FYS-MEK1110 - Oblig 1

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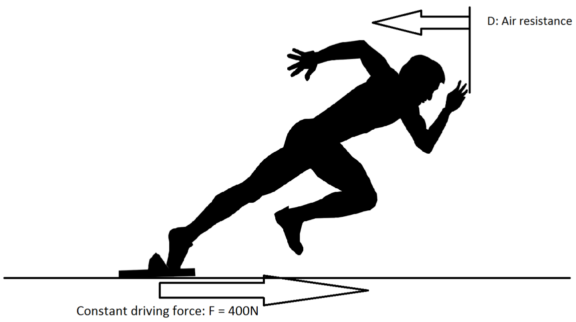
a)



Figur 1: The horizontal forces the sprinter is subject to are the driving force and the air resistance.

b)

Find the position, x(t)

$$a = \frac{F}{m}$$

$$v(t) = v_0 + \int a(t)dt = v_0 + \frac{1}{m}\int F dt = v_0 + \frac{Ft}{m}$$

$$v_0 = 0 \Rightarrow v(t) = \frac{Ft}{m}$$

$$x(t) = x_0 + \int v(t)dt = x_0 + \frac{F}{m}\int t dt = x_0 + \frac{Ft^2}{2m}$$

$$x_0 = 0 \Rightarrow x(t) = \frac{Ft^2}{2m}$$

c)

Show that the sprinter uses t=6.3s to reach the 100m line.
Position, x(t), is to be equal 100m

$$x(t) = 100m, F = 400N, m = 80kg$$

$$100m = \frac{400N * t^2}{2 * 80kg}$$

$$\Rightarrow t^2 = \frac{16000kgm}{400kgm/s^2} = 40s^2$$

$$\Rightarrow t = \sqrt{400s^2} = 6.3s$$

d)

Find an expression for the acceleration of the runner.
Air resistance described by a square law:

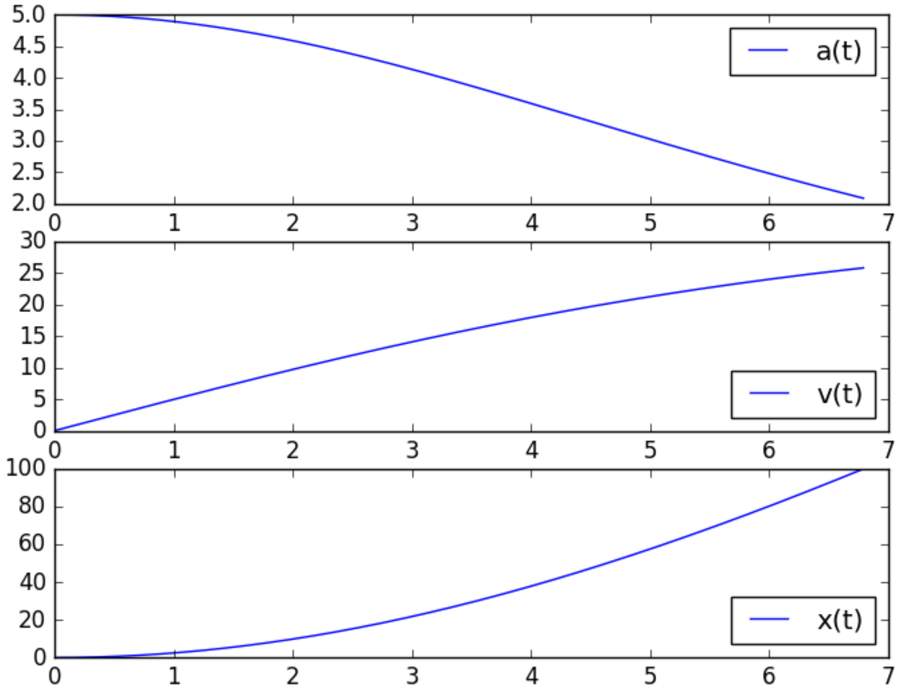
$$D = (1/2)pC_d A(v - w)^2$$

$p = 1.293kg.m^3, A = 0.45m^2, C_d = 1.2, w = 0m/s$

$$F_{net} = F - D = ma$$

$$\Rightarrow a(t) = \frac{F - D}{m} = \frac{400N - (\frac{1}{2}p * C_d * A)v(t)^2}{m}$$

