Fys-Mek1110 - Oblig 3

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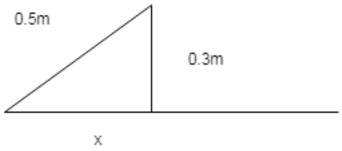
a)



Calculating the horizontal position:

$$x^2 + 0.3^2 = 0.5^2 \rightarrow x_0 = 0.4$$

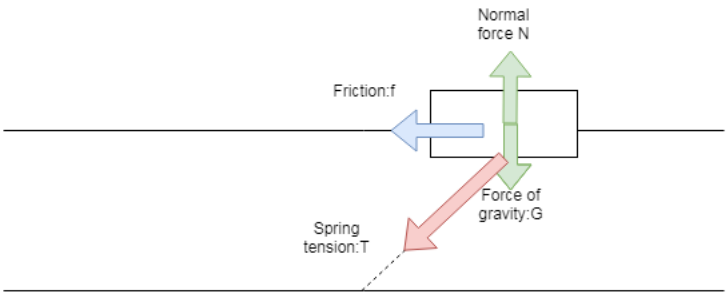
b)

The lenght of the spring at a position x:

$$h^2 + x^2 = l^2 \rightarrow l = \sqrt{0.3^2 + x^2}$$

h = 0.3 is constant.

c)

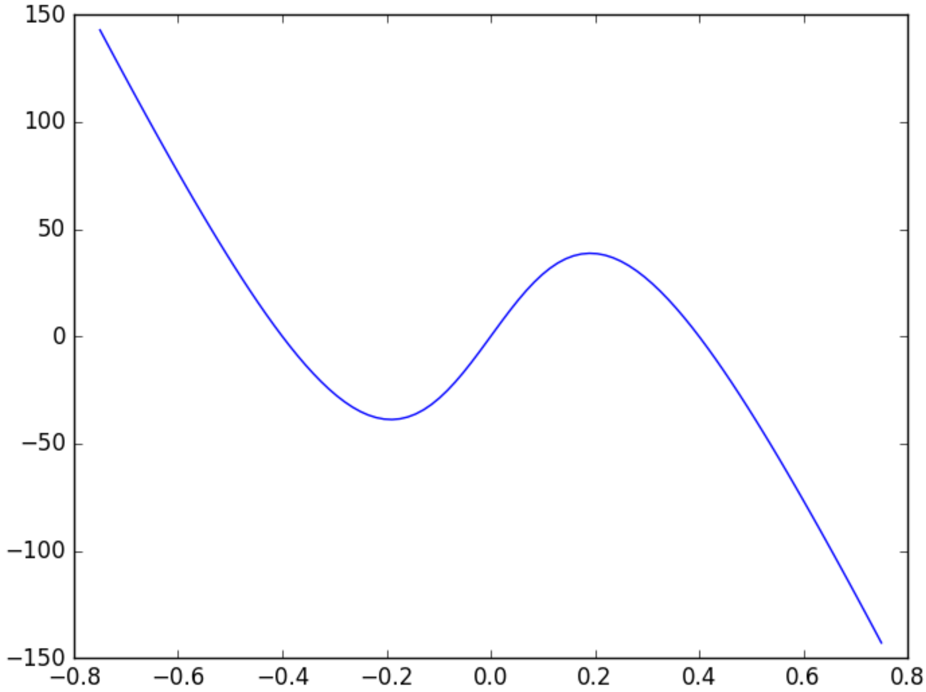


d)

$$F_x = F * \vec{i} = -k(r - l_0)\frac{\vec{r}}{r} * \vec{i} = -k(\sqrt{x^2 + h^2} - l_0)\frac{x\vec{i} + h\vec{j}}{\sqrt{x^2 + h^2}}\vec{i}$$

