MEK1100 - Oblig 1

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Oppgave 1a)

Finner t_m:

$$y(t) = v_0 t sin(\theta) - \frac{1}{2}gt^2 = 0$$

$$\rightarrow v_0 t sin(\theta) = \frac{1}{2}gt^2 \rightarrow \frac{2v_0 sin(\theta)}{g} = \frac{t^2}{t} = t_m$$

$$x(t_m) = v_0 \frac{2v_0 sin(\theta)}{g} cos(\theta) = \frac{V_0^2}{g} sin(2\theta)$$

b)

$$x^* = \frac{x}{x_m} = \frac{v_0 t cos(\theta)}{\frac{v_0^2}{g} sin(2\theta)} = \frac{gtcos(\theta)}{v_0 sin(2\theta)}$$

$$y^* = \frac{y}{x_m} = \frac{v_0 t sin(\theta)}{\frac{v_0^2}{g} sin(2\theta)} - \frac{\frac{1}{2}gt^2}{\frac{v0}{g} sin(2\theta)} = \frac{gtsin(\theta)}{v_0 sin(2\theta)} - \frac{g^2 t^2}{2v_0^2 sin(2\theta)}$$

$$t^* = \frac{t}{t_m} = \frac{t}{\frac{2}{g} v_0 sin(\theta)} = \frac{gt}{2v_0 sin(\theta)}$$

c)

Gjorde et forsøk, men må si jeg ikke helt skjønner hva som skal gjøres her.

```
from pylab import *

g = 9.81
v0 = 2
theta = pi/4.0

time = 20
dt = 0.001
n = int(round(time/dt))
t = zeros(n,float)
x = zeros(n,float)
y = zeros(n,float)
```

```python
for i in range(n-1):
    t1 = dt
    t[i] = t1*g/2*v0*sin(theta)
    x[i] = (t[i]*g/v0)*(cos(theta)/sin(2*theta))
    y[i] = t[i]*g/v0*(sin(theta)/sin(2*theta)-g**2*t[i]**2/2*v0**2*sin(2*theta))

plot(x,y)
show()
```

Oppgave 2a)

Hastighetsfeltet:

$$\vec{v} = v_x\vec{i} + v_y\vec{j} = xy\vec{i} + y\vec{j}$$

$$v_x = xy, v_y = y$$

Setter inn i $v_x dy = v_y dx$:

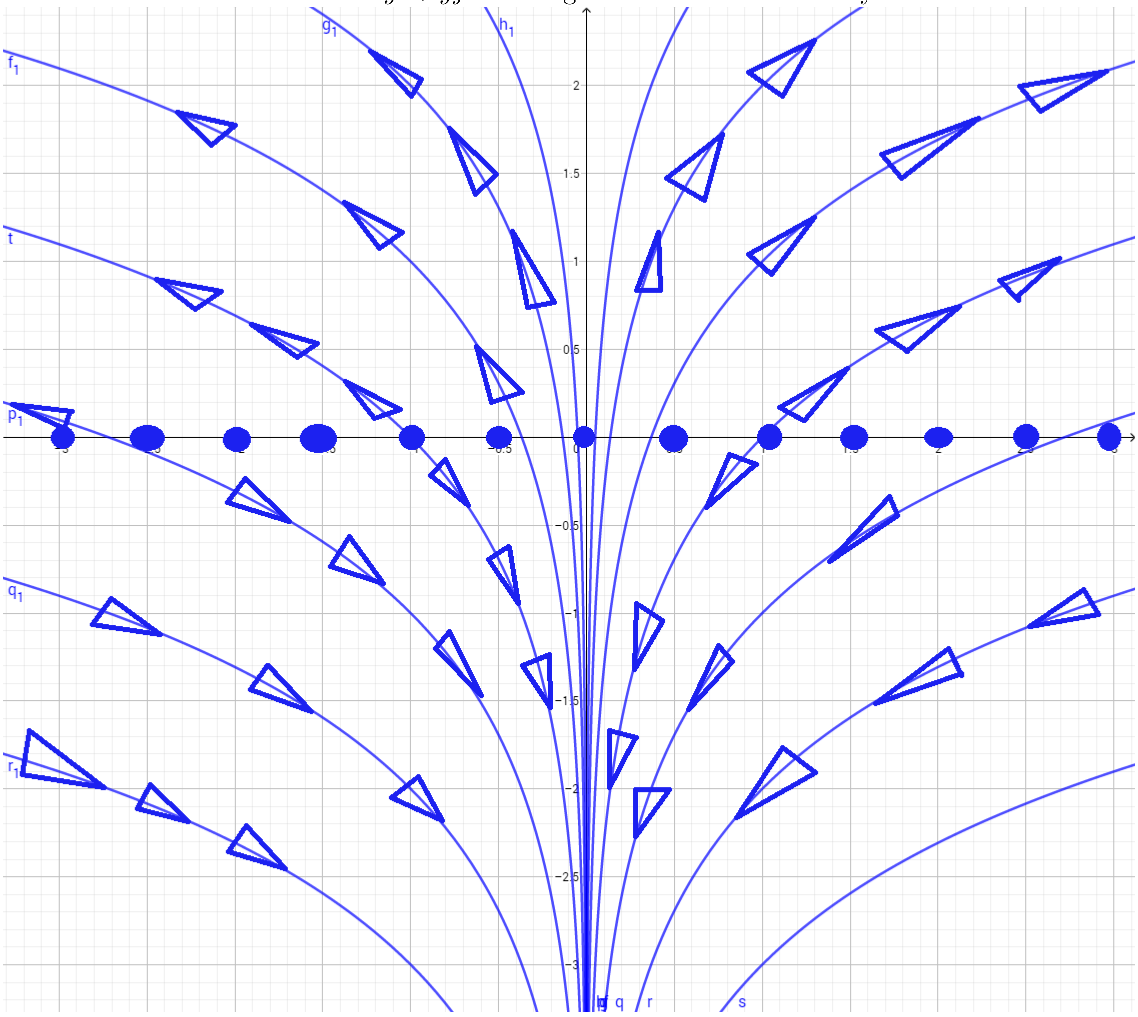
$$xydy = ydx$$

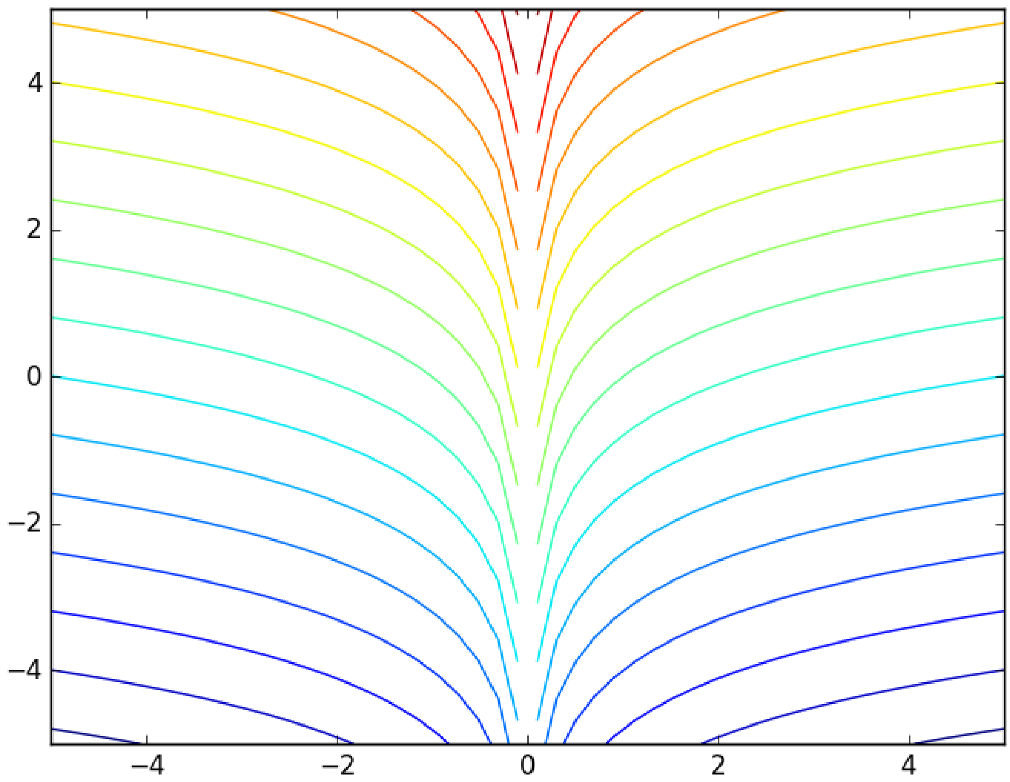
$$\rightarrow \frac{y}{y}dy = \frac{1}{x}dx \rightarrow \int dy \int \frac{1}{x}dx =$$

