# Pre-Registration: Systematic Characterisation of Velocity-Curvature Power Law Analysis Protocols Across Parameter Space

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NB OSF <https://osf.io/dwxa2/> github <https://github.com/dagmarfraser/velocity-curvature-power-law-simulation> UBIRA/edata <https://edata.bham.ac.uk> tested but not uploaded yet

## Abstract

The power law of Lacquaniti et al (1983), relating the tangential velocity to curvature (v ∝ κ^(-β), where β ≈ 1/3) may encode a fundamental principle governing biological movement. Fraser et al. (2025) demonstrated that legacy protocols systematically bias towards β ≈ 1/3, potentially skewing measurement of the underlying generator. Fraser and colleagues introduced a vetted protocol that avoids this bias. However, neither legacy nor vetted protocol is characterised across a wide range of biologically plausible parameters. This methodological gap undermines potential diagnostic applications of the power law where divergences of β, |Δβ| = 0.03-0.04, are calculated between autistic and neurotypical populations (Cook et al., 2023; Fourie et al., 2024).

We present a comprehensive characterisation framework, systematically evaluating performance across 14.7 million parameter combinations; across analysis protocols and kinematic parameters. Simulated data with known ground truth, allows Linear Mixed Effects Models to map the effective limits of different analytical approaches, identifying optimal strategies for specific measurement contexts.

This investigation establishes the first evidence-based protocol selection framework for velocity-curvature power law analysis, providing essential guidance for translating kinematic principles into reliable kinematic assessments.

## 1. The Velocity-Curvature Power Law: From Fundamental Principle to Kinematic Assessment

### 1.1 Theoretical Foundation

The velocity-curvature power law, codified by Lacquaniti et al. (1983), describes a remarkably consistent relationship governing biological movement across scales and species. This kinematic invariant relates tangential velocity to trajectory curvature through a power law:

v(t) = VGF × κ(t)^(-β)

where v represents tangential velocity, κ denotes curvature, VGF is the constant velocity gain factor, and β typically approximates 1/3. This "one-third power law" emerges across diverse motor behaviours; from eye saccades, to drawing movements, to bodily trajectories. The law emerges in non-human species suggesting a fundamental computational principle underlying motor control (Flash, 2021).

### 1.2 Methodological Concerns in Power Law Analysis

Fraser et al. (2025) reported that conventional analytical protocols contain systematic biases that artificially reduced deviations from β ≈ 1/3 regardless of underlying signal characteristics. Their investigation identified three critical failure modes:

1. **Differentiation artefacts:** Finite differences differentiation amplifies noise, creating artificial power law confirmations at higher magnitudes
2. **Filtering distortions:** Butterworth filters, ostensibly employed to address noise alter the underlying signal, creating spurious power law results of one-third
3. **Log-transform / linear regression complications:** Logarithmic projection, required for linear regression, fundamentally alters noise distributions, minimising deviations from one-third

The methodological implications of this uncertainty are profound: the β range compression around one-third, introduced by legacy protocols may mask genuine biological variation, at best rendering conservative the observed divergences in existing literature, at worst explaining the prior ubiquity of the one-third power law.

Fraser and colleagues demonstrated these biases for β = 0 (i.e. constant velocity regardless of curvature), yielding spurious β ≈ 1/3 through the legacy protocol. The authors in turn proposed a vetted protocol employing Savitzky-Golay smoothing differential filters and non-linear regression to minimise these distortions. This vetted protocol was demonstrated to avoid spurious one-third confirmation. However yet a full characterisation of protocol behaviour across a spectrum of β is required to assess if and where any protocol perform adequately, not merely avoiding spurious 1/3 compliance.