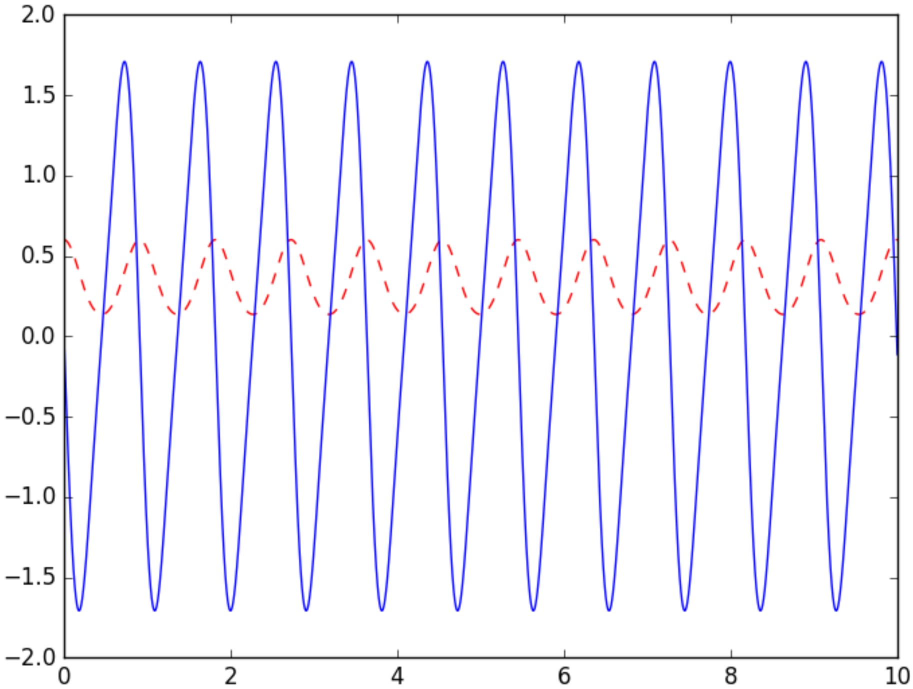
$$\rightarrow F_x = -kx(1 - \frac{l_0}{\sqrt{x^2 + h^2}})$$

e)



At the two extremes the spring is exerting a force in the opposite direction when it is stretched. While closer to zero the spring is compressed and is exerting a force in the direction it is positioned compared to 0 or the attachment point.

f)

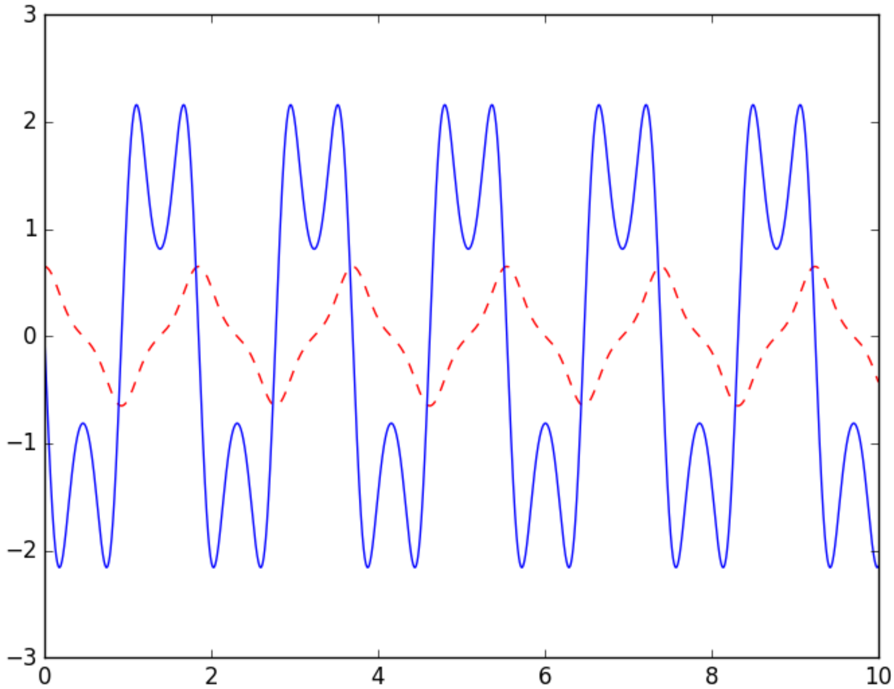


Motion is red line and velocity is blue.
The velocity of the cylinder is not high enough to overcome the force exerted by the compression of the spring when approaching zero. It therefore reverses.

```python
from pylab import *

k = 500       #N/m
h = 0.3       #m
l0 = 0.5      #m
m = 5         #kg
mu = 0.05

time = 10
dt = 0.001
n = int(ceil(time/dt))
t = zeros((n),float)
r = zeros(n,float)
v = zeros(n,float)

t[0] = 0.0
r[0] = 0.65
v[0] = 0.0

for i in range(n-1):
    x = r[i]
    Fx = -k*x*(1-l0/sqrt(x**2+h**2))
    a = Fx/m
    v[i+1] = v[i] + a*dt
    r[i+1] = r[i] + v[i+1]*dt
```

```
    t[i+1] =  t[i] + dt

plot(t,r,"--r",t,v,"-b")
show()
```

g)