$$y = ln(x) + C$$

b)

Vi ser fra $\vec{v} = xy\vec{i} + y\vec{j}$ at hastighetsfeltet er lik null når y=0.





Strømlinjer tegnet ved hjelp av python. Kode under.

```
from pylab import *

tx = linspace(-5,5)
ty= linspace(-5,5)
x,y = meshgrid(tx,ty)

c = y - log(-x)       #y = ln(x)+c
c2 = y - log(x)
plt.contour(x,y,c,20)
plt.contour(x,y,c2,20)
show()
```

c)

Regner ut strømfunksjonen.

$$xy = -\frac{d\psi}{dy} \rightarrow \int \frac{d\psi}{dy} dy = -x \int y dy$$

$$\rightarrow \psi = -x\frac{y^2}{2} + C(x)$$

$$y = \frac{d\psi}{dx} \rightarrow \int \frac{d\psi}{dx} dx = \int y dx$$

$$\rightarrow \psi = yx + C(y)$$

Ingen c(x) og c(y) kan gjøre $-x\frac{y^2}{2} + C(x) = yx + C(y)$
Det finnes ingen strømfunksjon.

Oppgave 3a)

$$v_x = cos(x)sin(y), v_y = -sin(x)cos(y)$$

Finner divergensen:

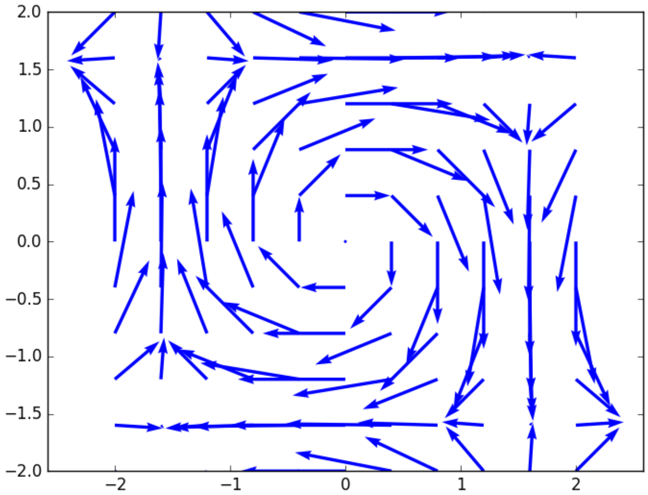
$$\nabla \cdot \vec{v} = \frac{dv_x}{dx} + \frac{dv_y}{dy} = -sin(x)sin(y) + sin(x)sin(y) = 0$$

Finner virvlingen:

$$\nabla \times \vec{v} = (\frac{dv_y}{dx} - \frac{dv_x}{dy})\vec{k} = (-2cos(x)cos(y))\vec{k}$$

b)

