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Supplementary Material
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The chapter "Non-coding Options for Data Analysis" contains contributions by Ms. Jenelle Millison, UTSA alumna.

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Preface

Data is everywhere, therefore data should be for everyone.

There is a constellation of resources that teach different aspects of data science, with promises ranging from learning skills instantly without mathematics, programming, or statistics, to mathematical methods that require a hopelessly long chain of prerequisites. Those resources fulfill specific needs and specialized niches. This book, however, attempts something different.

This book is intended for data analysts. We take a holistic approach trying to survey the most elementary building blocks in data analysis (quantitative methods), and their connection to the larger enterprise of data science. That is why we refer to the contents of this book as *Data Analytics*: the quantitative aspects of data analysis plus all else that makes data analysis possible.

The collection of knowledge skills and abilities required to become a competent data analyst is broad and difficult to acquire. It has foundational components related to mathematics, statistics, and computer science. It also has operational components related to information technology, scientific data analysis, and data engineering. Finally, there is a translational dimension related to ethical and legal considerations, as well as interdisciplinary collaborations.

The dominating paradigm in interdisciplinary education is inherently inefficient. It consists of teaching the same thing to each student at the same pace within a classroom with well defined initial end ending points analogous to a tree structure; however: (i) students in these settings usually come from different disciplines, thus have different (often non-overlapping) backgrounds, and (ii) curricula in interdisciplinary fields are comprised by subject matters drawn from different (often traditionally disconnected) areas. This book presents a model of experiential learning for data science in which the metaphor of the tree is replaced by a dense rhizome-like network that does not privilege a particular path, but instead offers a milieu

for traversal. In practice, it is the student during the learning process who makes an abstract knowledge network come to a unique realization.

There are multiple data projects associated with this book. They are referenced throughout the chapters, but are contained in different data volumes. Each data volume has to do with complex and heterogeneous public data in instances in which either data reveals an injustice, or the way data is collected and/or used is conducive to injustice. The rationale for this approach is that ethics problems, particularly related to injustice, tend to provoke a decisive response from students. Note that the concept of justice is rooted in specific moral systems, which vary in time and space. Within any given society there are groups that consider a given outcome just whereas a different group might consider the same outcome to be the apex of injustice. Experience has shown that most students are driven to learn about instances of injustice either to fight it, to try to debunk it, or to simply learn about the world. However, students must learn the techniques of data science to uncover the facts. We could call this method *"learning through passion."*

A competent data analyst never claims to have the true answer; they claim to have a correct answer, that is, a solution and insight generated by following a well-documented and reproducible set of steps. They know that insight and knowledge can change in time as perspectives evolve, new data becomes available, or additional interpretation informs new routes of analysis. Informed data analysts recognize the pitfalls of introducing one's values into data analysis or presentation, even when such values are aligned with the mission in which they find themselves in, and they take steps to make such biases explicit.

The material presented here assumes that students have foundational knowledge of linear algebra, calculus and computer programming. The first example presents a computer program to multiply matrices, not as step-by-step operations on rows and columns of matrices, but instead using defined operators. After that, all data analysis problems are error minimization problems, which necessitates the use of calculus. Ideally, the reader will know multivariate calculus, but given a foundation of calculus it is possible to acquire the basic multivariable skills here.

This book is programming language-independent. When a programming task is presented, examples will be presented in MAT-

LAB/Octave, Python, R, and C++. Data analysts are encouraged to learn in depth at least one programming language, and become familiar with as many programming languages as needed for the environment in which they will operate. Furthermore, a data analysis pipeline might be composed of elements created with different programming languages.

In closing, the author thanks the reader for attempting the voyage that follows. It might not be comfortable. The fact that injustice was chosen as the backbone to articulate concepts does not mean that we adopt a grim perspective of reality. Nor is this a book of lamentations. In borrowing from poetry, we embrace the Ulysses from Konstantinos Kavafis' "Ithaca" (1911), *"When you depart for Ithaca, wish for the road to be long, full of adventure, full of knowledge. Fear not the Laestrygonians and the Cyclopes, nor the angry Poseidon."* Let us depart now from safe harbors.

Prof. Juan B. Gutiérrez,
San Antonio, Texas, 2023.

Contents

Chapter 1

A Roadmap for Data Analytics

What is Data Analytics? In the context of this book, we use this name to refer to data analysis (quantitative methods), and their connection to the larger enterprise of data science through other areas such as ethics, code versioning (or, more general, configuration management), solution architecture, reporting, etc. In brief, *Data Analytics* is the quantitative aspects of data analysis plus all else that makes that analysis possible.

Data analysts employs advanced mathematical, statistical and computational techniques to tackle problems, evaluates alternatives, and executes solutions. They consume extensive data sources to address questions through exploratory data analysis, probabilistic modeling, or predictive analysis. Effectively communicating insights to stakeholders requires statistically rigorous visualizations and narratives. The role on occasion demands algorithm design to unearth data insights. Additionally, quantitative methods and software tools are utilized to construct computer programs, while designing coherent data structures for data access and analytics needs. Ethical and legal considerations are imperative throughout a project's duration.

The typical components that define data analytics start with data collection, where raw data is gathered from diverse, often multimodal (e.g. text, video, time series, etc.) sources. This is followed by the step of data cleaning, where inaccuracies and inconsistencies are weeded out. Once cleaned, the data is then explored to understand its structure, discern trends, and identify potential anomalies. The subsequent phase involves data modeling, where statistical or computational algorithms are applied to extract specific insights or predictions.

There are several several categories in data analytics. Descriptive analytics focuses on shedding light on past events, answering the question of what happened. Diagnostic analytics delves deeper, probing why a particular event occurred. Predictive analytics, as the name suggests, forecasts future events or trends. Lastly, prescriptive analytics provides recommendations on actions that should be taken based on the data's insights.

A myriad of tools and technologies facilitate data analytics. Simple tasks may utilize spreadsheets such as Microsoft Excel or Google Sheets. For more intricate tasks, programming languages like Python and R are prevalent. Visualization tools such as Tableau and Power BI assist in representing data in understandable formats. Furthermore, databases like SQL and NoSQL variants (e.g., MongoDB, Cassandra) play a role in storing and querying vast amounts of data.

Data analytics finds applications in a vast array of sectors. In the business domain, it's pivotal for sales forecasting, customer segmentation, and inventory management. Healthcare professionals employ it for predicting disease outbreaks and patient outcomes. The finance sector leverages it for fraud detection and credit scoring. Even in sports, it's harnessed to analyze player performances and optimize game strategies.

However, the journey in data analytics isn't devoid of challenges. Data privacy and security concerns loom large, emphasizing the need for ethical data usage and protection. Ensuring data quality, i.e., its accuracy and consistency, remains a persistent challenge. Moreover, there's a palpable skill gap, necessitating the need for professionals who can analyze increasingly intricate datasets.

Looking ahead, the future of data analytics is promising and dynamic. Big data analytics is emerging as a field focusing on handling and analyzing exceptionally large datasets. Real-time analytics, which pertains to analyzing data as it is generated or procured, is gaining traction. Additionally, augmented analytics, which utilizes machine learning to automate insights, is another promising frontier.

Data analytics is inherently value-driven, vital for extracting value from vast datasets. It's an ever-evolving field, adapting and growing with technological and business advancements. Its applicability, spanning various sectors and industries, underlines its significance in today's data-driven world.

Describe in your own words what is data analytics and whay and how it is different from data analysis. Record your explanation as

answer #1.

1.1 Data collection

The oldest artifact known is the Blombos ocher plaque, named after the Blombosfontein Nature Reserve, 300 km east of Cape Town on coastline of South Africa. In 2002, two pieces of ochre, an iron-rich mineral, were recovered exhibiting deliberate and complex geometric patterns dated to 70,000 to 100,000 years old [Hen+02]. Although we will never know for sure whether this particular instrument was used to count celestial events, livestock, time, or nothing at all, the following 80,000 years would see an increased level of sophistication in quantity representation, with varying degree of accuracy.

Figure 1.1-1: Engraved ochre found in the Blombos Cave, South Africa. This stone is suspected to be one of the earliest counting devices [Com17].

The concept and utility of data have evolved significantly throughout human history [Nef04; Pos72; RS97]. In ancient civilizations, data primarily existed in the form of physical tallies or rudimentary records. The ancient Sumerians, for instance, used cuneiform script on clay tablets to record trade transactions and inventories. With the invention of the printing press in the 15th century, data dissemination became more widespread, leading to an era of information sharing and record-keeping [Eis80].

The 20th century ushered in groundbreaking advancements in data storage and computation. The invention of the electronic computer in the 1940s provided a platform for structured data storage and complex computations [Cer03]. By the late 20th century, the rise of the internet and the proliferation of digital devices produced an explosion in the volume of data generated, stored, and analyzed. This digital revolution laid the groundwork for what is now known as “Big Data” [MSC12].

Today, with the emergence of technologies such as the Internet of Things (IoT), artificial intelligence, and cloud computing, data generation and processing capabilities are expanding at an unprecedented rate. The evolution from rudimentary tallies to complex digital datasets has revolutionized various sectors, including business, healthcare, and science, to name a few [GR12].

1.2 Computational Machines

Humans have been using devices to aid in computations for millennia. A machine explicitly designated to aid computations was the abacus. Its exact origins are unknown, although there are clear archeological findings in Mesopotamia (2700-2300 BCE), Persia (600 BCE), Greece (300 BCE), and China (200 BCE). The latter, known as Suan Pan, is still in use today; methods have been developed to perform multiplication, division, addition, subtraction, square root, and cube root operations.

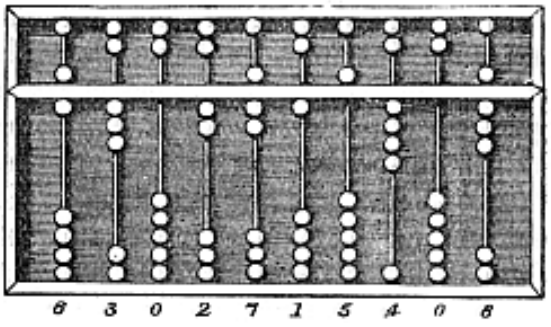


Figure 1.2-2: Suan Pan (the number represented in the picture is 6,302,715,408). This image appeared in the article for "abacus", 9^{th} edition Encyclopedia Britannica, volume 1 (1875) [Com16].

The Antikythera mechanism (150-100 BCE) was a machine designed to predict solar eclipses, along with the astronomical positions of several planets and the moon according to the theories of its day [Fre+06]. The results of the manipulation of this mechanism are now known to be inaccurate due to the limited understanding about planetary movement 2,000 years ago. The Antikythera mechanism is the first computing machine for which we can claim that computational errors rendered its use limited, whether the first users were aware of this limitation or not. The sophistication of the Antikythera mechanism was lost and would not occur again for at least 14 centuries,

with the emergence of watch makers in Europe.

Figure 1.2-3: The Antikythera mechanism. This image shows the largest gear in the mechanism, approximately 140 millimeters (5.5 in) in diameter [Com15b]. This mechanism is suspected to be the first mechanical computer.

The industrial revolution in Great Britain imposed demands on engineers to perform more and better calculations. This came with the certainty that dominion over the entire universe was within reach thanks to technification, and that calculations would lift off the veil of the mysteries of the universe. The zeitgeist was accurately captured by Pierre-Simon Laplace, in his *Essai philosophique sur les probabilités*, 1814 [Lap25, pages 3-4]:

> We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes

This paragraph, usually known as *Laplace's demon*, was the first mention of scientific determinism. It provides at the same time the notion of the the computation of everything (later embraced by Konrad Zuse, see below), and its possible implausibility.

It was until 1822 CE that a fully mechanized computing machine was designed by Charles Babbage in Great Britain, 1822 [Bab22]. Babbage's differential engine was intended to compute divided differences, an algorithm used at that time to calculate tables of logarithm and trigonometric functions. The design called the attention of governmental agencies, but Babbage was unable to deliver the promised product due to his interest in a more general analytical engine capable of more general operations. In the end, Babbage built neither but is commonly credited with the invention of both. The Difference Engine was finished in 1991 by the London Science Museum, and was shown in the exhibit *Computing* until September 1^{st}, 2015 [Mus91].

It took over a century for the next major development to occur. On the one hand, Alan Turing proposed in 1936 the concept of a computer of general purpose, what we call today a Turing Machine. Turing formalized the concepts of algorithm and computation. He worked in a computer used to decrypt German communications during World War II. Shortly after, Konrad Zuse created the Z1 in 1938 in his parents' living room in Hamburg. It was an electro-mechanical computer. The third version of this machine, named Z3 was the first programmable, fully automatic computer, with 2,000 relays, 22-bit word, operating at 10 Hz. Its development was suspended due to WWII. Zuse was the first to propose in 1967 that the entire universe is a computational machine; this is not far from Laplace's demon, although Zuse never credited him.

Turing is often cited as the father of computer science, although the numerical representation used in Zuse's Z3 is nearly identical to the current standard for floating point operations [Zus69], as described later in this chapter.

World War II catalyzed a fast advance of computing. International Business Machines' (IBM) Automatic Sequence Controlled Calculator (ASCC), known as Mark by Harvard staff (hence the name Harvard Mark I, II, III, and IV), was completed in 1944. The ENIAC is regarded as the first electronic computer. It was completed in 1946 for the US Army, Ordnance Corps.

The age of modern computing starts with the IBM System/360 (S/360), a family of commercial computers sold between 1965 and 1978. It was important because it standardized the use of 8 bits = 1 byte as the unit of memory, as described in the next section. Shortly after, Intel launched the first 8-bit microprocessor, named the 8008, in 1972; it was followed by the first 16-bit microprocessor, named the

8080, in 1974. In 1978, Intel released the 8086, a 16-bit processor that gave rise to the x86 architecture. In 2003, AMD launched the first x86 64-bit processor. As of 2017, the vast majority of computers (personal and servers) are based on the x86 architecture. Phones and tablets use a different architecture, but also based on either 32 or 62-bit platforms.

1.3 Bits, Bytes, and Floating Point

Computers have a finite number of ways to represent numbers. That is, they can only approximate a small subset of real numbers. Understanding this is an essential component of conducting numerical operations: You need to understand how and why numerical errors accumulate, and when they interfere with a calculation.

The unit of computing representation is a **bit** (unit represented by b), i.e. a circuit that is either open or closed. Eight bits are grouped into one **byte** (unit represented by B). This number evolved from computer architectures in the early days of computing, and was first standardized by ISO/IEC 2382-1:1993. In one byte, positions go from 0 to 7, usually represented from right to left. A 1 in the first position represents 2^0, in the second position is 2^1, and so on.

