FYS-MEK1110 - Oblig 2

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a)

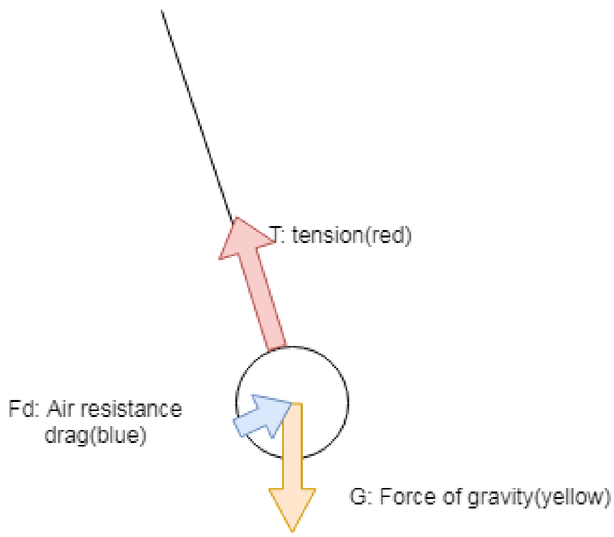


Figure 1: The forces acting upon the pendulum are the force of gravity, tension from the rope and the air resistance.

b)

The forces acting upon the pendulum excluding air resistance is the force of gravity(G) and tension from the rope(T).

$$\sum \vec{F}_{net} = \vec{G} + \vec{T} = -mg\vec{j} - k(r - L_0)\frac{\vec{r}}{r}.$$

c)

Rewriting the expression of the external forces on component form.

$$\vec{r} = x(t)\vec{i} + y(t)\vec{j}.$$

$$\vec{F_x} = \vec{F_{net}}\vec{i} = -mg\vec{j} * \vec{i} - k(r - L_0)\frac{1}{r}(x(t)\vec{i} * \vec{i} + y(t)\vec{j} * \vec{i} = 0 - k(r - L_0)\frac{x(t)}{r} = -k(r - L_0)\frac{x(t)}{r}$$

$$\vec{F_y} = \vec{F_{net}}\vec{j} = -mg\vec{j} * \vec{j} - k(r - L_0)\frac{1}{r}(x(t)\vec{i} * \vec{j} + y(t)\vec{j} * \vec{j} = -mg - k(r - L_0)\frac{y(t)}{r}$$

d)

No, the angle θ does not describe the position of the pendulum in this case. Due to the use of a spring the lenght of the rope is not fixed, so there is a need to describe the lenght aswell as the angle with the vertical.

e)

$$m\vec{a} = -mg\vec{j} - k(r - L_0)\frac{\vec{r}}{r} \to m(0) = -mg\vec{j} - k(r - L0)\frac{\vec{r}}{r} \to \vec{r} = (\frac{mgr}{-k(r - L_0})\vec{j}$$

When increasing the value of k the tension in the rope increases and the ball will be at rest at a higher level in the vertical dimension. As seen mathematically when k is increased the factor of the unit vector $\vec{j}$ becomes smaller.

f)

Finding an expression for the acceleration,$\vec{a}$.

$$m\vec{a(t)} = -mg\vec{j} - k(r - L_0)\frac{\vec{r(t)}}{r} \to \vec{a(t)} = -g\vec{j} - \frac{k}{rm}(r - L_0)\vec{r(t)}$$

Using the component forces from c)

$$\vec{a(t)} = \frac{\vec{F_x}}{m} + \frac{\vec{F_y}}{m} = -\frac{k}{rm}(r - L_0)x(t) - g - \frac{k}{rm}(r - L_0)y(t) = -g - \frac{k}{rm}(r - L_0)(x(t) + y(t))$$

$$= -g - \frac{k}{\sqrt{x^2 + y^2}}(\sqrt{x^2 + y^2} - L_0)(x(t) + y(t))$$

g)

The inital value problem to solve for the motion of the ball.

$$\frac{d^2\vec{r(t)}}{dt^2} = a\left(\vec{r(t)}, r(t), t, \frac{d\vec{r(t)}}{dt}\right) = -g\vec{j} - \frac{k}{r(t)m}(r(t) - L_0)\vec{r(t)}$$