Velocity in blue, position in red.
Here the cylinder has enough energy to pass over the point where the spring is compressing. A dip in velocity is seen when the cylinder is passing these points.

h)

$$F_y = F * \vec{j} = -k(r - l_0)\frac{\vec{r}}{r}\vec{j} = -k(\sqrt{x^2 + h^2} - l_0)\frac{x\vec{i} + h\vec{j}}{\sqrt{x^2 + h^2}}\vec{j}$$

$$\rightarrow -kh(1 - \frac{l_0}{\sqrt{x^2 + h^2}})$$

i)

The vertical forces:

$$F_y + G + F_n = am$$

F_n is the normal force. Assuming the cylinder is at rest and acceleration is 0:

$$F_y + G + F_n = 0 \to F_n = gm + -kh(1 - \frac{0.5}{0.5}) \to F_n = gm$$

j)

The normal force at a position x:

$$F_n(x) = gm + kh\left(1 - \frac{l_0}{\sqrt{x^2 + h^2}}\right)$$

k)

$$F_n = mg + kh\left(1 - \frac{l_0}{\sqrt{x^2 + h^2}}\right) = 0$$

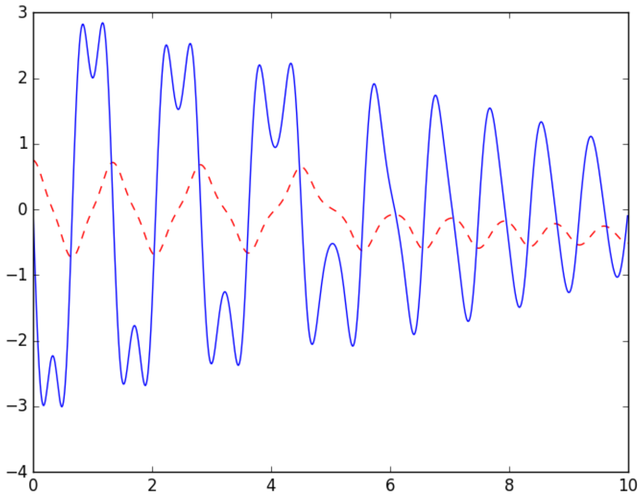
$$\to mg + kh = \frac{khl_0}{\sqrt{x^2 + h^2}} \to \sqrt{x^2 + h^2} = \frac{khl_0}{mg + kh} = \frac{l_0}{\frac{mg}{kh} + \frac{kh}{kh}}$$

$$\to x^2 + h^2 = \left(\frac{l_0}{1 + \frac{mg}{kh}}\right)^2 \to x = \sqrt{\left(\frac{l_0}{1 + \frac{mg}{kh}}\right)^2 - h^2}$$

l)

A modified formula for acceleration, friction added.

$$a = \frac{F_x}{m} + \frac{f}{m} = \frac{-kx}{m}\left(1 - \frac{l_0}{\sqrt{x^2 + h^2}}\right) + \frac{\mu}{m}\left(gm + kh\left(1 - \frac{l_0}{\sqrt{x^2 + h^2}}\right)\right)$$

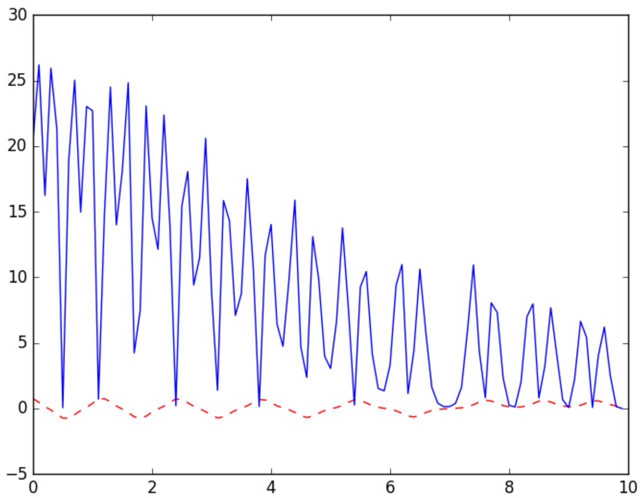


Velocity in blue, position in red.

```
from pylab import *

k = 500       #N/m
h = 0.3       #m
l0 = 0.5      #m
m = 5         #kg
g = 9.81
mu = 0.05

time = 10
dt = 0.01
n = int(ceil(time/dt))
t = zeros((n),float)
r = zeros(n,float)
v = zeros(n,float)
K = zeros(n,float)

t[0] = 0.0
r[0] = 0.75
v[0] = 0.0

for i in range(n-1):
    x = r[i]
    Fn = mu*g*m + mu*k*h*(1-l0/sqrt(x**2+h**2))
    Fx = -k*x*(1-l0/sqrt(x**2+h**2))
    a = Fx/m + -sign(v[i])*Fn/m
    v[i+1] = v[i] + a*dt
    K[i] = 0.5*m*v[i+1]**2
    r[i+1] = r[i] + v[i+1]*dt
    t[i+1] =  t[i] + dt

plot(t,r,"--r",t,v,"-b")
show()
```