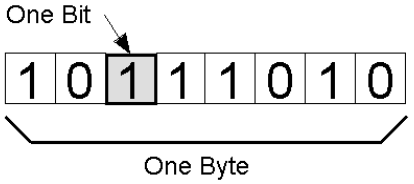


Figure 1.3-4: Reading from left to right, the example represents: $2^1+2^3+2^4+2^5+2^7 = 2+8+16+32+128 = 186$ when we read bits from right to left; this is the standard way to represent bits. If you read from left to right, you would obtain $2^0+2^2+2^3+2^4+2^6 = 1+4+8+16+64 = 93$. Thus, representation matters, and you need to know the convention.

A naive way to represent numbers with a computer is to allocate N decimal digits, for each number. For example:

$$012.345$$

This example represents a 6-digit fixed-point system. This 6-digit fixed-point system results in errors in some sum cases:

$$999.999 + 999.999 = \texttt{OVERFLOW}$$

Also errors occur in multiplication and division

$$013.000/003.000 = 004.333$$

In this case, we lose all digits after the third decimal.

We can allow the decimal point to "float" to gain accuracy with the same number of digits. For example:

$$013.000/003.000 = 4.33333$$

We could even represent larger numbers than 6 digits allowed us before:

$$999.999 \times 999.999 = 9.99998 \times 10^5$$

1.3.1 The IEEE Standard for Floating-Point Arithmetic

The IEEE Standard for Floating-Point Arithmetic (IEEE 754) is a technical standard for floating-point computation established in 1985 by the Institute of Electrical and Electronics Engineers (IEEE).

- Finite numbers may be either base 2 (binary) or base 10 (decimal).
- Each finite number is described by three integers:
  1. s = a sign (zero or one),
  2. c = a significand (or 'coefficient'),
  3. q = an exponent.
- The numerical value of a finite number is $(-1)^s \times c \times bq$ where b is the base (2 or 10), also called radix.

The three fields in an IEEE-754 float. Binary floating-point numbers are stored in a sign-magnitude form where the most significant bit is the *sign* bit, *exponent* is the biased exponent, and *fraction* is the significand minus the most significant bit.

One of the most surprising aspects of floating-point arithmetic is that addition is not commutative. Hence, $1-1/3-2/3 \neq 1-2/3-1/3$

$$1 - 1/3 - 2/3 = 1.1102e - 16$$

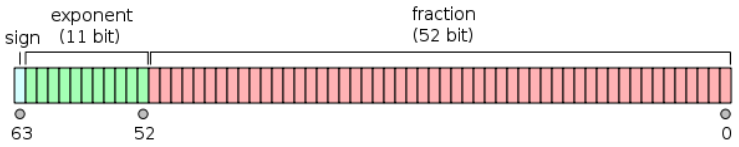


Figure 1.3-5: Representation of a double precision number [Com15a].

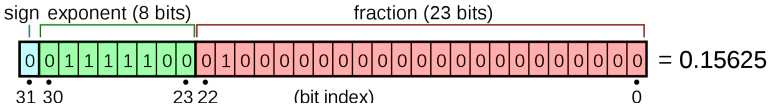


Figure 1.3-6: Representation of a single precision number [Com09]

$$1 - 2/3 - 1/3 = 5.5511e - 17$$

The following instruction compares whether $10^{10} + 1 - 10^{10}$ equals the value 1. Using elementary arithmetic, the answer should be a resounding *yes*. In the Python language, the exponent is represented by a double asterisk, like in Fortran, instead of the carat (^) sign used in most other computing platforms. The double equal sign tests whether the two sides are equal, and returns a Boolean value of *true* or *false*. Hence, the question is translated into the following instruction:

```
10**30 + 1 - 10**30 == 1
```

This particular instruction returns a Boolean value of *true* in Python and *false* in Matlab (the correct syntax is $10^10+1-10^10$ == 1). However, the following statement:

```
10**30 + 1. - 10**30 == 1
```

returns *false* in Python. Why? Record your explanation as answer #2.

An operation like

```
10**20 + 1 - 10**20 == 1
```

returns different results in Matlab, C, Fortran (*false*) vs. Python (*true*). It might be tempting to declare Python the winner in numerical computation, but what truly happens is that Python does not follow the IEEE-754 standard strictly. To learn about more subtleties of floating point in Python, go to `https://docs.python.org/3/tutorial/floatingpoint.html` In most cases, deviations from the standard in the Python interpreter are harmless and have an ideological origin rather than a technical one. However, it is conceivable

that extreme algorithms developed under the IEEE standard would have to be carefully adapted in Python. There are alternatives, but they are beyond the scope of this lab. For now, it suffices to say that there are landmines in the Python landscape.

We can explore further discrepancies in extreme floating point values by exploring in two numerical environments the results of $10 \cdot 10^b + 1 - 10 \cdot 10^b$ for different values of b. Download and install Python, Matlab, R, or compile a C/C++ program that executes the same operations. Compare what happens in these environments when you choose $b \in \{10, 1e2, 1e10, 1e100, 1e1000\}$. Describe the discrepancies, and analyze. Record your explanation as answer #3.

1.3.2 Mathematical Representation of the IEEE Standard for Floating-Point Arithmetic

If we want to represent a real number with a finite number N of memory positions, we can reserve one position for the sign, $N-k-1$ for the integer part, and k for the digits beyond the separator or fractional part. The notation conventionally adopted is [QSS10]:

$$x = (-1)^s[a_{N-2}a_{N-3}...a_k.a_{k-1}a_{k-2}...a_0] = (-1)^s \cdot \beta^{-k} \sum_{j=0}^{N-2} a_j \beta^j \quad (1.1)$$

where β is the base of the representation; s is either 1 0r 0; it could be any number, but usually it is 2 or 10.

The floating point representation in equation 1.1 limits the minimum or maximum number unless some scaling is allowed. In such case, a *floating point* representation is given by

$$x = (-1)^s \cdot [0.a_1a_2...a_t] \cdot \beta^{e-t} = (-1)^s \cdot m \cdot \beta^{e-t} \quad (1.2)$$

where $t \in \mathbb{N}$ is the number of significant digits a_i with $0 \leq a_i \leq \beta - 1$, $m = a_1a_2...a_t$ is an integer number called *mantissa* [1] with $0 \leq m \leq$

[1] It is important to understand the context in which the word *mantissa* is used. The word *mantissa* in computer science is defined as the part of a floating-point number representing the significant digits of that number multiplied by the base raised to the exponent to give the actual value of the number; it is often used as a synonym for *significand*. This, however, departs from the historical mathematical tradition because *mantissa* was first defined as the part after the separator (or fractional part) of a logarithm. Thus, in mathematics, a *mantissa* is the logarithm of a *significand*.

$\beta^t - 1$ and e is an integer number called *exponent* with $L \leq e \leq U$ (typically $L \leq 0$ and $U \geq 0$). If we require $a_1 \neq 0$ and $m \geq \beta^{t-1}$ then the representation in equation 1.2 is called *normalized*.

The total number of normalized floating point numbers of a double precision number $\mathbb{F}$ that can be represented with the IEEE standard is given by equation 1.3:

$$\begin{aligned} card\mathbb{F} &= 2(\beta - 1)\beta^{t-1}(U - L + 1) + 1 \\ &= 2(2 - 1)2^{52-1}(1023 - (-1022) + 1) + 1 \\ &= 9.214 \times 10^{18} \end{aligned} \tag{1.3}$$

We can approximate this value by noticing that a double precision number is represented by 64 bits, starting from zero. Therefore,

$$card\mathbb{F} \approx 2^{63} \approx 9.223 \times 10^{18}$$

A better approximation results invariably in equation 1.3.

Chapter 2

Non-coding Options for Data Analysis

There are numerous non-coding alternatives available for conducting data analysis. These tools offer graphical user interfaces (GUIs) that enable users to perform data analysis without the need to write code. They are popular among individuals who may not have coding experience or who prefer a more visual approach. Here are some of the prominent non-coding alternatives for data analysis:

1. Microsoft Excel: It is one of the most widely-used tools for basic data analysis. Offers functions for statistics, data visualization, and data manipulation. Pivot tables allow for powerful data summarization and aggregation.

2. Google Sheets: Similar to Excel but cloud-based. Provides data analysis functions, charting, and collaboration features.

3. Tableau: A powerful data visualization and business intelligence tool. It wnables users to create interactive dashboards and visualizations. It can connect to a variety of data sources.

4. Power BI: Microsoft's business analytics tool. Offers data visualization, report sharing, and enterprise-level capabilities. Integrates well with other Microsoft products.

5. SPSS: Statistical software widely used in social sciences and marketing research. Provides a GUI for data manipulation, statistical tests, and modeling.

6. QlikView/Qlik Sense: Data visualization and business intelligence tools. Known for their associative data modeling and in-memory data processing, allowing for interactive dashboards.

7. Knime: Open-source data analytics, reporting, and integration platform. Uses a modular data pipelining concept, allowing users to visually create data flows.

8. RapidMiner: A platform for data science and machine learning. Offers a visual workflow designer for data preprocessing, modeling, and deployment.

9. Stata: Statistical software widely used in economics and social sciences. While it has a coding aspect, many of its functions can be accessed and executed via menus and dialogs.

10. SAS Enterprise Guide: GUI-based interface to the SAS system. Allows users to conduct advanced analytics, business intelligence, and data management without writing SAS code.

11. CODAP (Common Online Data Analysis Platform): Web-based platform for data exploration primarily used for educational purposes. Offers interactive, dynamic, and visual interfaces for data analysis.

12. TIBCO Spotfire: Data visualization and analytics software. Supports creating dashboards, analytical applications, and sophisticated visualizations without coding.

Many of these tools provide a combination of drag-and-drop interfaces, menus, and dialogs that abstract away the coding aspect. While they are user-friendly and can be powerful, it's essential to understand that the flexibility and customization potential might be limited compared to coding-based solutions. For advanced analyses or specific tasks, coding might still be necessary, even with these tools. However, they offer a great starting point and cover a wide range of analytical needs for many users.

2.1 Visualizing Individual Items on a Map using CODAP

CODAP (Common Online Data Analysis Platform) is a free, open-source platform for exploring and analyzing data, particularly for educational purposes. It's web-based, so users don't need to install anything to use it. CODAP is designed with a focus on dynamic, interactive, and visual interfaces to support students in learning data analysis and statistics.

CODAP offers draggable data points, sliders, and other dynamic features that make it engaging for learners to play with data. While it can be used in various educational contexts, one of its primary goals is to support science and math education in middle and high schools.

Teachers and educators often use CODAP as an integral part of their curriculum to help students gain a hands-on understanding of data analysis concepts and techniques.

To visualize a list of items on a map using a list of coordinates in CODAP, follow these steps:

1. Importing the Data
   (a) Open the CODAP website and create a new document.
   (b) If you have your data in an external file (like a CSV), drag and drop it into CODAP, or use the "Import Data" feature. Your data should include columns for latitude, longitude, and any other attributes of the items you want to display.
2. Organizing the Data
   (a) Ensure that your dataset is visible in the data table view (Figure 2.1-1). It should appear as a table with rows representing each item and columns representing attributes like latitude, longitude, etc.
3. Adding a Map
   (a) From the toolbar at the top, click the "Map" button to add a new map to the document (Figure 2.1-2).
   (b) The map should automatically recognize the latitude and longitude columns and begin to plot points for each item.

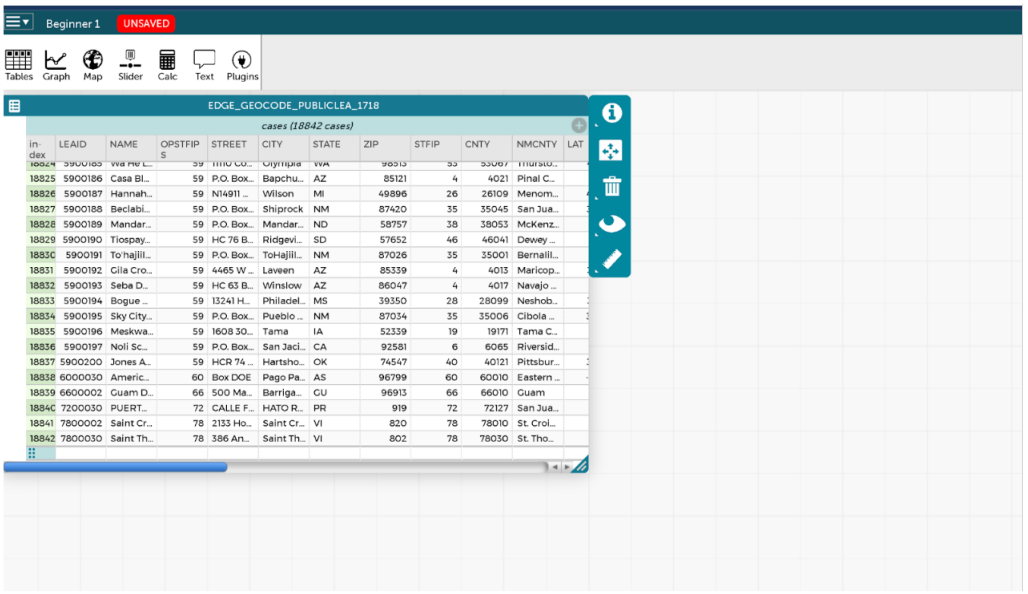


Figure 2.1-1: CODAP's Data Table View

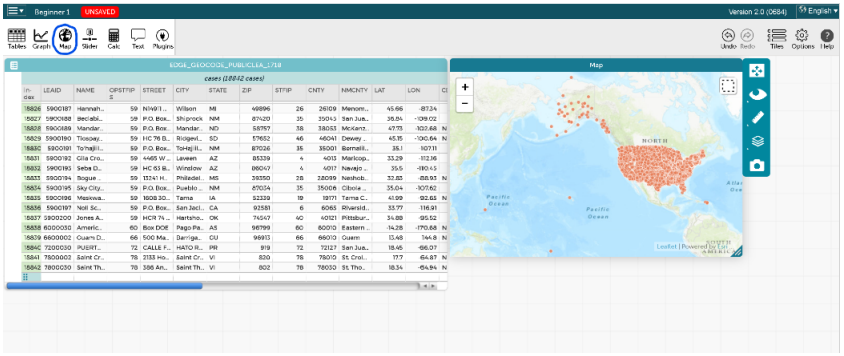


Figure 2.1-2: Adding a Map in CODAP

(c) If the map doesn't automatically recognize your coordinates, manually set the attributes for latitude and longitude by following the instructions below to drag the 'LEAID' feature onto the map.

4. Mapping the Data

(a) Drag the feature you wish to graph (click and drag the feature name like 'State' in Figure 2.1-3) onto the map.

5. Customizing the Map

(a) Using the settings (the panel on the right side of the map), you can adjust the appearance of the points, set colors based on attributes, add labels, and more. This is shown in Figure 2.1-4.