Initial conditions:

$$v(\vec{t_0}) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, r(\vec{t_0}) = \begin{bmatrix} 1/2 \\ -\sqrt{3}/2 \end{bmatrix}.$$

(h)

Equations to solve the problem numerically with Euler-Cromer's method.

$$\vec{v}(t+\Delta t) = \vec{v}(t) + \vec{a}(t)\Delta t = \vec{v}(t) + \left(-g\vec{j} - \frac{k}{r(t)m}(r(t) - L_0)\vec{r}(t)\right)\Delta t$$

$$\vec{r}(t+\Delta) = \vec{r}(t) + \vec{v}(t+\Delta t)\Delta t = \vec{r}(t) + \left[\vec{v}(t) + \left(-g\vec{j} - \frac{k}{r(t)m}(r(t) - L_0)\vec{r}(t)\right)\Delta t\right]\Delta t$$

$$= \vec{r}(t) + \vec{v}(t)\Delta t + \left(-g\vec{j} - \frac{k}{r(t)m}(r(t) - L_0)\vec{r}(t)\right)\Delta t^2$$

i)

Program that solves for the motion of the ball.

```python
from pylab import *
g = 9.81
m = 0.1       #kg
L0 = 1        #m
k = 200     #N/m
theta = 30  #degrees
x0 = sin(pi/6.0)
y0 = -cos(pi/6.0)

time = 10
dt = 0.001
n = int(ceil(time/dt))
t = zeros((n,1),float)
r = zeros((n,2),float)
v = zeros((n,2),float)
a = zeros((n,2),float)

t[0] = 0.0
r[0,:] = array([x0,y0])
v[0,:] = array([0.0,0.0])

for i in range(n-1):
    rr = norm(r[i,:])
    a[i,:] = -g*array([0,1]) - k*(rr-L0)*r[i,:]/rr*m
    v[i+1,:] = v[i,:] + a[i,:]*dt
    r[i+1,:] = r[i,:] + v[i+1,:]*dt
    t[i+1] = t[i] + dt

plot(r[:,0],r[:,1],"--o",markersize=10,markevery=r[:,0].size)
xlabel(["x(t)"])
ylabel(["y(t)"])
show()
```

j)

The pendulum moves lower from it's starting position due to a fairly elastic rope. And it moves in two alternating paths when returning from a leftward swing.

