Q1.

1) $m(a+bX) = a + b \times m(X)$

$m(a+bX) = \frac{1}{N}\sum_{i=1}^{N}(a+bx_i)$

$m(a+bX) = \frac{1}{N}\sum_{i=1}^{N} a + \frac{1}{N}\sum_{i=1}^{N} bx_i$

$\sum_{i=1}^{N} a = Na$

$m(a+bX) = a + b \times \frac{1}{N}\sum_{i=1}^{N} x_i = a + bm(x)$

2) $cov(X, a+bY) = b \times cov(X, Y)$

$cov(X, Y) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))(y_i - m(Y))$

$cov(X, a+bY) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(a + by_i - m(a+bY))$

$m(a+bY) = a + bm(Y)$

$cov(X, a+bY) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))(a + by_i - (a + bm(Y)))$

$cov(X, a+bY) = 1/N \sum_{i=1}^{N}(x_i - m(x))(b(y_i - m(Y)))$

$cov(X, a+bY) = b \times 1/N \sum_{i=1}^{N}(x_i - m(x))(y_i - m(Y))$

$cov(X, a+bY) = b \times cov(X, Y)$

3) $cov(a+bX, a+bX) = b^2 cov(X, X) \rightarrow cov(x, x) = s^2$

$cov(x, x) = 1/N \sum_{i=1}^{N}(x_i - m(x))^2$

$\hookrightarrow s^2 = cov(X, X)$

$cov(a+bX, a+bX) = b^2 cov(X, X)$

4) $g(x)$ being a non-decreasing function persevres the order of the value. Meaning $g(\tilde{x}) = g(\tilde{x})$, so the median of the changed function equals the median of the original function. This is true for all quartiles because they depend on the ordering of values.

The IQR, however will see a slight shift depending on the value that it is being scaled by.

5) No, it is not true that $m(g(x)) = g(m(x))$ is always true for a non-decreasing $g()$. This is the case because the sample mean does not apply a function to the mean and instead just sums the values up.