Tegner strømvektorer ved hjelp av python.



c)

Finner sirkulasjonen om randa til kvadratet ved å regne ut linjeintegralet til de fire sidene.

$$\int_{\pi/2}^{-\pi/2} -sin(\frac{-\pi}{2})cos(y)dy = \int_{\pi/2}^{-\pi/2} cos(y)dy = [sin(y)]_{\pi/2}^{-\pi/2} = -2$$

$$\int_{-\pi/2}^{\pi/2} -sin(\frac{\pi}{2})cos(y)dy = \int_{-\pi/2}^{\pi/2} -cos(y)dy = [-sin(y)]_{-\pi/2}^{\pi/2} = -2$$

$$\int_{-\pi/2}^{\pi/2} sin(\frac{-\pi}{2})cos(x)dx = \int_{-\pi/2}^{\pi/2} -cos(x)dx = [-sin(x)]_{-\pi/2}^{\pi/2} = -2$$

$$\int_{\pi/2}^{-\pi/2} sin(\frac{\pi}{2})cos(x)dx = \int_{\pi/2}^{-\pi/2} cos(x)dx = [sin(x)]_{\pi/2}^{-\pi/2} = -2$$

Dette gir sirkulasjon: $-2-2-2-2=-8$.

d)

Finner strømfunksjonen:

$$\frac{\delta\psi}{\delta y} = -cos(x)sin(y)$$

$$\frac{\delta\psi}{\delta x} = -sin(x)cos(y)$$

Integrerer:

$$\int \frac{\delta\psi}{\delta y}dy = \psi = -cos(x)\int sin(y)dy = cos(x)cos(y) + C(x)$$

$$\int \frac{\delta\psi}{\delta x}dx = \psi - cos(y)\int sin(x)dx = cos(y)cos(x) + C(y)$$

$$cos(x)cos(y) + C(x) = cos(x)cos(y) + C(y) \rightarrow C(x) - C(y) = 0$$

Som gir:

$$\psi = cos(x)cos(y)$$

e)

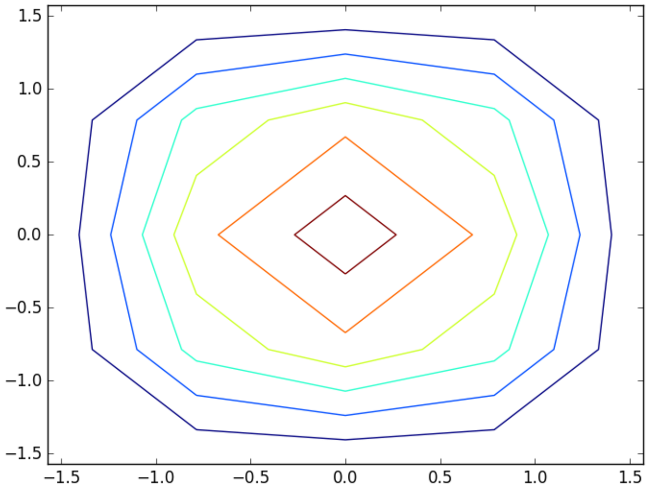
Taylorutvikling av andre orden rundt origo(0,0):