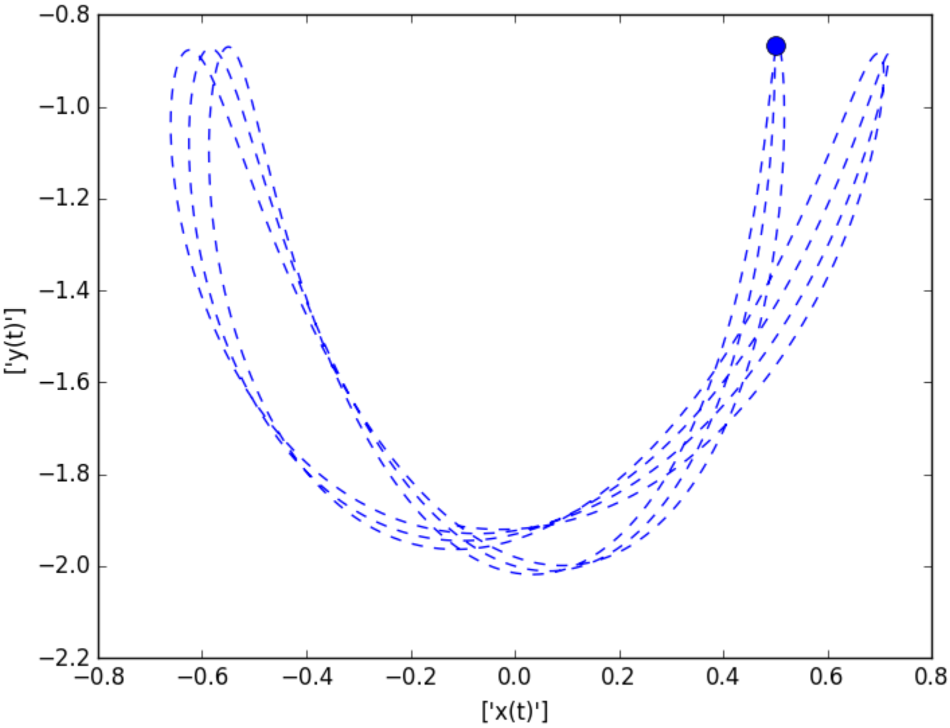
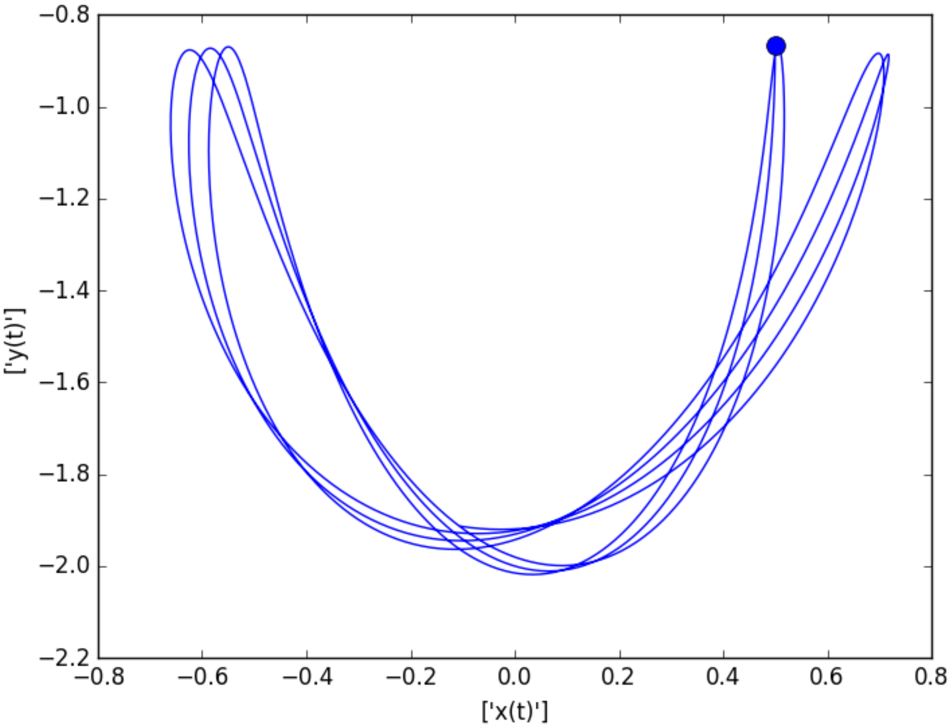
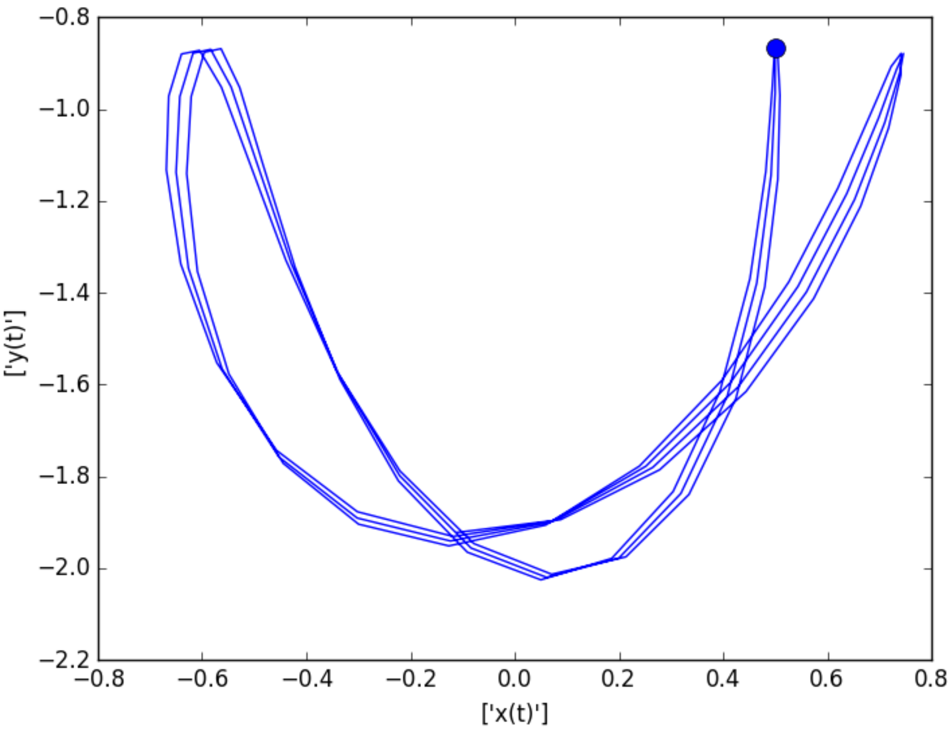