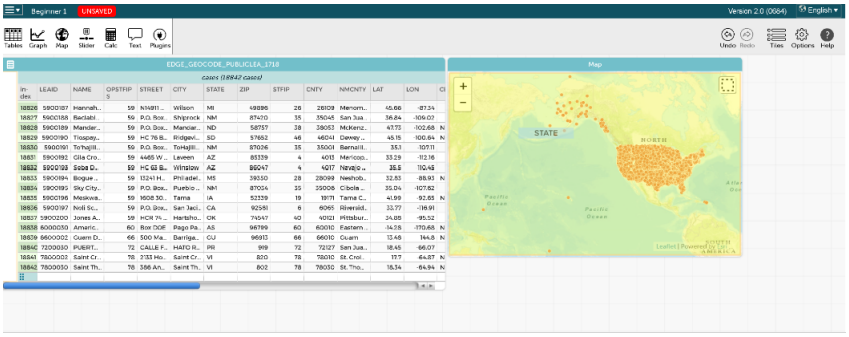


Figure 2.1-3: Graphing a Feature from a Data Table on a Map in CODAP

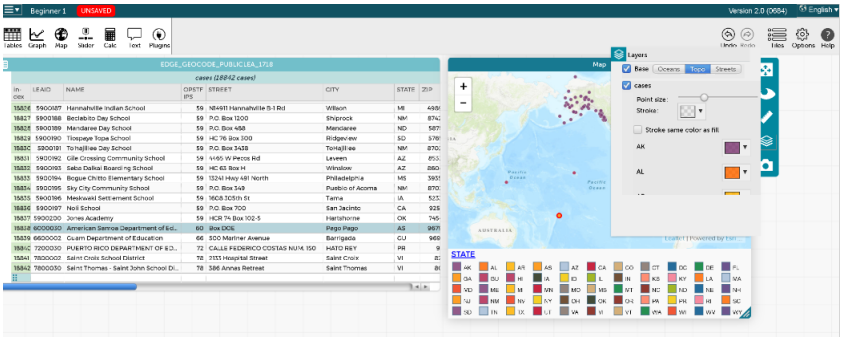


Figure 2.1-4: Customizing a Map in CODAP

(b) If your data includes other attributes (like categories or numerical values), you can often use these to change the appearance of the data points on the map - for example, different colors for different categories.

6. Exploring and Interacting

(a) You can zoom in and out of the map, click on individual data points to see more details, and even filter the data to only show certain items on the map. For example, you can see in Figure 2.1-4, a data point in America Samoa (near the bottom of the map) has been selected and the corresponding row in the data table is highlighted.

(b) If your dataset includes time data, you can also create animations or sliders to see how the items change over time on the map.

CODAP is designed to be interactive, so it's encouraged to play around and explore different visualizations and settings to get the most insight from your data.

2.2 Visualizing County Occurrences using CODAP and Excel

A common task is to visualize quantities in the US by county. For this example, assume that you have an Excel file containing the columns `CNTY` and `STFIP`, which represent the county and state FIPS code.

The Federal Information Processing Standard Publication 6-4 (FIPS 6-4) was a five-digit Federal Information Processing Standards code which uniquely identified counties and county equivalents in the United States, certain U.S. possessions, and certain freely associated states.

> *FIPS codes are numbers which uniquely identify geographic areas. The number of digits in FIPS codes vary depending on the level of geography. State-level FIPS codes have two digits, county-level FIPS codes have five digits of which the first two are the FIPS code of the state to which the county belongs.*

The US Department of Commerce announced in 2008 that FIPS 6-4 was one of ten FIPS standards withdrawn by the department's National Institute of Standards and Technology (NIST). Deemed obsolete, NIST replaced FIPS 6-4 with "INCITS 31 - 2009" codes. Many existing databases use FIPS codes; some newer databases use INCITS. You need to be aware of both.

1. Preparing the Excel File
   (a) Open your Excel file.
   (b) Make sure there are no blank rows or columns.
   (c) Save the file as a `.csv` (comma-separated values) file if it's not already in that format. You can do this by selecting `File > Save As`, choosing the `.csv` format from the dropdown list, and then clicking 'Save'.
2. Importing the Data into CODAP
   (a) Navigate to the CODAP website.
   (b) Click on `File > New Document` to start with a blank document.

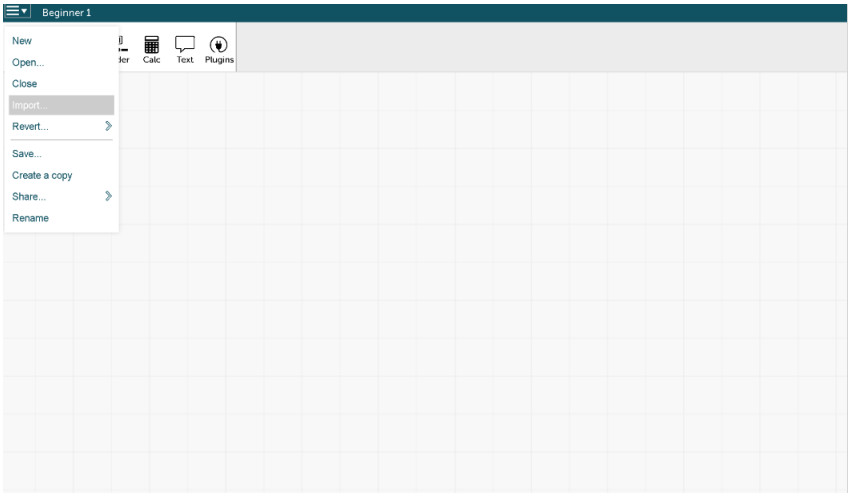


Figure 2.2-5: Importing Data into CODAP

(c) Click on the button with three horizontal lines (usually on the top left corner of the screen).

(d) Choose `Import Data` as shown in Figure 2.2-5.

(e) Choose your `.csv` file and import it.

3. Counting the Number of Rows for Each County

(a) Once your data is loaded, you should see the data table with the columns `CNTY` and `STFIP`.

(b) Click on the top of the `CNTY` column to select it.

(c) Drag the `CNTY` column to the leftmost part of the data table (as shown in Figure 2.2-6). This will group the data by county.

(d) A new attribute (or column) will appear, possibly named `Count`, showing the number of occurrences of each county.

4. Plotting the Data

(a) Click on `Graph` on the toolbar on the upper lefthand side of your screen.

(b) Drag the `CNTY` attribute to the x-axis. The result is shown in Figure 2.2-7

(c) Drag the `CNTY` attribute to map. The counties will now be colored based on the number of occurrences, with the color scale reflecting the data distribution.

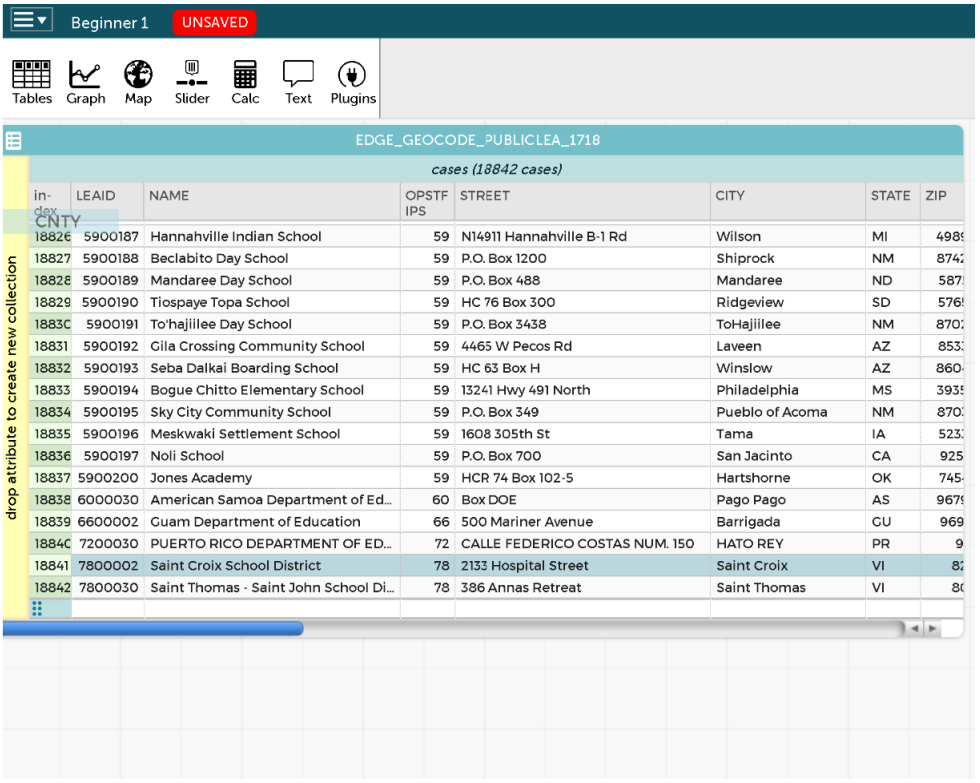


Figure 2.2-6: Create a Collection in CODAP by Dragging the Feature to the Left of the Table

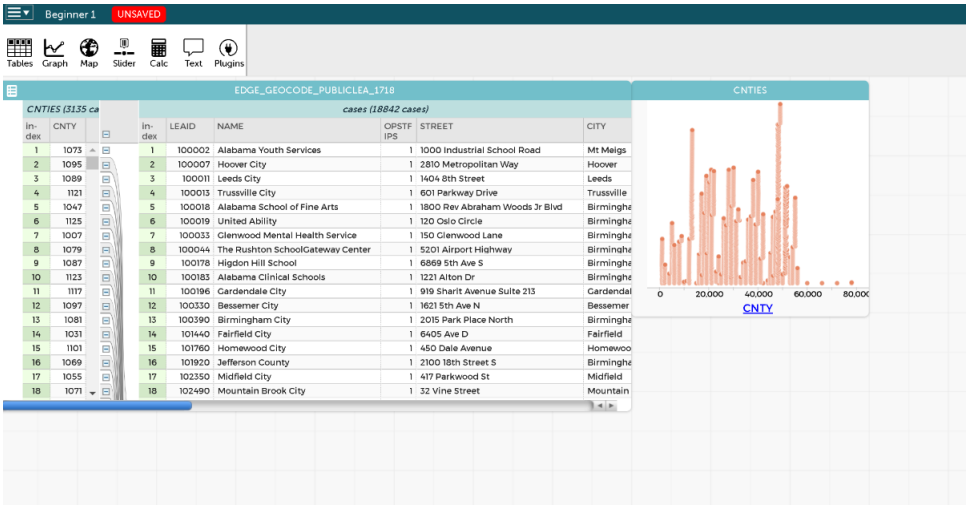


Figure 2.2-7: Create a Graph Showing a Distribution in CODAP by Dragging the Field Containing Counts onto a Graph Object

5. Customizing the Visualization (Optional)

   (a) You can further customize the graph by adjusting axis labels, color scales, etc., using the various options available in CODAP's interface.

   (b) There are also many ways to achieve this count of counties. Another method is to create a new "Count" feature using a function defined in the CODAP Documentation and graph the "Count" feature.

Chapter 3

Introduction to Programming Environments

3.1 Introduction

There is a large number of programming languages. For the purpose of data analytics, we will distinguish two main families of languages: complied vs. scripting languages. They represent two different paradigms of programming language design, each with its own advantages and trade-offs. The key difference lies in when and how the source code you write is translated into machine code that the computer can understand and execute.

In the realm of data analytics, the choice between using a compiled language like C++ and a scripting language like Python often comes down to the specifics of the project at hand, including its complexity, required execution speed, and the need for direct hardware access.

3.1.1 Compiled Languages

Compiled languages are those where the source code is entirely translated into machine code before execution. This translation is done by a compiler, a program that takes the entire source code as an input and generates an executable file. Examples of compiled languages include C++, Java, Rust, Go, and Swift.

Compiled languages offer several important benefits including:

- Performance: Since the code is precompiled into machine code, compiled programs usually run faster and more efficiently than

interpreted ones.

- Type checking: Many compiled languages have strict type systems, which catch many type-related errors at compile time, before the code is run.
- Security: The source code is not distributed with the software, which makes it harder for others to reverse engineer the software.

However, compiled languages also have some drawbacks:

- Development speed: The edit-compile-run cycle can be time consuming, particularly for larg software projects.
- Portability: The generated executable is usually specific to a type of processor and operating system. Availability for different operating systems or processors might require specific source code alterations and independent compilations.

Compiled languages like C++ are translated into machine code before execution, making them very fast because they're executed directly by the computer's hardware. This can be a significant advantage when dealing with huge datasets or when computation speed is a critical factor.

However, the syntax of languages like C++ is more complex and it may require more lines of code to achieve the same result as a scripting language. Moreover, C++ lacks many of the built-in data analysis functionalities found in languages such as Python, MATLAB or R, so it's necessary to use external libraries like Eigen or Armadillo for matrix operations and other complex computations.

3.1.2 Scripting Languages

Scripting languages, on the other hand, are typically interpreted from text into machine instructions, line-by-line of code just in time for execution. Examples of scripting languages include Python, JavaScript, Ruby, and PHP.

Scripting languages offer several advantages:

- Ease of use: Scripting languages are usually easier to learn and use. They have less verbose syntax and more built-in functionality.

- Flexibility: Since they are interpreted line-by-line, it's easier to make changes to your code and see their effect immediately without having to recompile.
- Portability: The same script can be run on any system that has the correct interpreter installed, without needing to change the code.

Among the drawbacks of scripting languages there ar issues related to:

- Performance: Interpreted languages are generally slower than compiled languages because of the overhead of interpreting code line-by-line during execution.
- Dependency on the interpreter: The code can only be run on systems that have the correct interpreter installed.

Scripting languages like Python, on the other hand, are interpreted languages. This means the code is read line-by-line and executed by an interpreter during runtime. While this can make scripting languages slower than compiled languages, they're often easier to write and read because of their high-level syntax.

Python is particularly well-suited for data analytics because it has numerous powerful libraries for data analysis (like Pandas, NumPy, and SciPy) and visualization (like Matplotlib and Seaborn). In addition, Python's simple, straightforward syntax makes it an excellent choice for quick prototyping and exploratory data analysis.

It must be noted that even with all the benefits of Python, the simplified MATLAB syntax for matrix operations facilitates algorithm development since the source code closely resembles mathematical notation.

3.1.3 Comparative Examples

Let us consider a simple example where we want to multiply two matrices A and B. The language examples are listed below in alphabetical oder by the name of the language.

