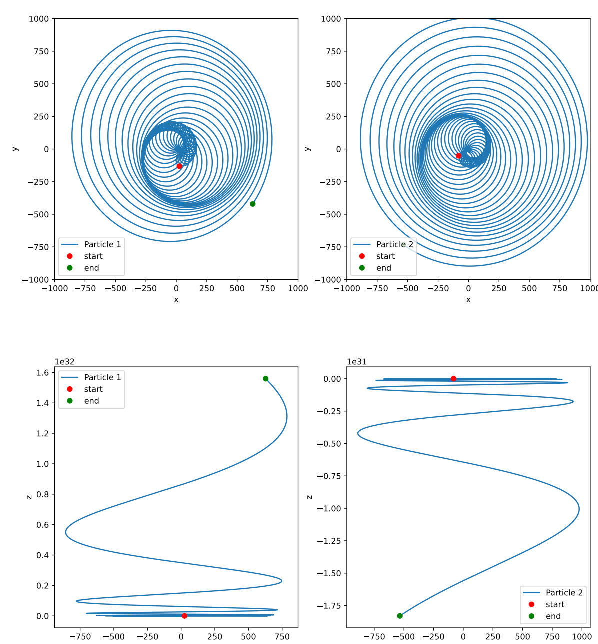

FIG. 4. xy and xz plots for two particle positions modeled with the Euler-Cromer method.

particle falls away. Here again a seemingly unrealistic

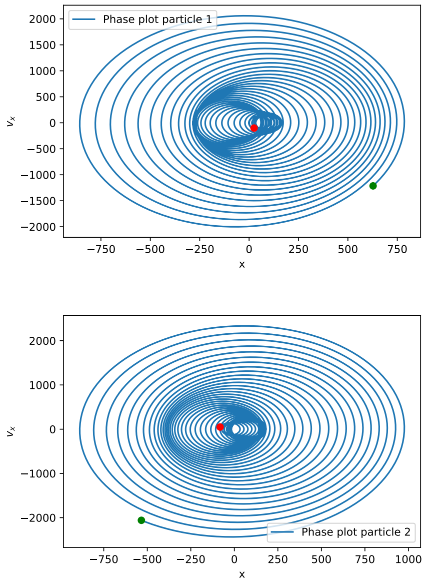

FIG. 5. Phase plots for particle 1 and 2.

scale of the particles path in z-direction.

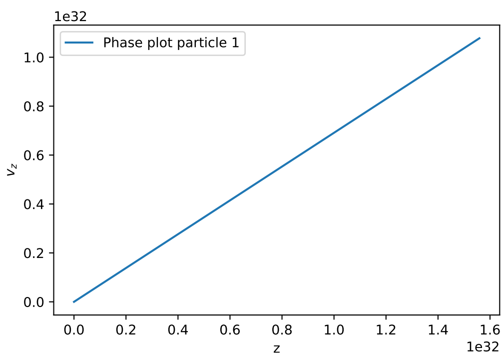

FIG. 6. Phase plot for z,v_z dimensions for the first particle.

The 3-D plots for the two particles shows very much the same picture as the 2-D plots where particles are trapped in the center of the system before spiraling out after some time. The plot for the two particle system without the particle interaction force seems to be exactly identical, which was unexpected. I am thinking maybe this could be due to some flaw in the code.

V. CONCLUSION

The numerical methods seemed to produce a somewhat realistic model. For the one particle system there seems to be an evolution of instability in the model, which seems to be wrong as the forces should be constant. The Runge Kutta method produces even more unrealistic results. The two particle system seems to be more stable

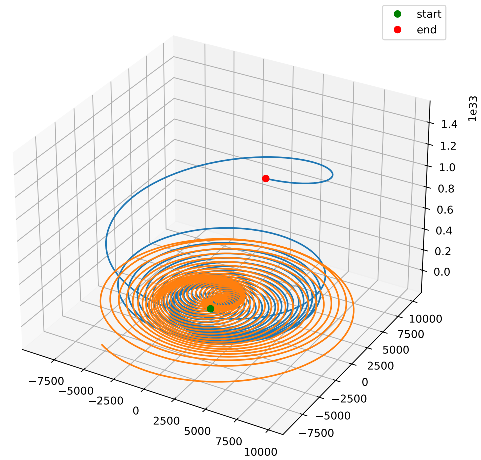

FIG. 7. 3-D plot of two particles using Euler-Cromer method.

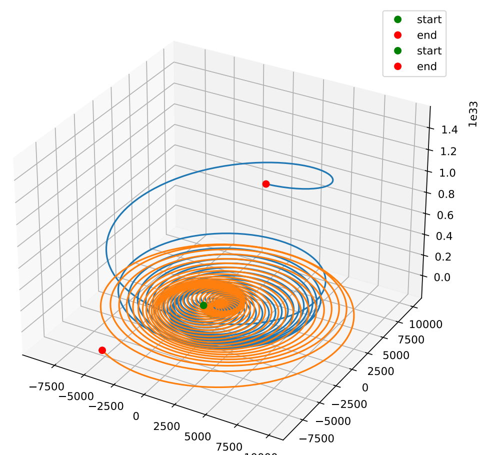

FIG. 8. 3-D plot of two particles without interactions using Euler-Cromer method.

for a longer period of time but these also evolve towards instability. There seems also to be no difference between the systems where the interacting force between particles are present and when not. This suggest some serious flaws in the implementation of the numberical methods.