e)




```python
from pylab import *

p = 1.293
w = -1
A = 0.45
Cd = 1.2
D = 0.5*p*Cd*A
time = 10
dt = 0.01
m = 80
F = 400

v0 = 0
x0 = 0

n = int(round(time/dt))
v = zeros(n,float)
a = zeros(n,float)
t = zeros(n,float)
x = zeros(n,float)

v[0] = v0
x[0] = x0

for i in range(n-1):
    a[i] = (F-D*v[i]**2)/m
    v[i+1] = v[i] + a[i]*dt
    x[i+1] = x[i] + v[i]*dt
    t[i+1] = t[i] + dt
```

```
    if x[i+1] >= 100:
        print "%.f meters is completed in %.2f seconds!" %(x[i+1], t[i+1])
        break

subplot(311)
plot(t[0:i+1],a[0:i+1])
legend(["a(t)"])
subplot(312)
plot(t[0:i+1],v[0:i+1])
legend(["v(t)"],loc=4)
subplot(313)
plot(t[0:i+1],x[0:i+1])
legend(["x(t)"],loc=4)
show()
```

I chose a time step that would give me an accurate measure of 100m

f)

By using the code in (e) I found the time for 100m to be 6.8 seconds.

g)

The terminal velocity is reached when acceleration reaches 0.

$$a(t) = \frac{F - (\frac{1}{2}p * C_d * A)v_T^2}{m} = 0$$

$$\Rightarrow \frac{F}{m} = \frac{(\frac{1}{2}p * C_d * A)v_T^2}{m}$$

$$\Rightarrow V_T^2 = \frac{F}{(\frac{1}{2}p * C_d * A)}$$

$$\Rightarrow v_T = \sqrt{\frac{2 * F}{(p * C_d * A)}}$$

```
for i in range(n-1):
    a[i] = (F-D*v[i]**2)/m
    v[i+1] = v[i] + a[i]*dt
    x[i+1] = x[i] + v[i]*dt
    t[i+1] = t[i] + dt
    if a[i]<0+1e-3:
        print v[i+1]
        break
```

By modifying the code as shown and increasing the time, I find an approximation of the terminal velocity within a tolerance of 10^{-3} around 0 acceleration.
The runner is approaching a terminal velocity of $33.85\frac{m}{s}$, this correspond to $121.86\frac{km}{h}$.I do not think this realistic.

h)

A new driving force consisting of $F = 400N$ and $f_V = 25.8sN/m$

$$F_D = F + F_V = F - f_V v$$

$$F - f_V v = ma$$

$$\Rightarrow a(t) = \frac{F - f_V v}{m}$$

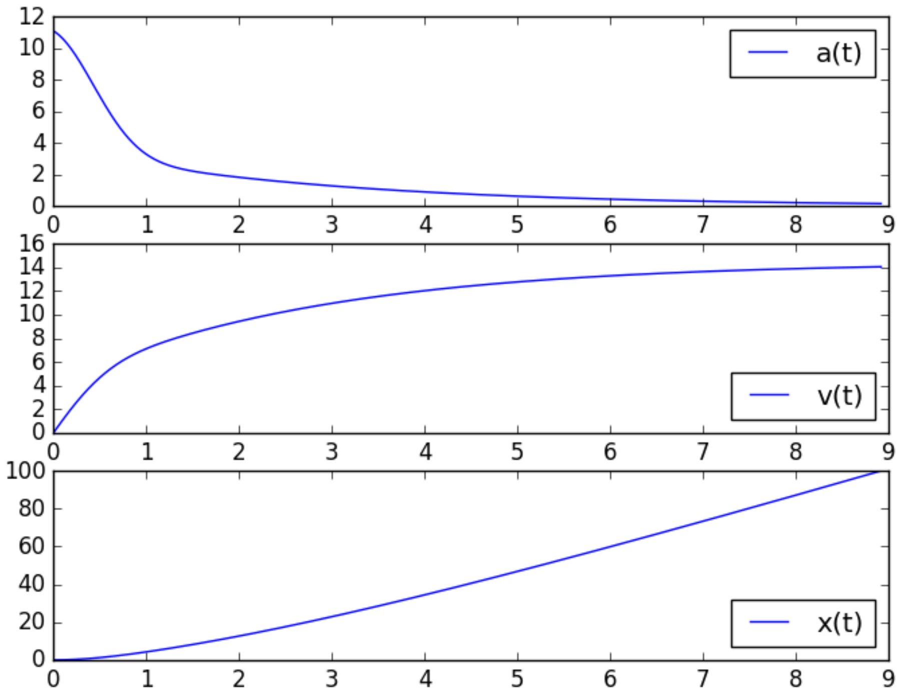
Finding maximum velocity

$$a(t) = \frac{F - f_V v}{m} = 0$$

$$\Rightarrow \frac{F_V v}{m} = \frac{F}{m} \Rightarrow f_V v = F$$

$$\Rightarrow v_T = \frac{F}{f_V} = \frac{400N}{25.8sN/m} = 15.5\frac{m}{s} = 55.8\frac{km}{h}$$

i)