In C++:

```
#include <Eigen/Dense>
#include <iostream>
```

```
3
4  int main() {
5      Eigen::Matrix2f A;
6      A << 1, 2,
7           3, 4;
8      Eigen::Matrix2f B;
9      B << 5, 6,
10          7, 8;
11     std::cout << "The product AB is:\n" << A * B;
12 }
```

../source/IntroProgMatMul.c

In MATLAB:

```
1 A = [1, 2; 3, 4];
2 B = [5, 6; 7, 8];
3 disp('The product AB is:')
4 disp(A * B)
```

../source/IntroProgMatMul.m

In R:

```
1 A <- matrix(c(1, 3, 2, 4), nrow=2, ncol=2, byrow = TRUE)
2 B <- matrix(c(5, 7, 6, 8), nrow=2, ncol=2, byrow = TRUE)
3
4 print("The product AB is:")
5 print(A %*% B)
```

../source/IntroProgMatMul.r

In Python:

```
1 import numpy as np
2
3 A = np.array([[1, 2], [3, 4]])
4 B = np.array([[5, 6], [7, 8]])
5 print("The product AB is:\n", np.matmul(A, B))
```

../source/IntroProgMatMul.py

From the above example, it's clear that MATLAB code is much more concise and easier to understand than C++. However, if we were dealing with very large matrices or needed to perform this operation many times in a loop, the C++ code might run faster due to being a compiled language.

3.1.4 What is Python?

Python is an *interpreted high-level* programming language for general-purpose computing.

- *Interpreted* means that a program in Python has to be decoded line by line by another program called the Python interpreter and translated into something a computer can understand.
- *High-level* means that there is a strong abstraction of the elements of the hardware of the computer; for instance, memory does not need to be pre-allocated in Python to create a matrix, whereas the C language (a low-level language) requires it. You can think of the level of the language as how close the instructions are to the metal of the machine.
- The process of language interpretation is called *compilation*, that is, the translation of one computer language into another language. This could happen multiple times until the instructions are reduced to a language called *assembler* or *assembly*; this code matches closely the architecture of the computer executing the program. For instance, Java programs are compiled into a language called Java bytecode, which the Java Virtual Machine translates into assembler.

3.1.5 Python Installation

Refer now to the software installation guide provided in a separate document.

Being an interpreted language, it means that Python requires an *interpreter*. This is what you install when you "*install Python*". A Python program is simply a text file that contains instructions the interpreter can understand one line (or block of lines) at a time. Since Python is open source, it has many contributors; this results in a robust ecosystem, but the drawback is that there are numerous software libraries that have to be installed while tracking a complex web of co-dependencies. Hence, there is some difficulty in installing a working Python environment to conduct quantitative analysis.

An easy way to use the Python language is through an *Integrated Development Environment*. There are many IDEs that can execute Python code. Being familiar with more than one environment is important as each has strengths and weaknesses. Furthermore, it is a common occurrence that Python programs appear to fail in one environment but work properly in another; this, of course, is a matter of configuration, but it can be a source of frustration in absence of

awareness of the potential sources of conflicts in software. Particularly, you want to know how to use tools frequently used in software development, and you need exposure to the multiple options you will find in academia, government and industry.

3.1.6 Executing Python Commands

Commands in Python are case-sensitive, that is, you must respect the capitalization of instructions as they appear in the documentation. The Python interpreter offers multiple mechanisms to interact with the environment. The most commonly used is the Python command line. You can start it form a command shell. Simply type `python`. The command line promt will change to `>>>`

You can enter Python commands in the command line prompt. Press *Enter* or *return* to execute commands. Like most other programming languages, Python provides mathematical expressions. The building blocks of expressions are variables, numbers, operators, scripts and functions

The basic operations are listed below.

Symbol	Operation
$+$	Addition
$-$	Subtraction
$\star$	Multiplication
$/$	Division
$\star\star$	Power

Some commands are just like you would type things in a calculator. If you want to refer the result of the command, you need to give it a name, or in other words, assign the result to a variable. The following command assigns the result to the variable s. (Note that Python didn't save the true answer 7/12 but rather a decimal approximation.)

```
>>> s = 1/3 + 1/4
```

If you want to know what the variable s contains, simply type its name in the command prompt:

```
>>> s
0.5833333333333333
```

If a command does not fit on one line, use a backslash (\) followed by Return/Enter to indicate that the statement continues on the

next line. For example,

```
>>> s = 1 - 1/2 + 1/3 -1/4 + 1/5 - 1/6 + 1/7 \
... :- 1/8 + 1/9 - 1/10 + 1/11 - 1/12
```

Type s and Enter to see the new value of s.

```
>>> s
0.6532106782106782
```

Use the double star(**) to raise something to an exponent.

```
>>> 89**2
7921
```

Imaginary numbers can be used with the constant j, which stands for $\sqrt{-1}$ by default.

```
>>> (1j)**2
(-1+0j)
```

Hence, you must be careful in using the variable j when dealing with complex numbers. What do you expect from the following operation? Can you explain what you observe? You might need to complete the entire chapter before you can answer this question. Record your explanation as answer #4.

```
>>> j = 10
>>> j = j*(-1)**(0.5)
>>> j
```

Does the result stored in the variable `j` match your expectation? Explain. Record your explanation as answer #5.

Python uses conventional decimal notation, with an optional decimal point and leading plus or minus sign, for numbers. Scientific notation uses the letter e or E to specify a power-of-ten scale factor. Imaginary numbers use i as a suffix. (This course does not use complex numbers, but you might generate errors with one.)

Some examples of legal numbers are:

1	-1	0.0001
9.87654321	1.2345e10	1.2345*10**10
1j	-2j	3e5j

When assigning names to variables or expressions, it is good practice to make the names useful. For example, in the second command above, we gave the result the name rhoSq to imply "rho squared".

Observe that you **CAN NOT** have spaces in variable names (i.e. no spaces in names on the left hand side of the equals sign). Giving useful names to your variables or output will help you keep track of what you are doing.

You can use the up and down arrow keys to easily recall and then edit a command with the left and right arrows. Try it. This is the fastest way to repeat Python commands you might have used in the past.

3.1.7 The VSC Python Editor: Executing A Simple Program

Download the file `lab1SimpleInstruction.py`. Open the file in VSC. The following instructions will appear in the editor:

```
k=0;
for i in range(1,11):
    k=k+1/10;
    print(k)
print(k==1)
```

You can click the *Run file* button on the toolbar (as shwon in Figure 3.1-1), or press F5. We call all these actions *to execute a program*. The program `lab1SimpleInstruction.py` adds ten times the number 0.1. The comparison of the number 1 against the sum returns a value of *false*, whereas elementary arithmetic tells us that adding the number 0.1 ten times should result in 1, and the returned value should be *true*. Why is this result false? Record your explanation as answer #6. Once the program is executed, the bottom portion of VSC shows a panel labeled TERMINAL, as well as other tabs labeled PROBLEMS, OUTPUT, and DEBUG CONSOLE, as shown in Figure 3.1-2

Note that you can enter commands in the terminal. For example, type `1+1` and hit enter. Try mathematical operations such as square root, or trigonometric functions. What challenges do you encouter? Record your explanation as answer #7.

In the program `lab1SimpleInstruction.py` all lines of code were executed sequentially, one at a time. This is what we call a **script file**. That is, when you execute a script file, you expect soemthing tangible to happen, whether it is a message on the screen, a file altereed, communication taking place, etc.

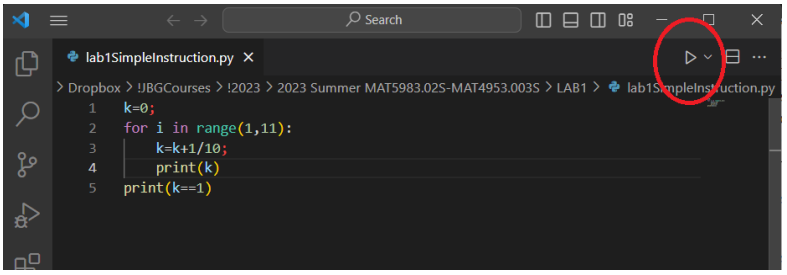


Figure 3.1-1: To execute a Python file in VSC, simply click on the Run button, represented by a triangle pointing to the right. It has been highlighted with a circle.

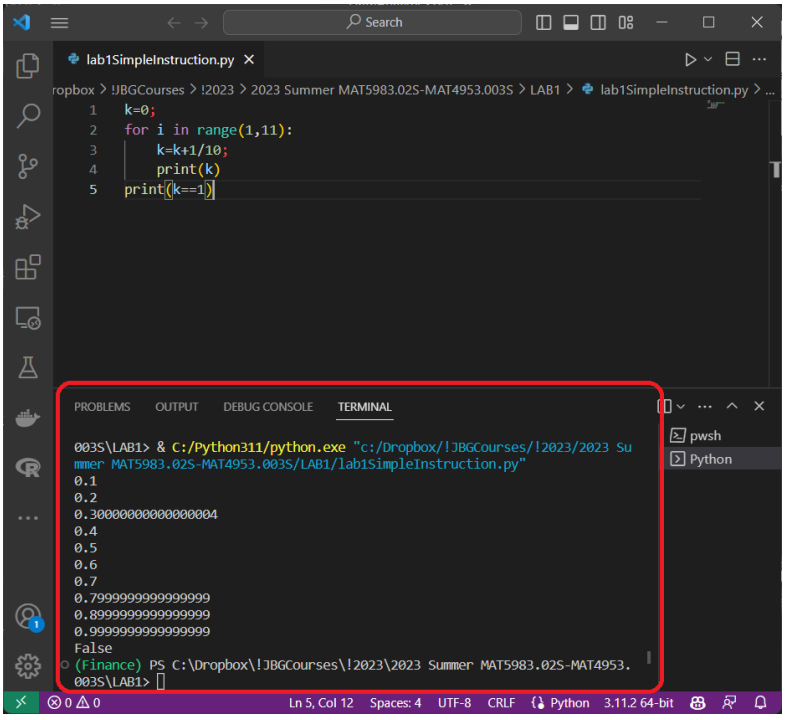


Figure 3.1-2: Executing a python program activates the command line. Results are printed in that panel. The command line is highlighted by the round box.

There is a way to use source code files to hold code that will be executed later, without necessarily executing any instruction. At this pointm, open the file `lab1Function.py`. You will see the following instructions:

```
def sum1(n):
    # sum1(n) computes the sum of 1 + 2 + ... + n
    total = 0;
    for i in range(1,n):
        total = total + i
    return total
```

This file defines a function named `sum1(n)`. The command `sum(3)` returns 6 since $1 + 2 + 3 = 6$. It is instructive to see how this code

does its work.

- The instruction `def` indicates the beginning of a *function*. Note that all instructions that are executed inside that function have a vertical alignment shifted to the right to indicate that all these commands are subordinate to `def`. This shift in vertical alignment is called indentation. It is part of the syntax of the Python programming language, therefore you must always treat indentations strictly.

- The instruction `# sum1(n)...` is a *comment*. Lines of code that start with the symbol `#` are not executed. The semicolon at the end is optional.

- The instruction `total = 0;` is an *assignment*. This line of code creates the variable `total` and assigns to it the value zero.

- The instruction `for i in range(1,n):` starts a *loop*. The subordinate instructions, identified by indentation, will be repeated n times. Particularly, the instruction `range(1,n)` creates a sequence of integers starting in 1 and ending in n. What is the outcome of `range(10,13)` ?

- The instruction `total = total + i` is an assignment. It is very important to emphasize that this is not an equation; if it were, `total` would cancel and `i` would be zero. What takes place is that for every repetition of the loop, the variable `total` is redefined to contain its previous value plus the value of `i`.

- The instruction `return total` is the output of the function. That is, if somehow we could execute this function like `x = sum1(3)`, the variable `x` would contain the value 6, because `sum1(3)` $= 1 + 2 + 3 = 6$.

Now, we will modify this file. Append at the end a program called `sum2` in which we sum integers squared. The next question has to be: How do we execute these functions?

The command `python [filename]` can be used to execute a Python program from the terminal, or any command line prompt. But if we run the following in the command prompt, nothing happens:

```
python lab1Function.py
```

Why? Record your explanation as answer #8.

Now, add the following line of code to the end of the file, **without indentation**:

```
print(sum1(2))
```

Execute the program. What was the result of `sum1(2)`? Is this what you expected? Why? Record your explanation as answer #9. Compute by hand what the output of `sum1(3)` should be.

Now change the last line in the program to read `print(sum1(2))`

What we did was simply to define a function and then use it. Note that if you place the previous line of code at the beginning of the file, the Python interpreter would raise an exception because the function `sum1` is not known to the interpreter yet... keep in mind that the Python interpreter executes one line of code at a time in sequential order (perhaps with branching and repetition, as we will explore later). Therefore, functions must be defined before they can be invoked within a file.

Did the progeam prodice the result you were expecting? In case it did not, enter the following instruction in the terminal: `man range`. This command shows the manual (`man`) for the function `range`. The fact that `range` behaves in certain ways does not force you to abide by that particular design. In fact, you can modify `sum1` so that the outcome corresponds to what a naive user would expect. To achieve this, you will need to change the limit of range inside the function. Implement it and explain. Record your explanation as answer #10.

3.1.8 Executing Functions

In the previous section, we added a single line of code at the end of the program to compute the function we programmed. However, there is a more interesting use case: Create a function that other programs can reuse. In order to use an existing function from a file, we must import that function into the Python environment. We already did something like this when we imported NumPy.

We will now import the function `sum1` from the file `lab1Function.py`. To do this, execute the following command:

```
import lab1function as LAB1
```

This instruction loads into memory a directory of all functions present in that file. Now you can invoke the functions you created. For example,

```
LAB1.sum2(3)
```

It is also possible to load into memory a single function from a file. We can accomplish this by executing

```
from lab1function import sum1
```

In this case, there is no alias associated with the function. To use, you can simply call the function with a parameter, as in `sum2(3)`

Do some research on how to load multiple functions with a single instruction. Explain. Record your explanation as answer #11.

3.2 Introduction to NumPy

Numerical operations can be programmed in Python. A library called `NumPy` encapsulates much needed functionality and has become the *de facto* standard. The basic data element in `NumPy` is a multidimensional array. This allows you to solve many technical computing problems, especially those with matrix and vector formulations, in a fraction of the time it would take to write a program in a low-level language such as C.

For example, at the Python prompt type in the following command and press Enter.

```
import numpy as np
```

This command loads into memory a reference to the NumPy library. Now you can access all functions in this library by using the prefix `np`; this is an *alias*. You can pick any alias you want, but it is customary to use `np` for NumPy. Now enter the following command:

```
np.sqrt(9)
```

The function `sqrt` computes the square root of a real number. Here is an example, and the resulting values.

```
>>> rho = (1+np.sqrt(5))/2
>>> rhoSq = rho**2
>>> a = rhoSq + rho
>>> a
4.23606797749979
```

However, you cannot use the function `sqrt` with negative numbers. What would you have to do to alloy Python to compute the square root of a negative number and return a complex number? Record your explanation as answer #12.