### 1.3 Methodological Stakes: Small Effects with Large Implications

Recent autism research has elevated the urgency of characterising the methodological precision of power law analyses. Cook et al. (2023) and Fourie et al. (2024) independently documented that autistic individuals demonstrate subtle but consistent deviations from canonical power law values:

* **Fourie et al. (2024):** Autistic children showed β = 0.36 versus neurotypical β = 0.33 (|Δβ| = 0.03, Cohen's d = 0.46)
* **Cook et al. (2023):** Adult autistics populations exhibited similar effect sizes (|Δβ| = 0.03-0.04)

This β exponent can purportedly be extracted from any curved movement, regardless of scale or duration. Therefore, these numerically small divergences offer potential non-invasive kinematic markers for neurodevelopmental conditions, though their broader significance requires systematic validation. However, detecting such subtle effects demands analytical precision. Characterizing precision within legacy or vetted protocols, across the full range of biologically plausible parameters, is the aim of this study.

## 2. Research Questions and Innovation

### 2.1 Primary Investigation

**Which analytical protocols enable reliable detection of the |Δβ| = 0.03-0.04 deviations documented in preliminary autism research, and how do protocol-parameter interactions determine measurement precision?**

### 2.2 Methodological Framework

We introduce a comprehensive characterisation framework that:

* Maps protocol performance boundaries across parameter space through systematic evaluation
* Identifies optimal analytical strategies for specific measurement contexts
* Quantifies uncertainty boundaries for each protocol-parameter combination
* Provides evidence-based guidance for future protocol selection

## 3. Ground Truth Parameter Space

### 3.1 Comprehensive Parameter Space

The comprehensive characterisation of the analytical protocols requires simulated elliptical trajectories with known β and VGF ground truth values, contaminated with noise. This ground truth approach enables a definitive assessment of protocol performance.

Synthetic trajectories are generated, and in turn examined, with the following parameters;

* Generative β values covering the empirical spectrum observed in biological movements by Huh and Sejnowski (2015)
* Velocity gain factors represent different movement speeds, from slower deliberate actions to rapid fluid movements; 0.5Hz to 2Hz ellipse trajectories, at β = 1/3. These values straddle a value under which power law may be more consistently recovered empirically i.e. when ellipse tracing approximates or exceeds ~1Hz (Matic and Gomez-Marin, 2023)
* Noise characteristics span from instrument precision (submillimetre) through average human motor variability for such tasks (3.3mm; Madirolas et al., 2022) to high measurement degradation (10mm). For computational tractability this wide range is sparse above 2mm. Noise is added as injected as per Maoz et al., (2005)
* Noise colours range from the white noise of instrumentation, the presumed pink of biological motion. Red and black noise are also considered, as after differentiation these noises are transformed into white and pink (e.g. coordinates to velocities). Noise generation employs the fractional differencing approach of Xu et al. (2022) for accurate 1/f^α noise across the complete spectrum from white (α=0) through black (α=3)
* Sampling rates span widely available instrumented tablets, e.g. 60Hz Android tablets, 120Hz for WACOM devices, to iPad Pro with proprietary stylus at 240Hz
* Protocol variations compare established legacy approaches against the improved vetted methods introduced by Fraser et al. (2025). See attached table

| **Parameter** | **Values** | **Count** | **Methodological Justification** |
| --- | --- | --- | --- |
| Sampling Rates (fs) | [60, 120, 240] Hz | 3 | Consumer tablet through professional motion capture spectrum |
| Generative β | 0:(2/3)/20:(2/3) | 21 | Empirical range observed by Huh and Sejnowski (2015) for shapes with angular frequency φ [0...6] |
| VGF Values | exp(4.5:0.1:5.8) | 14 | Velocity gain factors corresponding to ellipse tracing frequencies of ~0.5-2 Hz |
| Noise Colour (1/f^α) | 0:0.1:3.0 | 31 | White (α=0) replicating Maoz et al. (2005), Pink (α≈1) physiological noise, through to Black (α=3) accounting for α transformations after differentiation |
| Noise Magnitudes | [0:0.025:0.1, 0.25:0.25:2.25, 4, 6, 8, 10] mm | 18 | Instrument precision < 0.1mm, regression-safe < 2.25mm, to challenging measurement conditions up to 10mm |
| Filter Types | Legacy vs. Vetted | 2 | Butterworth + finite differences vs. Savitzky-Golay comparison |
| Regression Types | Linear, LMLS, IRLS | 3 | Legacy versus non-linear regression methods |
| Parameter Combination | 5 repetitions | 5 | Statistical reliability for precision assessment |