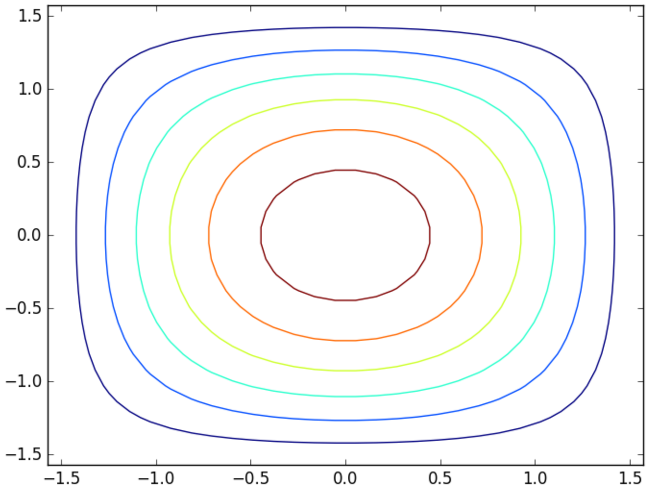
$$\psi(0,0) + \frac{\delta\psi}{\delta x}(x-0) + \frac{\delta\psi}{\delta y}(y-0) + \frac{1}{2}\frac{\delta^2\psi}{\delta x^2}(x-0)^2 + \frac{1}{2}\frac{\delta\psi}{\delta y^2}(y-0)^2 + \frac{\delta^2\psi}{\delta x\delta y}(x-0)(y-0)$$

$$= cos(0)cos(0) + -sin(0)cos(0)x + -cos(0)sin(0)y + \frac{1}{2} - cos(0)cos(0)x^2 + \frac{1}{2} - cos(0)cos(0)y^2 + sin(0)sin(0)xy$$

$$= 1 - \frac{x^2}{2} - \frac{y^2}{2}$$

Oppgave 4a)





Høyere verdier av n gir mer nøyaktig plot. Men er ikke helt sikker påhva som menes når jeg skal sammenholde plotene med punkt e).

```
from numpy import linspace, meshgrid, cos, pi
from pylab import *

def streamfun(n=20):
    '''Regner ut et grid og en str mfunksjon'''

    x=linspace(-0.5*pi,0.5*pi,n)
    #resultatet er en vektor med n elementer, fra -pi/2 til pi/2
    [X,Y] = meshgrid(x,x)
    psi=cos(X)*cos(Y)
```

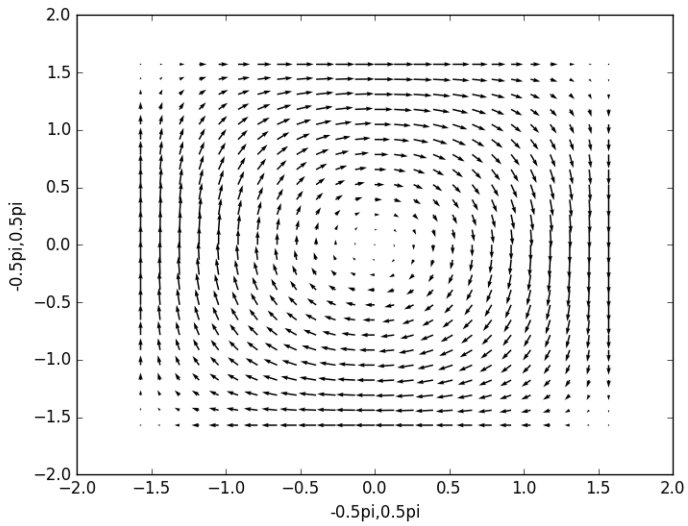
```

    return X, Y, psi

X30,Y30,psi30 = streamfun(30)
X5,Y5,psi5 = streamfun(5)
psiTaylor_5 = 1-X5**2-Y5**2
psiTaylor_30 = 1-X30**2-Y30**2

contour(X5,Y5,psi5)
xlabel("-0.5pi,0.5pi")
ylabel("-0.5pi,0.5pi")
show()
contour(X30,Y30,psi30)
xlabel("-0.5pi,0.5pi")
ylabel("-0.5pi,0.5pi")
show()
```

b)




```
from pylab import *

def velfield(n):
    t = linspace(-0.5*pi,0.5*pi,n)
    x,y = meshgrid(t,t,indexing="ij")

    vx = cos(x)*sin(y)
    vy = -sin(x)*cos(y)

    return x,y,vx,vy

x,y,u,v = velfield(25)

quiver(x,y,u,v)
xlabel("-0.5pi,0.5pi")
ylabel("-0.5pi,0.5pi")
show()
```