The library NumPy offers easy access to commonly used constants, such as:

Constant	Value
`np.e`	Returns the number e
`np.pi`	Returns the number π
`np.inf`	Returns the number ∞
`np.nan`	NaN, not-a-number

Expressions use the arithmetic operators and precedence rules you already know by now. There are also functions, such as cosine or sine. You surround the input to the function by parenthesis. With NumPy, we can now access commonly used mathematical functions. Among others:

Function	Operation
`np.abs(x)`	Absolute value $\lvert x \rvert$
`np.sqrt(x)`	Square root $\sqrt{x}$
`np.exp(x)`	Exponential function e^x
`np.log(x)`	$\log(x)$ where x is positive real number
`np.sin(x)`	$\sin(x)$ where x is assumed to be in radians
`np.cos(x)`	$\cos(x)$ where x is assumed to be in radians

The library NumPy has *many* functions. A comprehensive list can be found at `https://numpy.org/doc/stable/reference/`. Now enter the following command:

```
x = np.random.standard_normal(3)
```

There is no output. Why do you think this is the behavior? Record your explanation as answer #13. On the right-hand side of the Spyder window there is a tab called *Variable Explorer*. It will show the value of x. Alternatively, you can enter

```
print(x)
```

Alternatively, you can print the result directly without assigning it to a variable. Try

```
print(np.random.standard_normal(3))
```

Why are the results of this last operation different to the values stored in variable x? Record your explanation as answer #14.

3.2.1 Matrix Operations

The basic data element in NumPy is a multi-dimensional array. Special meaning is sometimes attached to 1-by-1 matrices, which are called scalars, and to matrices with only one row or column, which are called vectors. Python has other ways of storing both numeric and non-numeric data, but in the beginning, it is usually best to think of everything as a matrix.

Row vectors are delimited in notation by square brackets. A comma separates individual numbers in a row vector. Multiple row vectors of the same size are separated by commas, and are delimited by square brackets to define a matrix. It is common practice to use a capital letter when naming matrices in order to distinguish them from a single vector, scalar or variable value.

Now, we will create a vector and a matrix to demonstrate simple matrix operations

Function	Operation
`x = np.array([1,2,3,4])`	Creates the row vector $\mathbf{x} = [1, 2, 3, 4]$
`b = np.array([1,2,3])`	Creates the row vector $\mathbf{b} = [1, 2, 3]$
`A = np.array([[1,2,3],[4,5,6],[7,8,9]])`	Creates the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$
`B = np.ones([2,3])`	Creates the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$
`C = np.zeros([2,3])`	Creates the matrix $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

The following commands demonstrate basic vector manipulation cases:

Operation	Outcome
`x[-1]`	Returns 4, the last element of x
`x[1]`	Returns 2, the second element of x
`x[-2]`	Returns 3, the second to last element of x

The colon (:) operator is notable. In vector notation, it indicates a range of rows or columns. In Python, the first element of a vector has index 0; the second element has index 1, and so on. The colon operator $a : b$ starts at the row or column of index a and covers up to an index less than b. Thus, $A[0 : 2, 0 : 2]$ returns a matrix composed of rows 1 through 2, and columns 1 through 2 with respect to the matrix A.

The following commands demonstrate basic matrix manipulation cases:

Operation	Outcome
`A[1,2]`	Value of 2^{st} row and 3^{nd} column of A
`A[0:2,0:2]`	Returns $A = \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}$
`A[1:3,1:3]`	Returns $A = \begin{bmatrix} 5 & 6 \\ 8 & 9 \end{bmatrix}$

The matrix A has three rows and three columns. Then, why does $A[1 : 100, 1 : 100]$ not produce an error? Record your explanation as answer #15.

The size or dimension of a matrix refers to the number of rows and number of columns a matrix has. The `shape` command returns the number of rows and columns of a matrix. Notice that the result is a vector - a one-dimensional matrix with 2 values where the first is the number of rows and the second value is the number of columns. By accessing each element of this vector, we can access each dimension individually.

Using the matrix A of our previous examples,

```
>>> y = A.shape()
>>> y
(3,3)
>>> y[0]
3
>>> y[1]
3
```

The following example will illustrate a subtlety in copying matrices:

```
>>> D = A

array([[1, 2, 3],
       [4, 5, 6],
       [7, 8, 9]])
>>> D[0,0] = 9
>>> A

array([[9, 2, 3],
       [4, 5, 6],
       [7, 8, 9]])
```

Note that by updating a value in D, the corresponding value in A was changed. Why? What would you need to do to prevent an update in D to change A? Record your explanation as answer #16.

The basic matrix operations are listed below, using the matrices in the previous examples.

Operation	Outcome
$A.T$	Transpose of A.
$A.conj().T$	Conjugate transpose of A.
$A@B.T$	Matrix multiplication $A \cdot B$.
$A \star A$	Element-wise multiplication
A/A	Element-wise division
$A \star\star 2$	Element-wise exponentiation

NumPy gives an easy way of solving systems of linear equations, Given the matrix A and the vector $\mathbf{b}$ from the previous examples, the linear system $A\mathbf{x} = \mathbf{b}$ can be solved by invoking `np.linalg.solve` as follows

```
>>> x = np.linalg.solve(A,b)
array([-0.23333333,  0.46666667,  0.1       ])
```

It is easy to use Matrix multiplication $A\mathbf{x}$ to check the answer

```
>>> A@x
array([1., 2., 3.])
```

Chapter 4

Linear Operations with Data

Data can usually be represented by a *feature matrix* in which each row is an observation and each column is a variable. A trivial example is a physician's office; annual exams for every patient include variables such as glucose level, cholesterol levels, etc. In total dozens of variables are defined. In this case, all patients in the physician's office could be represented by a table, i.e., a matrix, in which each patient would be a row, and the value for each clinical variable would be represented by individual columns. In this section we explore the most basic technique for linear manipulation of this feature matrix.

4.1 Principal Component Analysis

Let us try to represent data in a feature matrix with the best possible *projection* of all observations $\mathbf{X} = \{\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_k, \cdots, \mathbf{x}_n\}$, along a unitary vector $\mathbf{e}$ onto a vector $\mathbf{m}$ that occupies the same dimensional space. Here the subtlety is that asking about the optimal projection is not the same as asking about the single data vector (i.e. observation) that represents best the data; the latter would simply be the average of all observations in $\mathbf{X}$.

The projection of an observation $\mathbf{x}_k$ onto a vectoJr $\mathbf{m}$ along a unitary vector $\mathbf{e}$ does not necessarily produce an equality. An amount of error, denoted by ξ, might occur, as shown in in Figure 4.1-1. This can be characterized by $\mathbf{x}_k + \xi = \mathbf{m} + a_k\mathbf{e}$, where a_k is a scalar that stretches the unitary vector $\mathbf{e}$ from $\mathbf{x}_k$ to the direction of the vector $\mathbf{m}$. It is evident that when $\xi = 0$, then

$$a_k = \mathbf{e}^t(\mathbf{x}_k - \mathbf{m}). \tag{4.1}$$

Our goal is to minimize the error ξ for all elements of $\mathbf{X}$. We can find an optimal set of coefficients a_k by minimizing the error criterion function given by

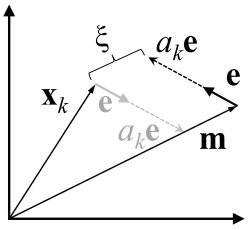


Figure 4.1-1: Projection of an observation $\mathbf{x}_k$ along an arbitrary unitary vector $\mathbf{e}$ onto a vector $\mathbf{m}$.

$$\begin{aligned} J_0(a_1, a_2, \cdots, a_n, \mathbf{e}) &= \sum_{k=1}^{n} \|(\mathbf{m} + a_k\mathbf{e}) - \mathbf{x}_k\|^2 \\ &= \sum_{k=1}^{n} \|a_k\mathbf{e} - (\mathbf{x}_k - \mathbf{m})\|^2 \\ &= \sum_{k=1}^{n} a_k^2 \|\mathbf{e}\|^2 - 2\sum_{k=1}^{n} a_k\mathbf{e}^t(\mathbf{x}_k - \mathbf{m}) + \sum_{k=1}^{n} \|\mathbf{x}_k - \mathbf{m}\|^2 . \end{aligned}$$

We will seek the direction of $\mathbf{e}$ that results in no error. Thus, replacing a_k given by Equation 4.1, and since $\|\mathbf{e}\|^2 = 1$, then J is transformed into a function that no longer depends on a_k

$$\begin{aligned} J_0(\mathbf{e}) &= \sum_{k=1}^{n} \left(\mathbf{e}^t(\mathbf{x}_k - \mathbf{m})\right)^2 - 2\sum_{k=1}^{n} \left(\mathbf{e}^t(\mathbf{x}_k - \mathbf{m})\right)^2 + \sum_{k=1}^{n} \|\mathbf{x}_k - \mathbf{m}\|^2 \\ &= -\sum_{k=1}^{n} \left(\mathbf{e}^t(\mathbf{x}_k - \mathbf{m})\right)^2 + \sum_{k=1}^{n} \|\mathbf{x}_k - \mathbf{m}\|^2 . \\ &= -\sum_{k=1}^{n} \mathbf{e}^t(\mathbf{x}_k - \mathbf{m})(\mathbf{x}_k - \mathbf{m})^t\mathbf{e} + \sum_{k=1}^{n} \|\mathbf{x}_k - \mathbf{m}\|^2 . \end{aligned}$$

Recall that the scatter matrix, defined as $n-1$ times the covariance matrix, is given by $\mathbf{S} = \sum_{k=1}^{n}(\mathbf{x}_k - \mathbf{m})(\mathbf{x}_k - \mathbf{m})^t$. Thus, the objective function J is the constrained optimization problem

$$J_0(\mathbf{e}) = \mathbf{e}^t\mathbf{S}\mathbf{e} + \sum_{k=1}^{n} \|\mathbf{x}_k - \mathbf{m}\|^2$$

subject to the constraint $\|\mathbf{e}\|^2 = 1$. Since the term $\sum_{k=1}^{n} \|\mathbf{x}_k - \mathbf{m}\|^2$ is always positive, the optimal $\mathbf{e}$ is found by minimizing

$$J_0(\mathbf{e}) = \mathbf{e}^t\mathbf{S}\mathbf{e}, \text{ subject to } \|\mathbf{e}\|^2 = 1. \tag{4.2}$$

Equation 4.3 is a constrained optimization problem, which can be converted into an unconstrained optimization problem by adding to the original problem the constraint equated to zero, multiplied by a Langrange multiplier. In this case, the constraint $\|\mathbf{e}\|^2 = 1$ can be expressed as $\|\mathbf{e}\|^2 - 1 = \mathbf{e}^t\mathbf{e} - 1 = 0$. Thus, we transform the problem of minimizing $J_0(\mathbf{e})$ into the problem of minimizing the following unconstrained problem:

$$L_0(\mathbf{e}, \lambda) = \mathbf{e}^t\mathbf{S}\mathbf{e} - \lambda(\mathbf{e}^t\mathbf{e} - 1). \tag{4.3}$$

Now that we have an unconstrainted optimization problem, we can find the extrema by simply deriving and equating to zero, as follows:

$$\frac{\partial L_0(\mathbf{e}, \lambda)}{\partial \mathbf{e}} = 2\mathbf{S}\mathbf{e} - 2\lambda\mathbf{e} = 0 \tag{4.4}$$

From Equation 4.4 we see that $\mathbf{S}\mathbf{e} = \lambda\mathbf{e}$, thus $\mathbf{e}$ must be an eigenvector, and the minimization problem is simply an eigenvalue problem.

4.2 Numerical Implementation of Principal Component Analysis

Generate Data

The first step is to generate as an examnple two distributions of size 5×50.

```
import numpy as np

np.random.seed(42)
X1 = np.random.rand(5, 50)
X2 = X1 + 1
```

Standardize the Data

For PCA to work efficiently, the data must be standardized.

```
def standardize_data(X):
    mean = np.mean(X, axis=1, keepdims=True)
    std_dev = np.std(X, axis=1, keepdims=True)
    return (X - mean) / std_dev

X1_standardized = standardize_data(X1)
X2_standardized = standardize_data(X2)
```

Compute the Covariance Matrix

Compute the covariance matrix from the standardized data.

```
def compute_covariance_matrix(X):
    return np.dot(X, X.T) / X.shape[1]

cov_matrix1 = compute_covariance_matrix(X1_standardized)
cov_matrix2 = compute_covariance_matrix(X2_standardized)
```

Compute Eigenvalues and Eigenvectors

Obtain the eigenvalues and eigenvectors from the covariance matrix.

```
eigenvalues1, eigenvectors1 = np.linalg.eig(cov_matrix1)
eigenvalues2, eigenvectors2 = np.linalg.eig(cov_matrix2)
```

Sort Eigenvalues and Choose Top Eigenvectors

Sort the eigenvalues and choose the top eigenvectors for dimensionality reduction.

```
sorted_indices1 = np.argsort(eigenvalues1)[::-1]
top_2_eigenvectors1 = eigenvectors1[:,
   sorted_indices1[:2]]

sorted_indices2 = np.argsort(eigenvalues2)[::-1]
top_2_eigenvectors2 = eigenvectors2[:,
   sorted_indices2[:2]]
```

Transform the Original Data

Transform the data into the new subspace defined by the top eigenvectors.

```
Y1 = np.dot(top_2_eigenvectors1.T, X1_standardized)
Y2 = np.dot(top_2_eigenvectors2.T, X2_standardized)
```

Visualization

Finally, visualize the transformed data using plots.

```
import matplotlib.pyplot as plt

plt.scatter(Y1[0, :], Y1[1, :], label="Class 1")
plt.scatter(Y2[0, :], Y2[1, :], label="Class 2")
plt.legend()
plt.xlabel('Principal Component 1')
plt.ylabel('Principal Component 2')
plt.title('2D PCA')
plt.show()
```

To visualize in 3D, take the top 3 eigenvectors and use the 3D plotting tools in Matplotlib.

Chapter 5

Configuration Management

5.1 Introduction to Configuration Management

In data analysis, rigorous management of system configurations, which include both hardware and software components, is essential. This management is termed 'configuration management' (CM). CM ensures the optimal operation of systems and applications by organizing and documenting all software, hardware, settings, and associated records.

Data analysis tools and methodologies depend upon an integration of various software libraries, environment variables, system configurations, and data sources. A change in any of these elements can influence the results of a data analysis project. Therefore, maintaining records of system states and managing modifications is a *sine qua non* condition for the principles of reproducibility and scalability in data analysis projects.

Configuration management integrates multiple facets: (i) Source Control pertains to the versions and conditions of the source code used in data analysis projects, (ii) Software and Libraries concern the versions and configurations of software utilities, libraries, and programs employed in the data analysis projects, (iii) Environment Variables and System Configurations include the hardware framework, operating systems, runtime environments, and the established parameters within these arenas, (iv) Data Sources and Versions address the datasets employed for the training and evaluation of data analysis models, (v) Algorithm and Model Parameters cover the parameters and configurations within the machine learning algorithms

or statistical techniques implemented, and (vi) Documentation of the aforementioned elements is imperative to ensure the reproducibility, transferability, and scalability of the project.