**Total configurations:** 14,764,680 (including 5 repetitions per condition)

### 3.2 Precision Stratification

Systematic assessment of the adequacy of protocols will employ the ±0.03 precision boundary based on preliminary findings from autism research (Cook et al., 2023; Fourie et al., 2024). However it should be noted this precision boundary derives from legacy protocol calculations which compress the expressed range of β. Therefore, these thresholds potentially represent conservative estimates i.e. true biological differences may be larger. Additional precision boundaries will be empirically derived through bootstrap confidence interval analysis and cross-validation performance assessment across the 14.7M parameter combinations.

## 4. Validation Criteria

### 4.1 Parameter Space Validation

Prior to simulating ground truth trajectories, the computational tractability of such a large parameter space has been comprehensively validated through our 4-stage Model Adequacy Framework (ModelAdequacyMasterv002). The seven-way factorial mixed-effects model with full interaction structure and random intercepts for parameter combinations has been demonstrated to converge with test data. However convergence of is not guaranteed with the ground truth simulations. In anticipation this framework also demonstrates computational feasibility across tractability levels (1-9) which employ progressively smaller subsets of the parameter space. The true subset framework ensures that all tractability levels 1-8 use identical parameter values as exact subsets of Level 9, maintaining methodological consistency. Additionally this enables scalable analysis across different computational budgets.

### 4.2 Ground Truth Validation Criteria

This controlled ground truth generation approach provides the empirical foundation necessary to translate the observations of Δβ in clinical populations into a diagnostic tool.

* Verification of β recovery accuracy within ±0.03 for optimal conditions (white noise 0-0.1mm standard deviation), in which only instrument error is observed. Failure in this zone will suggest the method can never accurately recover β.
* Demonstration of noise resilience across physiological ranges (pink through black noise spectra). Failures in this zone will indicate that only biased estimate of β can be recovered for biological motion.
* Computational stability assessment across all parameter combinations. This will delineate the boundaries within which β can be recovered faithfully.

## 5. Four-Stage Progressive Analysis Framework

### Stage 1: Global Interaction Modelling

Fit comprehensive Linear Mixed Effects Model capturing all parameter interactions across the complete parameter space, relating them to divergences from the groundtruth generated beta:

δβ ~ βgenerated × VGF × samplingrate × filtertype × regressiontype × noisemagnitude × noisecolour + (1|parameter combination)

Where δβ is βgenerated - βrecovered.

This global model quantifies how each parameter and their interactions systematically affect β recovery accuracy. The random effect accounts for the five simulation repetitions per parameter combination, whilst the fixed effects reveal which measurement conditions introduce systematic bias versus random error. The comprehensive 7-way interaction structure identifies the specific parameter combinations where protocols fail and maintain precision.

### Stage 2: Systematic Adequacy Assessment

Stage 2 determines whether the global model adequately captures parameter recovery patterns or requires conditional analysis for specific parameter regions. Three empirical criteria identify model inadequacy:

**Coefficient Stability:** Bootstrap confidence intervals exceeding ±0.03 indicate unstable parameter estimates that cannot reliably detect autism-relevant effect sizes (Cook et al., 2023; Fourie et al., 2024).

**Residual Pattern Analysis:** Cohen's d > 0.5 for systematic deviations reveals parameter regions where the global model systematically under- or over-predicts β recovery accuracy.

**Prediction Accuracy Assessment:** Changepoint analysis of cross-validation R² distributions will allow identification of inflection points where model performance degrades significantly, thus indicating inadequate representation of underlying parameter relationships.

