# Quiz 1

Taylor Woodard

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## 1 Question 1

Given this model follows a Polya urn model, and given that we start with one orange and one purple ball, we can calculate the probability of having both 15 and $n - 15$ such that we have $n$ total balls in the urn.
Since we start with one orange and one purple, we can assume the urn follows a binomial distribution.
The probability of 15 orange balls is given by:

$$P(X = 15) = \binom{n-2}{15-1}\left(\frac{1}{2}\right)^{n-2} = \binom{n-2}{14}\left(\frac{1}{2}\right)^{n-2} \tag{1}$$

and the probability of $n - 15$ purple balls is given by the same equation, because the urn model is symmetric.

## 2 Question 2

Given the probability of a bit flips is p implies the probability of a bit remaining the same is $1 - p$.
Since we know that the bits are being sent in a sequence, we can assume the flips follow of a binomial distribution following:

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k} \tag{2}$$

If $k$ is even, then Bob will recieve the correct bit.
If $k$ is odd, then Bob will recieve the incorrect bit.
Therefore, the total probability of Bob recieving the bit is given by:

$$P(\text{correct bit}) = \sum_{\substack{k=0 \\ \text{k is even}}}^{n} \binom{n}{k} p^k (1-p)^{n-k} \tag{3}$$

and

$$P(\text{incorrect bit}) = \sum_{\substack{k=0 \\ \text{k is odd}}}^{n} \binom{n}{k} p^k (1-p)^{n-k} \tag{4}$$

Using the Biomial Identity we see:

$$P(\text{correct bit}) = \sum_{\substack{k=0 \\ \text{k is even}}}^{n} \binom{n}{k} p^k (1-p)^{n-k} = \frac{1 + (1-2p)^n}{2} \tag{5}$$

and

$$P(\text{incorrect bit}) = \sum_{\substack{k=0 \\ \text{k is odd}}}^{n} \binom{n}{k} p^k (1-p)^{n-k} = \frac{1 - (1-2p)^n}{2} \tag{6}$$

Therefore, the probability of Bob recieving the correct bit is:

$$P(\text{correct bit}) = \sum_{\substack{k=0 \\ \text{k is even}}}^{n} \binom{n}{k} p^k (1-p)^{n-k} = \frac{1 + (1-2p)^n}{2} \tag{7}$$