As data analysis projects progress, CM protocols facilitate structured development and oversight of modifications across these elements. This ensures clarity among team members and allows replication of work and revisitation of past projects. Configuration management acts as a comprehensive version control throughout the data analysis project continuum.

Subsequent sections will explore the significance of configuration management, its methodologies, challenges, and solutions within data analysis.

5.2 Source Control

Git, a widely used version control system, facilitates collaboration and code management in software development. Two prevalent strategies for incorporating changes in Git repositories are:

(i) **Direct Merge into the Main Version:** This approach involves making changes directly to the main branch (typically called ‘master‘ or ‘main‘) and then pushing those changes.

```
# Clone the repository
git clone <repository-url>

# Navigate to the repository directory
cd <repository-name>

# Make changes in the code

# Stage the changes
git add .

# Commit the changes
git commit -m "Commit message describing the changes"

# Push the changes directly to the main branch
git push origin main
```

(ii) **By Creating Branches:** Rather than making changes directly to the main branch, developers create a separate branch, make changes within that branch, and subsequently merge the branch back into the main branch.

```
# Clone the repository
git clone <repository-url>

# Navigate to the repository directory
cd <repository-name>

# Create a new branch
git checkout -b <branch-name>

# Make changes in the code

# Stage the changes
git add .

# Commit the changes
git commit -m "Commit message describing the changes"

# Push the branch to the remote repository
git push origin <branch-name>

# To merge the branch into the main branch
# First, switch to the main branch
git checkout main

# Merge the feature branch into the main branch
git merge <branch-name>

# Push the merged changes to the remote main branch
git push origin main
```

The decision between these methods often rests on the specific workflow requirements, team preferences, and the nature of the changes being introduced. Utilizing branches can be advantageous in larger projects or teams as it compartmentalizes changes, thereby minimizing conflicts and ensuring a stable main branch.

5.3 Software and Libraries

Software and libraries form the basis of most data analysis projects. They are tools that provide pre-written code to perform certain functions, thereby saving time and resources in the process of development. Understanding the role of software and libraries in the context of configuration management involves two main aspects: versions and configurations.

Versions: As developers improve and update software and libraries, they often release new versions. These new versions may include feature enhancements, bug fixes, security improvements, or even modifications in how certain functions operate. It is important to keep track of which version of a software or library used in a data analysis project because different versions may produce different results.

Configurations: Apart from versions, the configurations of software and libraries can significantly affect the results of a data analysis project. Configurations may involve settings that determine how a software or library behaves, or parameters that influence the operation of certain functions. Ensuring consistency in configurations is essential for the replicability and reliability of a project's outcomes.

Careful management of software and library versions and configurations ensures that data analysis work is reliable, replicable, and less prone to unexpected bugs or inconsistencies.

5.4 Environment variables and system configurations

Environment variables and system configurations are a wide range of settings and parameters that dictate how a system behaves when running a data analysis project. They are an essential part of configuration management because they impact the performance, reproducibility, and even the results of data analysis tasks.

- **Environment Variables:** These are dynamic variables that can affect the way running processes behave on a computer. They are part of the environment in which a process runs and can include anything from paths to certain directories, locale settings that determine language and format, or custom variables that specific data analysis projects might need. For instance, a project may need to connect to a database, and the connection string could be stored in an environment variable for security and ease of management.

- **System Configurations:** These refer to the specifics of the hardware environment and operating system (OS) in which a project runs. They can include the type of processor, the amount

of RAM, the operating system version, and more. The system configurations can impact the execution of a project. For example, an algorithm may run smoothly on a system with a large amount of RAM but crash on a system with less memory.

- **Runtime Environments:** These refer to the environment in which a computer program runs. This includes the type of interpreter for scripting languages, the versions of these interpreters, and other settings. Changes in the runtime environment can lead to different execution behavior for the same piece of code.
- **Parameters and Configurations within these environments:** These are specific settings that influence the behavior of software and libraries within the environments, like the number of threads allocated for an operation, memory limits for certain processes, or specific flags set in the operating environment.

In data analysis, being able to reproduce an analysis is absolutely essential, and this extends to being able to recreate the environment in which the analysis was originally run. Minor differences in these variables and configurations can sometimes lead to major differences in outcomes, making the project non-reproducible.

Effective configuration management ensures that these environment variables and system configurations are carefully controlled and documented. This means that anyone trying to replicate a project, or when moving a project to a different system, will have a record of the environment in which the code was originally run, reducing the chance of unexpected issues or results.

5.5 Data Sources and Versions in Data Analysis

This refers to the specific datasets used in various stages of the data analysis project, such as training, testing, and validation of models. This involves maintaining an understanding of not only where the data is sourced from, but also the specific versions or states of these datasets used at different points in time.

- **Data Sources:** These are the origins of your datasets. They can be databases, APIs, CSV files, Excel files, data scraped from

websites, or any other place where data can be stored. Keeping track of where your data is sourced from is necessary for reproducibility and transparency.

- **Data Versions:** This refers to the specific state of your dataset at a certain point in time. Datasets, especially in a live environment, can change over time as new data is added, erroneous entries are corrected, or outdated entries are removed. Each unique state of a dataset is a different version. Tracking these versions is critical because changes in data can lead to different model training results, and thus, different model performance. This is especially crucial in situations where a model is trained on a particular version of a dataset and is expected to perform consistently when the data is updated or changes.

Managing data sources and versions effectively ensures that the project is robust, reproducible, and reliable. For this purpose, tools like version control systems (e.g., Git), dataset versioning tools (e.g., DVC), or data cataloging tools can be utilized to keep track of the various versions of datasets used throughout a data analysis project.

5.6 Algorithm and Model Parameters in Data Analysis

Thsi area refers to the specific parameters and configurations used within the machine learning models or statistical methods applied in a project.

- **Algorithm Parameters:** These are the settings or options chosen when running an algorithm that can affect its behavior or outcome. This coudl be the step size for an integrator, the number of clusters in a clustering algorithm, the seed of a randomizer, or the learning rate in a neural network. Different configurations of these parameters can lead to different models, each with potentially different performance characteristics.
- **Model Parameters:** These are the parameters that the model learns from the training data. In a linear regression model, for example, the coefficients of the regression equation are model parameters that the algorithm estimates from the data. In a

neural network, the weights and biases are model parameters that get updated during the learning process.

- **Pipeline Configurations:** These refer to the settings and sequences of a data analysis pipeline. A pipeline consists of several steps such as data cleaning, feature extraction, model training, and model evaluation. Each step could have its own specific configurations. For instance, a pipeline configuration could define how missing data is handled during the cleaning process, how features are scaled or transformed, or how model performance is evaluated.

Effective configuration management of algorithm parameters, model parameters, and pipeline configurations is essential and necessary in data analysis. It ensures that experiments are repeatable and that different models and pipelines can be compared fairly. It can also facilitate collaboration, as others can replicate and build upon your work.

Chapter 6

Visualization for Data Analytics

Under Development...

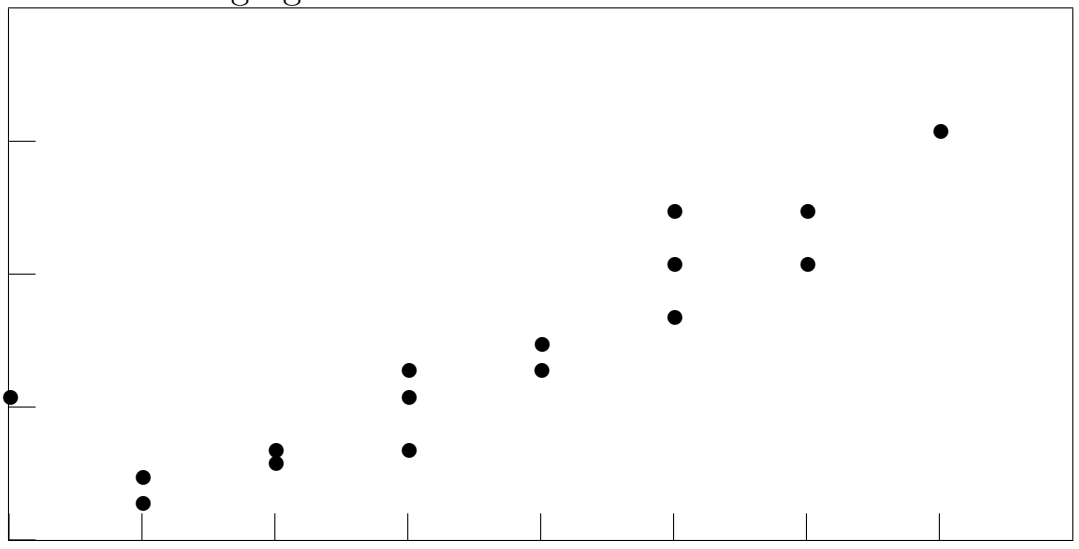
Chapter 7

Processing Time Series

7.1 Time Series

A quantity evolving in time is one of the most common ways to represent data available in public data sets. Let $i, m, n \in \mathbb{N}$, a *time series* consisting of n time points and m variables is a totally ordered set $T := \{(t_i, \mathbf{x}_i)\}$, such that $\mathbf{x}_i \in \mathbb{C}^m$ for $i \in [0, n]$.

Suppose that we obtain a data set from an experiment as shown in the following figure:



It is clear that there is no function which produces the given data.

Suppose we want to approximate a function $f(t)$ with a sum of a given set of functions multiplied by a constant, that is,

$$f(t) = c_1 u_1(t) + c_2 u_2(t) + \cdots + c_k u_k(t) + \cdots + c_n u_n(t) = \sum_{k=1}^{n} c_k u_k(t).$$

We call $u_k(t)$ a **basis function**. It is possible to determine optimal constants c_k by minimizing the error function:

$$J(c_1, \cdots, c_n) = \min_{c_1,\cdots,c_n} \int_\Omega \left(\sum_{k=1}^{n} c_k u_k(t) - f(t) \right)^2 dt.$$

We say that $J(c_1, \cdots, c_n)$ is optimal in a *least square error sense.*

To find this minimum, derive with respect to each c_k, and set the equation equal to zero:

$$\begin{aligned}
\frac{\partial J}{\partial c_k} =& \frac{\partial}{\partial c_k} \left(\int_\Omega \left(\sum_{k=1}^{n} c_k u_k(t) - f(t) \right)^2 dt \right) \\
=& \int_\Omega \left(\frac{\partial}{\partial c_k} \left(\sum_{k=1}^{n} c_k u_k(t) - f(t) \right)^2 dt \right) \\
=& \int_\Omega \left(2u_k(t) \left(\sum_{k=1}^{n} c_k u_k(t) - f(t) \right) dt \right) \\
=& 2 \int_\Omega c_1 u_1(t) u_k(t) dt + 2 \int_\Omega c_2 u_2(t) u_k(t) dt + \ldots 2 \int_\Omega c_k u_k^2(t) dt \ldots \\
& + 2 \int_\Omega c_n u_k(t) u_n(t) dt - 2 \int_\Omega u_k(t) f(t) dt. \\
=& 0
\end{aligned}$$

Let $(u_j, u_k) = \int_\Omega u_j(t) u_k(t) dt$. Rearranging the previous terms,

$$c_1 (u_1(t), u_1(t)) + \cdots c_k (u_1(t), u_k(t)) + \cdots + c_n (u_1(t), u_n(t)) = (u_k(t), f(t)) .$$

Thus, in matrix form,

$$\begin{bmatrix} (u_1, u_1) & \cdots & (u_1, u_n) \\ \vdots & \ddots & \vdots \\ (u_n, u_1) & \cdots & (u_n, u_n) \end{bmatrix} \begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix} = \begin{bmatrix} (u_1, f) \\ \vdots \\ (u_n, f) \end{bmatrix}.$$

The basis functions $u_k(t)$ can be arbitrary. If we select monomials, then $f(t) = c_0 + c_1 t + c_2 t^2 + \cdots + c_k t^k + \cdots + c_n t^n$. This results in traditional linear regression if the basis set is $\{c_0, c_1 t\}$, traditional quadratic regression if the basis set is $\{c_0, c_1 t, c_2 t^2\}$, Fourier series with basis $\{\cos(k\pi t/l)\}$, discrete Fourier transform follows with little

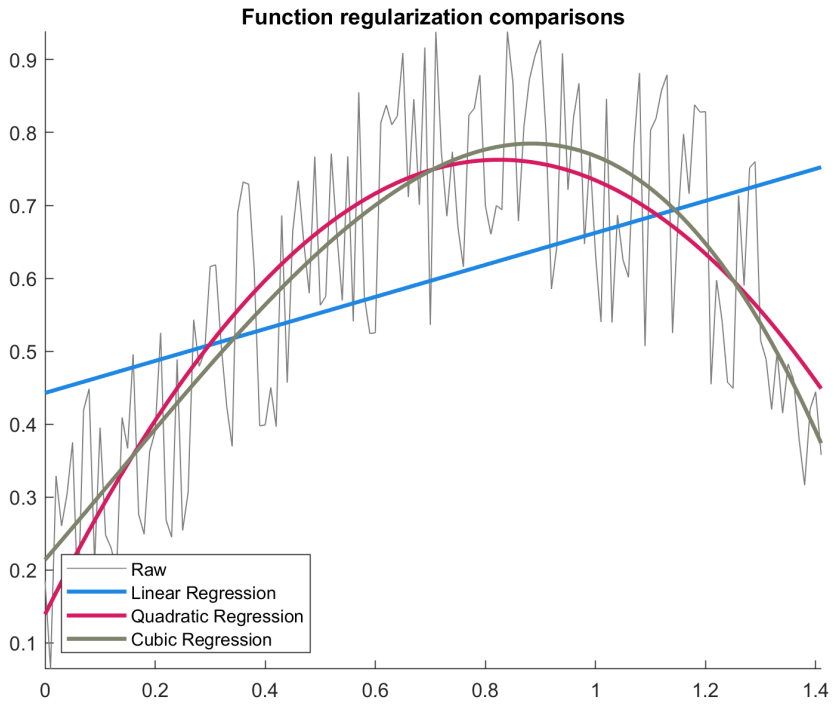


Figure 7.1-1: Multiple regression cases using monomials as basis functions

effort and so on. Figure 7.1-1 depicts multiple regression cases using monomials as basis functions.

We can select functions such that $(u_i, u_j) = 0$, $\forall i \neq j$. Such functions are called orthogonal functions, e.g. $\sin(t)$, $\cos(t)$, etc. Thus,

$$c_i = \frac{(f, u_i)}{(u_i, u_i)}. \tag{7.1}$$

The consequences of this method are far reaching. When $\sin(t)$ and $\cos(t)$ are used, this method produces the Fourier series for a finite interval of length l. If we study this problem in the limit when $l \to \infty$, the Fourier transform is produced.