Figure 2: The motion of the pendulum in xy-plane with starting position marked.

k)



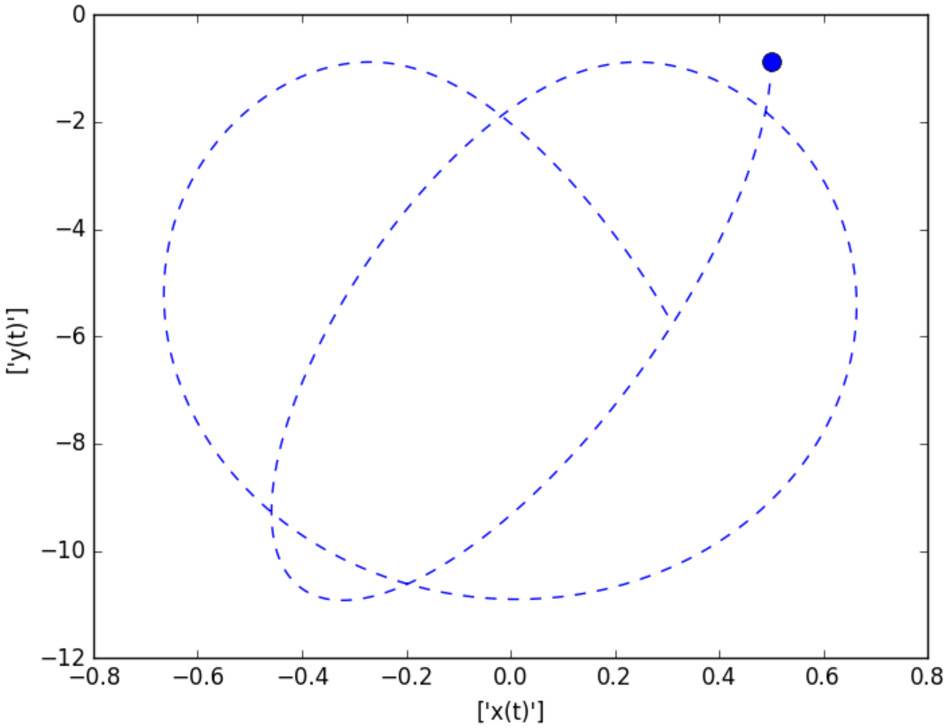
$\Delta t = 0.01$



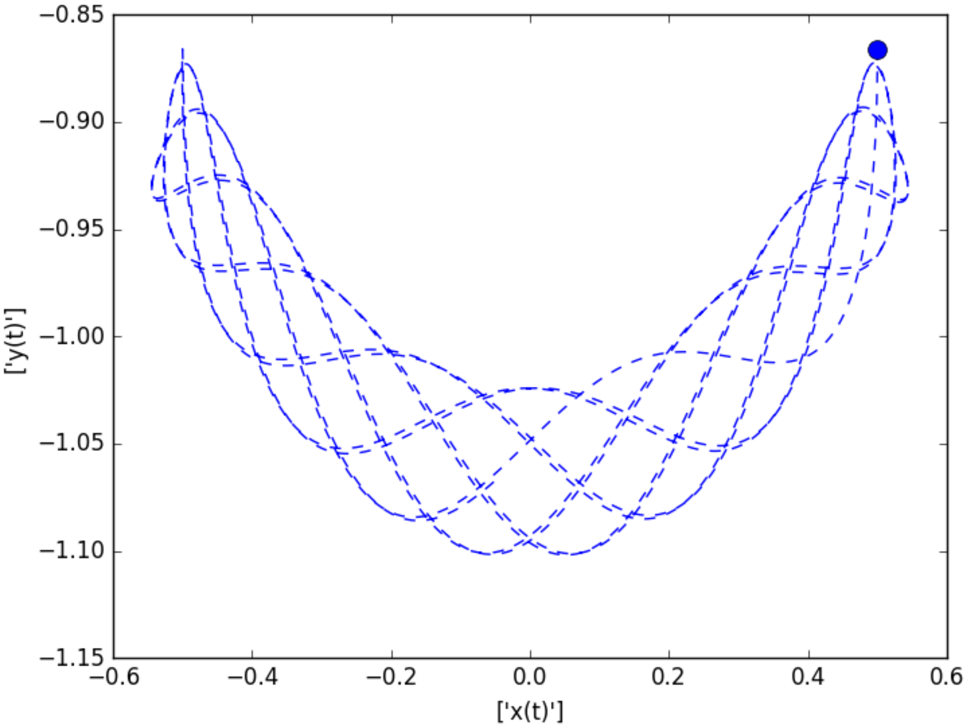
$\Delta t = 0.1$

The smoothness of the curves is reduced, especially for the curve where $\Delta t = 0.01$. This is due to a reduced number of data points produced by the program when the time increment is increased for the same lenght of time.

l)



$k = 20$

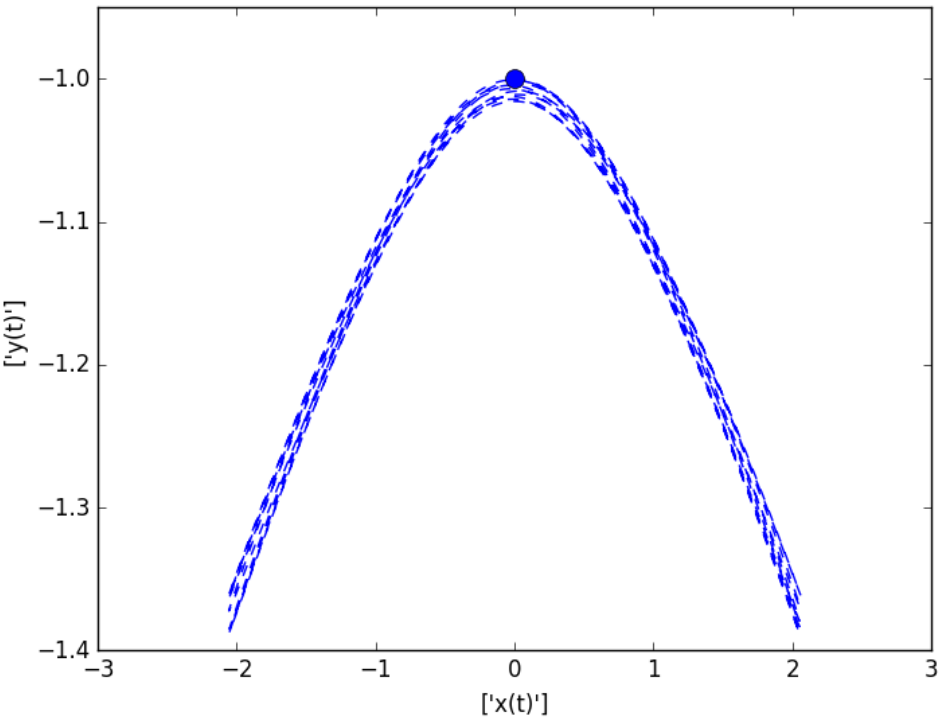


$k = 2000$

When k is reduced the pendulum accelerates in a direction for longer and therefore produce a wider swing than if k is increased. Given long enough time the swing of the lower k pendulum would cover a greater area. When increasing k the pendulum will move closer to a single path.

m)

```
for i in range(n-1):
    rr = norm(r[i,:])
    if L0>rr:
        k=0
    else:
        k=200
    a[i,:] = -g*array([0,1]) - k*(rr-L0)*r[i,:]/rr*m
    v[i+1,:] = v[i,:] + a[i,:]*dt
    r[i+1,:] = r[i,:] + v[i+1,:]*dt
    t[i+1] = t[i] + dt
```



With inital conditions:$\vec{v_0} = 6.0\frac{m}{s}\vec{i}$ and $\vec{r_0} = -L_0\vec{j}$
The pendulum turns very sharply lower when it reaches it's starting point, because all tension from the rope disappears when the lenght of the rope moves below the equilibrium lenght and gravity takes the pendulum lower.