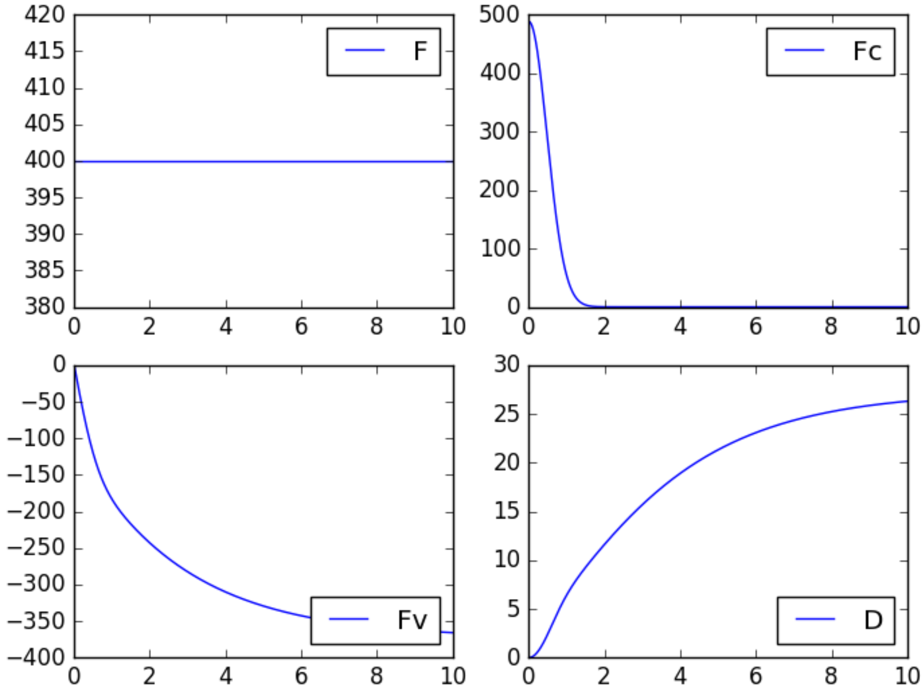


```
from pylab import *

p, w, Cd = 1.293, 0, 0.45
time, dt, m = 10, 0.01, 80
fc, tc, fv = 488, 0.67, 25.8
F = 400
A = 0.45

v0 = 0
x0 = 0

n = int(round(time/dt))
v = zeros(n,float)
a = zeros(n,float)
t = zeros(n,float)
x = zeros(n,float)

v[0] = v0
x[0] = x0

for i in range(n-1):
    Fc = fc*exp(-(t[i]/tc)**2)
    Fv = -fv*v[i]
    D = A*(1-0.25*exp(-(t[i]/tc)**2))*p*Cd*0.5*(v[i]-w)**2
    a[i] = (F + Fc + Fv - D)/m
    v[i+1] = v[i] + a[i]*dt
    x[i+1] = x[i] + v[i]*dt
    t[i+1] = t[i] + dt
    if x[i+1] >= 100:
        print "%.f meters is completed in %.2f seconds!" %(x[i+1], t[i+1])
        break

subplot(311)
plot(t[0:i+1],a[0:i+1])
legend(["a(t)"])
subplot(312)
plot(t[0:i+1],v[0:i+1])
legend(["v(t)"],loc=4)
subplot(313)
plot(t[0:i+1],x[0:i+1])
legend(["x(t)"],loc=4)
show()
```

j)

By using the code above I find he runs 100m in 8.93 seconds.

k)



F_c has a very big effect at the very start of the run, but tapers off very quickly. The drag force, D, has an increasing effect over the course of the run, but has relatively small effect compared to the driving force. F_v is by far the biggest influence over the course of the run. It reaches very big negative values over time and in the end negates very much of the constant driving force.

l)

If the runner had a hind wind with speed of $w = 1m/s$ his finish time would be 8.89 seconds. If on the other hand he was running into a wind of $1m/s$ his time would be 8.97 seconds. A difference of 0.08 seconds.