Example 1. Given $f(t) = t^2$, $t \in [0, 1]$, find c such that the function $g(t) = ct$ approximates $f(t)$ in a *least square error sense*.

$$c = \frac{(t^2,t)}{(t,t)} = \frac{\int_0^1 t^3 dt}{\int_0^1 t^2 dt} = \frac{3}{4}$$

Thus, $\frac{3}{4}t$ approximates t^2 optimally in a *least square error sense* in the interval $[0, 1]$.

These examples lead us to find functions approximately fitting the given data. Suppose that $\{(x_i, b_i) : i = 1, \cdots, n\}$ are given observation data from a physical model. Let $f(x) = a_1\phi_1(x) + \cdots + a_m\phi_m(x)$ be a function to represent the physical model. We are looking for coefficients $a_i, i = 1, \cdots, m$ such that

$$\min_{a_1,\cdots,a_m} \max_{1\le i\le n} \{|f(x_i) - b_i|\}, \tag{1}$$

or

$$\min_{a_1,\cdots,a_m} \sum_{i=1}^{n} |f(x_i) - b_i|, \tag{2}$$

or

$$\min_{a_1,\cdots,a_m} \sum_{i=1}^{n} (f(x_i) - b_i)^2. \tag{3}$$

In this lab, we mainly study the problem of finding $a_1, \cdots, a_m$ satisfying (3) which is called *Least Square Problem*. The other cases lead to different approximate schemes and will not be covered in this chapter.

Let us first give a remark on choosing the model function f. Usually, people use their experiences and the pattern of observation data to choose $\phi_i, i = 1, \cdots, m$. For example, if the pattern of the observation data behaviors like a periodic function, we choose $\sin kx$ and/or $\cos kx$ as ϕ_i. We may choose exponential functions $e^{c_i x}$ as ϕ_i if the asymptotic behavior of the physical model has exponential decay or growth.

7.2 Pseudoinverse

Time series, many times, as presented as a collection of values at specific time intervals with known associated outcomes. This problem is equivalent to the following matrix problem: find $\mathbf{x}^* \in \mathbb{R}^m$ such that

$$J(\mathbf{x}^*) = \|A\mathbf{x}^* - \mathbf{b}\|_2^2 \le \min_{\mathbf{x}\in\mathbb{R}^m} \|A\mathbf{x} - \mathbf{b}\|_2^2$$

since we may write $\mathbf{x} = (a_1, \cdots, a_m)^t$, A is a matrix of size $n \times m$ with row entries $\phi_i(x_j), 1 \le i \le m, 1 \le j \le n$ and $\mathbf{b} = (b_1, \cdots, b_n)^t$.

Now, for ease of notation, let's drop the $*$. The objective function $J(\mathbf{x})$ can be expressed as:

$$J(\mathbf{x}) = (A\mathbf{x} - b)^T(A\mathbf{x} - b) = \mathbf{x}^T A^T A\mathbf{x} - 2\mathbf{x}^T A^T \mathbf{b} + \mathbf{b}^T \mathbf{b}.$$

Now minimize $J(\mathbf{x})$:

$$\nabla J(\mathbf{x}) = 2A^T A\mathbf{x} - 2A^T \mathbf{b} = 0$$

Thus, we conclude that $\mathbf{x}$ satisfies the following

$$A^T A\mathbf{x} - A^T \mathbf{b} = 0.$$

which is called the normal equation.

It follows that

$$\begin{aligned} A^T A\mathbf{x} &= A^T \mathbf{b} \\ \mathbf{x} &= (A^T A)^{-1} A^T \mathbf{b} \\ \mathbf{x} &= A^\dagger \mathbf{b} \end{aligned}$$

where $A^\dagger = (A^T A)^{-1} A^T$ is known as the pseudo-inverse of A.

7.3 Fourier Transform

In our first brush with the integral approach in the previous section, we suggested that it should be possible to express a function as an infinite linear combination of the sinusoids $\{\sin kt, \cos kt\}$. Note that Euler's formula allows us to express each exponential function e^{ikt} in terms of $\sin kt$ and $\cos kt$ and conversely we can express both $\sin kt$ and $\cos kt$ in terms of the exponentials $\mathrm{e}^{ikt}, \mathrm{e}^{-ikt}$. Thus it should be possible to express each reasonable 2π-periodic function $f : \mathbb{R} \to \mathbb{C}$ as an infinite series

$$f(t) = \sum_{k=-\infty}^{\infty} a_k \mathrm{e}^{ikt}. \tag{7.2}$$

We have also introduced an inner product which makes the exponential functions appearing in Equation 7.2 mutually orthogonal, so it should be easy to recover the coefficients $\{a_k\}$ from f by taking inner products. Still there are delicate questions remaining, including the meaning of "reasonable" and how we even observe the infinitely

many values of f. The idea of the *discrete Fourier transform* is to make this whole process finite by restricting attention to the values of f at n equally spaced points in the interval $[0, 2\pi)$. This idea is known as *sampling*.

Example 1. Let's begin with the trivial case $n = 1$, i.e., we only want Equation 7.2 to hold when t is a multiple of 2π. Since all the functions f, e^{ikt} are 2π-periodic, Equation 7.2 reduces to $f(0) = \sum_{k=-\infty}^{\infty} a_k 1$. There are infinitely many unknowns here and we simplify the situation by taking $a_k = 0$ for $k \neq 0$. This leaves $a_0 = f(0)$. and we get the solution $a_k = \begin{cases} f(0), & k = 0 \\ 0, & \text{otherwise} \end{cases}$.

Let's next consider the case $n = 2$, which means we want Equation 7.2 to hold when $t = 0$ and when $t = \pi$. Since $\mathrm{e}^{ik\pi} = (-1)^k$, this translates to the system

$$f(0) = \sum_{k=-\infty}^{\infty} a_k (1)^k \tag{7.3}$$
$$f(\pi) = \sum_{k=-\infty}^{\infty} a_k (-1)^k \tag{7.4}$$

Since there are only two equations here, we again simplify the situation by taking all a_k except a_0, a_1 to be zero. This leaves the system

$$f(0) = a_0 + a_1 \qquad f(\pi) = a_0 - a_1 \tag{7.5}$$

which easily yields the solution $a_k = \begin{cases} \frac{f(0)+f(\pi)}{2}, & k = 0 \\ \frac{f(0)-f(\pi)}{2}, & k = 1 \\ 0, & \text{otherwise} \end{cases}$. which can be expressed matricially as

$$\begin{bmatrix} f(0) \\ f(\pi) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}. \tag{7.6}$$

The coefficient matrix here, denoted F_2, is referred to as a *discrete Fourier transform matrix*.

Example 2. Suppose $n = 4$, so we only care about the values of f at multiples of $\frac{\pi}{2}$. We can store this information as a vector in $\mathbb{C}^4$, namely $v_f := \begin{bmatrix} f(0) \\ f(\frac{\pi}{2}) \\ f(\pi) \\ f(\frac{3\pi}{2}) \end{bmatrix}$. In particular, our basic function e^{ikt} is stored

as $v_k := \begin{bmatrix} \mathrm{e}^{0} \\ \mathrm{e}^{\frac{ik\pi}{2}} \\ \mathrm{e}^{ik\pi} \\ \mathrm{e}^{\frac{3ik\pi}{2}} \end{bmatrix} = \begin{bmatrix} 1 \\ i^k \\ (-1)^k \\ (-i)^k \end{bmatrix}$. We should be able to express vectors $v \in \mathbb{C}^4$ as infinite linear combinations of the $\{v_k\}$. But $v_0 = v_4 = v_8 \ldots$ and in general v_k only depends on $k \mod 4$. Thus most of the vectors $\{v_k\}$ are redundant and we might hope to express v_f as a linear combination of the four vectors v_0, v_1, v_2, v_3. In other words, given v_f, we are looking for scalars a_0, a_1, a_2, a_3 satisfying

$$v_f = a_0 v_0 + a_1 v_1 + a_2 v_2 + a_3 v_3. \tag{7.7}$$

Take F_4 to be the four by four matrix whose columns are v_0, v_1, v_2, v_3 respectively; written out explicitly, we have $F_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix}$, and the preceding display can be abbreviated as

$$\begin{bmatrix} f(0) \\ f(\frac{\pi}{2}) \\ f(\pi) \\ f(\frac{3\pi}{2}) \end{bmatrix} = F_4 \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} \tag{7.8}$$

In fact the columns of F_4 are mutually orthogonal in $\mathbb{C}^4$, and thus they form a basis $\mathcal{B}$ for $\mathbb{C}^4$. We can summarize this discussion by noting that *every* $v \in \mathbb{C}^4$ satisfies $v = F_4[v]_{\mathcal{B}}$

(Multiplication by) F_4 is known as the *discrete Fourier transform* on $\mathbb{Z}_4$. Why $\mathbb{Z}_4$? Because as we have seen, while we can define v_k for every integer k, the actual value of v_k only depends on the equivalence class $k \mod 4$. For similar reasons, we will sometimes index entries of vectors in $\mathbb{C}^n$ and matrices in $M_n(\mathbb{C})$ by integers modulo n. In the current section, it will also be convenient to begin our indexing at 0, though we will not always do so later. Thus for the vector $\mathbf{x} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$, we think of a as its 0th or 3rd or 6th, etc. coordinate, b as its 1st or 4th, etc. coordinate, and c as its 2nd or 5th etc. coordinate. With this understanding, the (j, k) entry of the matrix F_4 is given by i^{jk}, with j, k ranging from 0 to 3 modulo 4.

More generally, we write $\omega = \omega_n := \mathrm{e}^{\frac{2\pi i}{n}}$ for the primitive nth root of unity and set $F_n := \left[\omega^{jk}\right]_{j,k=0}^{n-1}$. The *discrete Fourier transform (DFT) on* $\mathbb{Z}_n$ is implemented by multiplication by the matrix F_n. It allows us to satisfy Equation 7.2 when we only care about the values of f at multiples of $\frac{2\pi}{n}$.

Example 3. Compute F_3.

Solution. The primitive third root of the identity is $\omega = \omega_3 = \mathrm{e}^{\frac{2\pi i}{3}} = \cos(\frac{2\pi i}{3}) + i\sin(\frac{2\pi i}{3}) = \frac{-1+i\sqrt{3}}{2}$, so

$$F_3 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \frac{-1+i\sqrt{3}}{2} & \frac{-1-i\sqrt{3}}{2} \\ 1 & \frac{-1-i\sqrt{3}}{2} & \frac{-1+i\sqrt{3}}{2} \end{bmatrix}. \tag{7.9}$$

Chapter 8

Basic Operations in SQL

8.1 Introduction to PostgreSQL

Relational database management systems (RDMSs) are programs that manage databases. They have been the backbone for data operations in government and industry for several decades. The main sources for college-level training in RDMSs can be found in computer science departments, but most often (and usually at a greater depth) in a College of Business, in a major usually called Information Systems. At UTSA, the major is called *Bachelor of Business Administration Degree in Information Systems*.

Knowing how to use a relational database goes deeper than simply learning how to use a software tool. There are many open source and proprietary database management systems. The differences can be quite substantial in implementation, but all of them have a core of instructions that are compatible with Standard ISO/IEC 9075-1:2016[1] from the International Standards Organization (ISO) and the International Electrotechnical Commission (IEC).

The most important aspect of relational databases is information organization, and the architecture of a solution to access data. In this lab, we will explore basic operations related to importing data, querying data, and harmonizing multiple sources of heterogeneous data.

[1] `https://www.iso.org/standard/63555.html` accessed on March 15, 2021

8.2 Basic operations in a RDMS

We will focus on RDMS that support a high-level access language known as Structured Query Language, or SQL. This language is declarative, that is, a statement specifies *what* needs to be obtained, not *how* the data retrieval must be executed. The RDBMS interprets statements of the SQL language to perform the requested database access using sophisticated algorithms for data retrieval, but without exhibiting this inherent complexity.

A RDMS can be organized in *databases*, each one of which might contain one or more *schemas*. Each *schema* contains one or more *tables*. Each *table* contains a set of unique *rows*. Each *row* describes a different *entity*. Each *entity* is defined by a unique combination of attributes organized in *columns*.

Table 8.1 illustrates a student data organized in a table; this table is named 'STUDENT'. In this case, there is a row (or record) per student in 'STUDENT'. Usually, students have multiple records in a 'GRADE' table, as illustrated in Table 8.2

Table 8.1: Example of a data table named 'STUDENT'

ID	*NAME*	*ADDRESS*	*STATUS*
101	John	123 Merry Ln	fresh
102	Shawn	1 UTSA Circle	soph
110	Leonardo	100 Main St	soph
199	Sofia	200 High Rd	senior

Table 8.2: Example of a data table named 'GRADE'

ID	*LETTER*	*POINTS*
101	C	2
101	B	3
101	A	4
102	C	2
110	A	4
110	A	4
199	A+	4

An SQL query to retrieve students named John would be

```
select * from STUDENT where NAME = 'John'
```

The * after the keyword '*select*' means *all columns*; individual columns can be selected by name separated by commas, e.g. ID, NAME. The table name goes after the keyword '*from*'. Finally, a condition follows the keyword '*where*' using the column name.

The result of this query is shown in Table 8.3

Table 8.3: Result of query

ID	*NAME*	*ADDRESS*	*STATUS*
101	John	123 Merry Ln	fresh

It is possible to link multiple tables in a single query. For example,

```
select     NAME, avg(POINTS)
from       STUDENT S
           join GRADE G
           on S.ID = G.ID
where      ID > 105
group by   NAME
order by   avg(POINTS)  ASCENDING
```

What would be the result of this query? Write down the expected table. Record your explanation as answer #17. Search for information and explain each element of this query. Record your explanation as answer #18.

8.3 How to import a CSV file

This is many times the most difficult step in data analysis. Data is often presented in comma-separated values (CSV) format. PostgreSQL offers an easy way to import CSV files through the *COPY* instruction, provided that a table exists to hold the data. Hence, the first step is to create a table.

For instance, if you have a CSV file with three columns, the table could be something like:

```
CREATE TABLE MyDataTable (
  column1 SERIAL,
  column2 VARCHAR(50),
```

```
  column3 DATE,
  PRIMARY KEY (column1)
)
```

This instruction contains the statement *CREATE TABLE* which is part of a subset of the SQL language called Data Definition Language (DDL). Note that each column has to be defined with a data type. Once the table is created, you have to import the data with the *COPY* instruction.

```
COPY MyDataTable(column1, column2, dob, column3)
FROM 'MyDataTable.csv'
DELIMITER ','
CSV HEADER;
```

8.4 Character Encoding. A Challenge Importing Data into a RDBMS

File encoding is a method by which information is stored or presented in a file. Specifically, it deals with character sets and their representation in a binary format. Encoding provides a mechanism that allows us to map these sequences of bits to human-readable characters. In doing so, we can efficiently read, write, and store text in a manner that computers can precisely retrieve.