As the cylinder start at high energy it passes the points of spring compression easily, but as time goes by it becomes harder to traverse these points which is seen in the increasing dip in velocity at extremes of the velocity curves, this is due to the introduction of friction. Eventually the cylinder is not able to spend the energi to traverse the point of the spring compressing and is stuck osciliating between a spring extreme and a point of spring compression.

m)



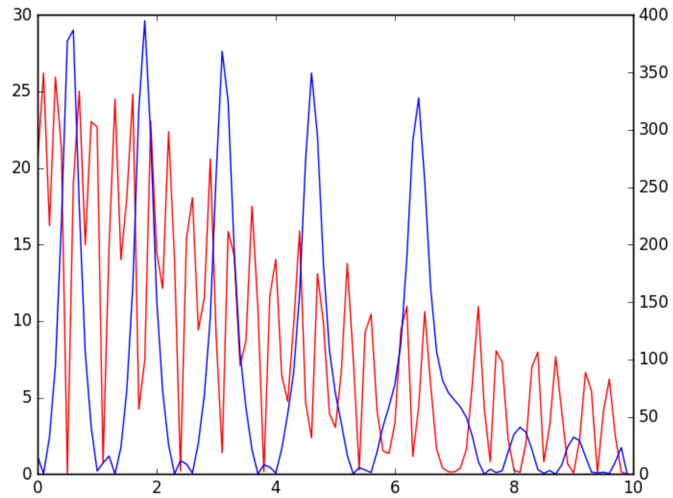
Kinetic energy in blue, position in red.
The gradual loss of kinetic energy due to friction as describes in l) is shown in this plot. The kinetic energy is gradually lost until it reaches a state where little energy is spent. The spikes to zero is explained by the cylinder reaching its extreme where velocity is zero. That all spikes down does not reach zero I think is because of the numerical method.

n)

```
from pylab import *

k = 500       #N/m
h = 0.3       #m
l0 = 0.5      #m
m = 5         #kg
g = 9.81
mu = 0.05

xval =linspace(0.4,0.75,1000)

Fn = mu*g*m + mu*k*h*(1-l0/sqrt(xval**2+h**2))
Fx = -k*xval*(1-l0/sqrt(xval**2+h**2))

y = Fx + Fn
w = trapz(y,x=xval)
print w
```

The work needed to bring cylinder from its equilibrium position of x=0.4 to x=0.75 is -22.3. I was expecting a positive number, and do not know how to comment the answer.

o)



Kinetic energy in red, potential energy in blue.

```
from pylab import *

k = 500      #N/m
h = 0.3      #m
l0 = 0.5     #m
m = 5        #kg
g = 9.81
mu = 0.05

time = 10
dt = 0.1
n = int(ceil(time/dt))
t = zeros((n),float)
r = zeros(n,float)
v = zeros(n,float)
K = zeros(n,float)
U = zeros(n,float)

t[0] = 0.0
r[0] = 0.75
v[0] = 0.0

for i in range(n-1):
    x = r[i]
    Fn = mu*g*m + mu*k*h*(1-l0/sqrt(x**2+h**2))
    Fx = -k*x*(1-l0/sqrt(x**2+h**2))
    a = Fx/m + -sign(v[i])*Fn/m
    v[i+1] = v[i] + a*dt
    K[i] = 0.5*m*v[i+1]**2
    U[i] = 0.5*k*(x-l0)**2
    r[i+1] = r[i] + v[i+1]*dt
    t[i+1] =  t[i] + dt

fig,ax1 = subplots()
ax2 = ax1.twinx()
ax1.plot(t,K,"-r")
ax2.plot(t,U,"-b")
show()
```

I am not sure what I am doing here. I defined U(x) as $\frac{1}{2}k(x-b)^2$ where $b = l_0$. I observe that the potential energy osciliates at a lower leven when the cylinder no longer is able to pass the point of spring compression.

p)

The equilibrium points are at the position where the spring is at its equilibrium lenght, $x = \pm 0.4$. And they are stable.