Parameter regions failing any criterion proceed to Stage 3 conditional analysis. Regions meeting all criteria confirm global model adequacy for those measurement conditions.

### Stage 3: Conditional Analysis

Stage 3 develops specialized models for parameter regions where the global model demonstrates inadequate performance. This targeted approach addresses the specific sources of model failure identified in Stage 2 rather than imposing a single analytical framework across the entire parameter space.

Conditional analysis isolates problematic parameter combinations and develops region-specific models that account for the unique interaction patterns within those measurement contexts. For example, high-noise conditions may require different analytical structures than low-noise scenarios, or specific combinations of sampling rate and filter type may exhibit non-linear relationships not captured by the global model.

Each conditional model undergoes rigorous validation through regional cross-validation to ensure improved performance over the global approach. Only conditional models demonstrating statistically significant improvement proceed to Stage 4 integration.

### Stage 4: Integrated Assessment Framework

Stage 4 synthesizes the global and conditional models into a unified decision framework that optimizes analytical approach selection based on specific experimental conditions. This integration provides researchers with evidence-based guidance for choosing between global model predictions and conditional model recommendations depending on their measurement parameters.

The framework generates precision-stratified protocol recommendations that account for the uncertainty inherent in different measurement contexts. These recommendations include quantified confidence bounds that enable researchers to assess whether their experimental conditions support reliable detection of the small effect sizes relevant to neurodevelopmental research applications.

## 6. Expected Outcomes and Research Translation

### 6.1 Anticipated Results

Based on Fraser et al. (2025), we anticipate systematic protocol-parameter interactions: vetted protocols maintaining precision across broader parameter ranges, whilst legacy approaches demonstrate progressive degradation with increasing noise magnitude and decreasing noise colour (i.e. as α trends to 0). Global modelling will characterise these performance boundaries, with conditional analysis providing targeted optimisation for challenging measurement regimes.

Specifically, we expect:

* For white noise (α=0) at magnitudes exceeding instrument precision (>0.1mm), progressive β estimation degradation
* For pink noise (α≈1) beyond average human error (>3.3mm), reliability boundaries for both protocols
* For red noise (α=2) in challenging ranges (>10mm), limited protocol utility
* For black noise (α=3), potential adequacy across all magnitudes due to spectral characteristics

These patterns will manifest as improved noise magnitude tolerance and lower noise colour resilience in vetted protocols compared to legacy methods.

### 6.2 Methodological Decision Support

Deliverables include:

* **Protocol selection flowchart** based on experimental conditions
* **Performance boundary maps** delineating protocol operational ranges
* **Precision lookup tables** for common paradigms
* **Uncertainty quantification tools** for individual assessments

## 7. Significance and Future Directions

This investigation provides the first systematic characterisation of velocity-curvature power law analysis protocols across comprehensive parameter space. By mapping where methodological approaches succeed and fail, we enable evidence-based protocol selection for detecting small-effect kinematic markers essential to research translation.

Our framework extends beyond autism applications to any condition affecting movement kinematics. The open-source implementation ensures reproducibility whilst the parametric characterisation approach establishes a methodological paradigm for validating complex biomarker protocols under realistic measurement conditions.

## 8. Timeline

**Weeks 1-2:** Finalize framework validation ✓ COMPLETE - 4-stage Model Adequacy Framework validated (computational feasibility confirmed across tractability levels 1-9)  
**Weeks 3-5:** Execute 14.7M parameter simulations using validated crash-safe toolchain (210 compute hours)  
**Weeks 6-8:** Progressive precision analysis and threshold investigation  
**Week 9:** Integration and documentation of methodological translation framework

## 9. Open Science Commitment

All materials will be publicly available via OSF (CC BY 4.0):

* Complete MATLAB simulation toolchain
* Simulation database with characterisation metadata
* Precision stratification modules
* Methodological decision support algorithms

## References