One of the earliest character encoding standards is known as ASCII, which stands for the American Standard Code for Information Interchange. ASCII encapsulates each character within a 7-bit integer representation. Its primary design was intended for the English language, thus it envelops 128 characters which include English alphabets, numbers, and a variety of punctuation marks.

However, as the need for more diverse character sets arose, particularly for languages other than English, extensions to ASCII were introduced. The ISO-8859 series is an excellent example of this. Each variant of ISO-8859 was curated for different languages or sets of languages. A notable instance is ISO-8859-1, often referred to as Latin-1, which was crafted to cater to several Western European languages.

The advent of the Unicode standard brought with it UTF-8, a variable-length encoding with the capability to represent any character within the Unicode paradigm. A salient feature of UTF-8 is

its backward compatibility with ASCII, a trait that played a pivotal role in its rise to dominance, especially in the World Wide Web. The Unicode standard also brought forth other encoding forms, namely UTF-16 and UTF-32. While UTF-16 employs variable-length encoding, UTF-32 opts for a fixed-length approach. Both these forms, unlike UTF-8, are adept at representing characters beyond the ASCII spectrum more efficiently but sacrifice backward compatibility with ASCII in the process.

One might wonder about the ramifications of mismatched encoding. Characters may appear misplaced, be replaced with placeholders such as the ubiquitous  , or even result in gibberish output. This phenomenon is colloquially termed "mojibake". Mismatched encoding can severely hinder the usability fo data.

Contemporary text editors and Integrated Development Environments (IDEs) are equipped with auto-detection features for file encodings. Moreover, they grant users the autonomy to specify the encoding when accessing a file. For those who prefer the command line, tools like 'file' on UNIX-based systems can sometimes discern the encoding of a given file. As data and text are increasingly shared and transferred between systems or software, it becomes paramount to maintain encoding consistency or to adeptly perform a conversion when needed.

Adhering to universal encodings, like UTF-8, is now considered a best practice.

8.5 Handling Encoding Issues in PostgreSQL across Operating Systems

When importing a CSV file into PostgreSQL, one might encounter an encoding issue. On Linux and macOS, you can use the `file` command to determine the encoding:

```
file -i /path/to/your/csvfile.csv
```

The output might indicate an encoding like `charset=iso-8859-1`.

On Windows (also LKinux & Mac), you can use the Python tool `chardet`:

```
pip install chardet
chardetect /path/to/your/csvfile.csv
```

If the file isn't in UTF-8 encoding, convert it using the `iconv` command:

```
iconv -f ORIGINAL_ENCODING -t UTF-8
   /path/to/your/csvfile.csv -o
   /path/to/your/newfile.csv
```

Replace `ORIGINAL_ENCODING` with the encoding you identified.
If you don't have `iconv` on Windows:

- Open the file in Notepad++.
- From the top menu, select "Encoding" and take note of the current encoding.
- Choose "Convert to UTF-8" from the "Encoding" menu.
- Save the file.

After conversion, you can proceed with importing the file into PostgreSQL.

If you prefer not to change the file's encoding:

```
SET CLIENT_ENCODING TO 'ORIGINAL_ENCODING';
COPY school_data FROM 'C:\path\to\your\csvfile.csv'
   DELIMITER ',' CSV HEADER;
SET CLIENT_ENCODING TO 'UTF8';
```

Chapter 9

Using Relational Databases in Data Analysis

9.1 Sending SQL statements from Python

PostgreSQL and Python –and in general relational database management systems (RDBMSs) and general-purpose programming languages– are different computer programs developed by separate communities for different purposes. However, they complement each other remarkably well for data analysis operations.

Managing a large text file to conduct data analysis is an immensely inefficient process, hence the need for an RDMS. At the same time, conducting relatively simple mathematical operations in a RDMS, e.g. matrix multiplication, becomes a hard task. The point of equilibrium with the tools we are using is to conduct data operations (i.e. search and retrieval, update, delete, aggregate) with an RDMS, and then perform data analysis operations with Python.

The first step needed to access data in PostgreSQL is to import the *psycopg2* library. We will also use *numpy* This can be accomplished with the import command:

```
1 import psycopg2
2 import numpy as np
```

Assume you ahve created a schema called *USA*, and in it a table called *natalityConf*. To access data in that table, a connection has to be created to the database. Let's assume that there is a user called *postgresql* within PostgreSQL, with the associated password *SARS-CoV-2*. A connection to the database is created with the following

instruction:

```
conn = psycopg2.connect("dbname=USA user=postgres
   password=SARS-CoV-2")
```

The connection by itself does nothing visible; however it is a data structure that exists in memory and contains all the information needed to exchange data with PostgreSQL. In order to use it, we need to create an instance of the *cursor*:

```
cur = conn.cursor()
```

At this point we have all we need to send a request for data to PostgreSQL from within a Python environment. In this case, we will request as an example the distinct values in the column *countyfips*, which contains the FIPS codes for every county in the database.

To obtain the unique FIPS codes in the table, execute the following instruction:

```
x = cur.execute('''select distinct countyfips,
                county , "state" from
                   "USA"."natalityConf";''')
```

To see the results of the previous query, you have to fetch the data. Execute the following instruction:

```
rows = cur.fetchall()
```

The variable *rows* contains the data as a list. The following instruction will take the first element of every row to create a numeric vector with which we will be able to conduct linear algebra operations:

```
# Now, how to extract all members of a tuple
n = 0 # this is the column number
y = [x[n] for x in rows]

# If the field is numeric, you might want to convert it
   to a matrix
z = np.asarray(y)
```

The variable z contains a numeric array. Now, data analysis can begin.

9.2 Accelerating SQL statements

To accelerate data retrieval, you will need to create an *index* in the database. Think of this as an analog to a phone guide; it simply

contains information sorted in a way that makes search and retrieval faster. With a phone guide, you might be able to find the phone of a specific business because the listing is sorted in alphabetical order. However, if you wanted to find all the business in e.g. Merry Street, the standard phone guide would not be the best instrument because you would have to read the entire guide line by line to ensure you capture all the entries that match the search criteria; you would need a different *index*.

The following command in our imaginary table

```
x = cur.execute('''CREATE INDEX "idxFIPS"
    ON "USA"."natalityConf" USING btree
    (countyfips ASC NULLS LAST)
    TABLESPACE pg_default;''')
```

is equivalent to opening PGadmin and executing the following command:

```
CREATE INDEX "idxFIPS"
    ON "USA"."natalityConf" USING btree
    (countyfips ASC NULLS LAST)
    TABLESPACE pg_default;
```

Chapter 10

Nonlinear Discriminants

10.1 Neural Networks

This lab[1] deals with simple problem: Given a family Ω of observations, $\mathbf{X}(t)$, and a finite number C of classes Ω can be assigned to, and assuming there is a feature vector $\mathbf{x}$ in a nth dimensional feature space such that each element of $\mathbf{x}$ represent a characteristic of $\mathbf{X}(t)$, we want to find a discriminant function $g_i(\mathbf{x})$, $i = 1, 2 \ldots C$ that assigns $\mathbf{x}$ to class w_i if $g_i(\mathbf{x}) < g_j(\mathbf{x})$ for all $j \neq i$. We define the *error* in this operation as the probability or assigning the wrong class w_i to $\mathbf{x}$. We define the *decision region* in the space of $\mathbf{x}$, as a set of boundaries that divide the feature space into segments such that each segment corresponds to either one class w_i or no class at all.

With a clever choice of nonlinear functions, it is possible to obtain arbitrary decision regions, in particular those leading to minimum error. These nonlinear functions are well defined by neural networks. In general, a *neural network* is composed of a group of connected *neurons*. A single neuron can be connected to many other neurons, so that the overall structure of the network can be very complex.

Neural network algorithms can be roughly divided into two classes.

- Supervised neural networks are given data in the form of inputs and targets, the targets being a *teacher*'s specification of what the neural network's response to the input should be. The central idea of supervised neural networks is this: Given examples of a

[1]This section is an excerpt from "*Information theory, inference, and learning algorithms*" by David J.C. MacKay, *in memoriam*, used with permission from the author.

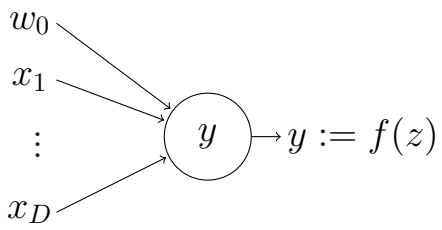


Figure 10.1-1: Single processing unit and its components. The activation function is denoted by f and applied on the actual input z of the unit to form its output $y = f(z)$. $x_1, \ldots, x_D$ represent input from other units within the network; w_0 is called bias and represents an external input to the unit. All inputs are mapped onto the actual input z using the propagation rule.

relationship between an input vector $\mathbf{x}$, and a target t, we hope to make the neural network 'learn' a model of the relationship between $\mathbf{x}$ and t. A successfully trained network will, for any given $\mathbf{x}$, give an output y that is close (in some sense) to the target value t. Training the network involves searching in the weight space of the network for a value of $\mathbf{w}$ that produces a function that fits the provided training data well.

- Unsupervised neural networks are given data in an undivided form – simply a set of examples. Some learning algorithms are intended simply to memorize these data in such a way that the examples can be recalled in the future. Other algorithms are intended to 'generalize', to discover 'patterns' in the data, or extract the underlying 'features' from them. Some unsupervised algorithms are able to make predictions, for example, some algorithms can 'fill in' missing variables in an example (and so can also be viewed as supervised networks).

10.1.1 The single neuron

A single neuron has a number I of *inputs* x_i and one output which we will here call y. (See figure 10.1-2.) Associated with each input is a weight w_i $(i = 1, \ldots, I)$. There may be an additional parameter w_0 of the neuron called a *bias* which we may view as being the weight associated with an input x_0 which is permanently set to 1. The single neuron is a *feed forward* device, i.e. the connections are directed from the inputs to the output of the neuron.

The activity rule has two steps.

- First, in response to the imposed inputs x, we compute the *ac-*

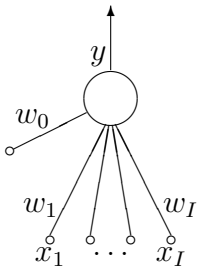


Figure 10.1-2: A single neuron

tivation of the neuron,

$$a = \sum_i w_i x_i, \tag{10.1}$$

where the sum is over $i = 0, \ldots, I$ if there is a bias and $i = 1, \ldots, I$ otherwise.

- Second, the *output* y is set as a function $f(a)$ of the activation. The output is also called the *activity* of the neuron. Notice it is different to the activation a.

activation activity
$a \rightarrow y(a)$

There are several possible *activation functions*; here are the most popular.

- Deterministic activation functions:

  1. Linear.
  $$y(a) = a.$$
  2. Sigmoid (logistic function).
  $$y(a) = \frac{1}{1 + e^{-a}} \qquad (y \in (0, 1)).$$
  3. Sigmoid (tanh).
  $$y(a) = \tanh(a) \qquad (y \in (-1, 1)).$$
  4. Threshold function.
  $$y(a) = \Theta(a) \equiv \begin{cases} 1 & a > 0 \\ -1 & a \leq 0. \end{cases}$$

- Stochastic activation functions: y is stochastically selected from ± 1.

  1. Heat bath.

$$y(a) = \begin{cases} 1 & \text{with probability } \dfrac{1}{1+e^{-a}} \\ -1 & \text{otherwise.} \end{cases}$$

  2. The Metropolis rule produces the output in a way that depends on the previous output state y:

     Compute $\Delta = ay$
     If $\Delta \leq 0$, flip y to the other state
     Else flip y to the other state with probability $e^{-\Delta}$.

A neural network implements a function $y(\mathbf{x}; \mathbf{w})$; the 'output' of the network, y, is a nonlinear function of the 'inputs' $\mathbf{x}$; this function is parameterized by 'weights' $\mathbf{w}$.

An example of a very simple neural network is the following function of $\mathbf{x}$, which produces an output between 0 and 1:

$$y(\mathbf{x}; \mathbf{w}) = \frac{1}{1+e^{-\mathbf{w}\cdot\mathbf{x}}}. \tag{10.2}$$

For convenience let us study the case where the input vector $\mathbf{x}$ and the parameter vector $\mathbf{w}$ are both two-dimensional: $\mathbf{x} = (x_1, x_2)$, $\mathbf{w} = (w_1, w_2)$. Then we can spell out the function performed by the neuron thus:

$$y(\mathbf{x}; \mathbf{w}) = \frac{1}{1+e^{-(w_1 x_1 + w_2 x_2)}}. \tag{10.3}$$

We now introduce the idea of *weight space*, that is, the parameter space of the network. In this case, there are two parameters w_1 and w_2, so the weight space is two dimensional. Each *point* in $\mathbf{w}$ space corresponds to a *function* of $\mathbf{x}$. Notice that the gain of the sigmoid function (the gradient of the ramp) increases as the magnitude of $\mathbf{w}$ increases.

Typically an *objective function* or *error function* is defined, as a function of $\mathbf{w}$, to measure how well the network with weights set to $\mathbf{w}$ solves the task. The objective function is a sum of terms, one for

each input/target pair $\mathbf{x}, t$, measuring how close the output $y(\mathbf{x}; \mathbf{w})$ is to the target t.

The training process is an exercise in function *minimization*, i.e., adjusting $\mathbf{w}$ in such a way as to find a $\mathbf{w}$ that minimizes the objective function. Many function-minimization algorithms make use not only of the objective function, but also its *gradient* with respect to the parameters $\mathbf{w}$. For general feed forward neural networks the *backpropagation* algorithm efficiently evaluates the gradient of the output y with respect to the parameters $\mathbf{w}$, and thence the gradient of the objective function with respect to $\mathbf{w}$.

10.1.2 A Didactic Implementation

Programming an artificial neural network is a trivial task. I have made available to you a MATLAB and a Python program that computes a simple ANN. Try to use it.

Acknowledgements

Acknowledgements for the Rowdy Datathon must go beyond mere formalities, for the event is a monumental effort that reflects a commitment to making data accessible and comprehensible to all.

Special thanks are due to the Student Chapter of the Association for Computing Machinery at the University of Texas at San Antonio: Roni Maddox, Poonacha Cheppudira, UTSA alumna Jenelle Millison, and many others. The event would have not been possible without the support of the School of Data Science at UTSA and the National Security Agency. To all contributors, your collective efforts have culminated in a Datathon that serves as both a platform for applied data science and as an educational crucible for emerging data analysts.

The phrase "Data is everywhere, therefore data should be for everyone" is not merely a tagline; it encapsulates the philosophy that guided us through countless meetings, debugging sessions, manual tests, and problem-solving endeavors. This project is not isolated but forms a part of our larger mission: to guide aspiring data analysts through the multi-faceted landscape of data science. The Datathon aims to bridge the gaps in a fragmented educational landscape, offering a holistic view of data analytics that is sorely needed in the field. Thank you for your time, your expertise, and most importantly, your unwavering commitment to the democratization of data.

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