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Assignment questions

(1)

Soln:

$T(n) = 3T(n-1) + 12n$, given $T(0) = 5$

$T(2) = ?$

$T(1) = 3.T(1-1) + 12\times1$
$= 3.T(0) + 12$
$= 3\times5 + 12$
$= 15 + 12 = 27$

now.
$T(2) = 3T(2-1) + 12\times2$
$= 3T(1) + 24$
$= 3\times27 + 24$
$= 81 + 24$
$= 105$
$\therefore T(2) = 105$

(2.)

(a) $T(n) = T(n-1) + c$

Soln:
$T(n) = T(n-1) + c$ —— (1)
$\therefore T(n-1) = T(n-1-1) + c$
$= T(n-2) + c$
$\therefore T(n) = T(n-2) + 2c$ —— (2)

$T(n-2) = T(n-3) + c$
$\therefore T(n) = T(n-3) + 3c$ —— (3)

↓

$\Big\downarrow$ k times

$$T(n) = T(n-k) + k.c$$

$n - k = 1$

or $k = n-1$ Substituting in $T(n)$

$$T(n) = T(n-(n-1)) + (n-1).c$$
$$= T(\cancel{n}-\cancel{n}+1) + (n-1)c$$
$$= T(1) + (n-1)c$$
$$= 1 + n.c - c$$
$$= n.c - c + 1$$

$T(n) = O(n)$ = Time Complexity

(b) $T(n) = 2T(n/2) + n$

Soln:-

$$T(n) = 2T(n/2) + n \quad \text{--- (1)}$$

$$T(n/2) = 2.T\left(\frac{n}{2^2}\right) + \frac{n}{2}$$

$$T(n) = 2.2.T\left(\frac{n}{4}\right) + \frac{n}{2} + n$$

$$T(n/4) = 2.T\left(\frac{n}{8}\right) + \frac{n}{4}$$

$$T(n) = 2^3 T\left(\frac{n}{8}\right) + \frac{n}{4} + \frac{n}{2} + n$$

$\vdots$

k times

$$T(n) = 2^k \cdot T\left(\frac{n}{2^k}\right) + \frac{n}{2^{k-1}} + \cancel{\frac{n}{2^{k-2}}} + \ldots + \frac{n}{2} + n$$

$$\frac{n}{2^k} = 1$$

$$n = 2^k$$

$$\log_2 n = \log_2 2^k$$

$$k = \log_2 n$$

$$T(n) = 2^{\log_2 n} \cdot T\left(\frac{n}{2^{\log_2 n}}\right) + \frac{n}{2^{\log_2 n - 1}} + \ldots + \frac{n}{2} + n$$

$$= n^{\log_2 2} \cdot T\left(\frac{n}{n^{\log_2 2}}\right) + \frac{n}{2^{\log_2 n - 1}} + \ldots + \frac{n}{2} + n$$

$$= n \cdot 1 + \frac{n}{2^{\log_2 n - 1}} + \ldots + \frac{n}{2} + n$$

$$= n\left(1 + \frac{1}{2^{\log_2 n - 1}} + \ldots \frac{1}{2} + 1\right)$$

$T(n) = O(n)$ = Time complexity

(c) $T(n) = 2T(n/2) + C$

Soln:-

$$T(n) = 2T(n/2) + C \longrightarrow (1)$$

$$T(n/2) = 2T(n/2^2) + C$$

$$T(n) = 2^2 T\left(\frac{n}{2^2}\right) + 2C \longrightarrow (2)$$

$$T(n/3) = 2T(n/2^3) + C$$

$$T(n) = 2^3 T\left(\frac{n}{2^3}\right) + 3C \longrightarrow (3)$$

⋮

k times

$$T(n) = 2^k T\left(\frac{n}{2^k}\right) + k \cdot C$$

$$\frac{n}{2^k} = 1$$

$$n = 2^k$$

$$\log_2 n = \log_2 2^k$$

$$k = \log_2 n$$

$$\therefore T(n) = 2^{\log_2 n} T\left(\frac{n}{2^{\log_2 n}}\right) + \log_2 n \cdot c$$

$$= n \cdot T\left(\frac{n}{n^{\log_2 2}}\right) + \log_2 n \cdot c$$

$$= n \cdot T(1) + \log_2 n \cdot c$$

$$= n + \log_2 n \cdot c$$

Time complexity $= T(n) = O(n)$

(d) $T(n) = T(n/2) + c$

Soln:-

$$T(n) = T(n/2) + c \quad \text{--- (1)}$$

$$T(n/2) = T(n/2^2) + c$$

$$T(n) = T(n/2^2) + 2c$$

$$T(n/4) = T(n/2^3) + c$$

$$T(n) = T(n/2^3) + 3c$$

⋮ k - times

$$T(n) = T\left(\frac{n}{2^k}\right) + k \cdot c$$

$$\frac{n}{2^k} = 1$$

or $n = 2^k$.

or $\log_2 n = \log_2 2^k$

or $\log_2 n = k \cdot \log_2 2$

or $k = \log_2 n$.

$$T(n) = T\left(\frac{n}{2^{\log_2 n}}\right) + \log_2 n \cdot c$$

$$= T\left(\frac{n}{n^{\log_2 2}}\right) + \log_2 n \cdot c$$

$$= T(1) + \log_2 n \cdot c$$

$$= 1 + \log_2 n \cdot c$$

$\therefore T(n) =$ Time complexity $= O(\log_2 n)$

(3)

(a) $2T(n-1) + 1 = T(n)$

$\therefore T(n) = 2T(n-1) + 1$

$T(n-1) = 2T(n-2) + 1$

$T(n) = 2^2 T(n-2) + 1 + 1$

$T(n-2) = 2T(n-3) + 1$

$T(n) = 2^3 T(n-3) + 3$

$\vdots$ k times

$T(n) = 2^k T(n-k) + k$

$n-k = 1$

or $k = n-1$

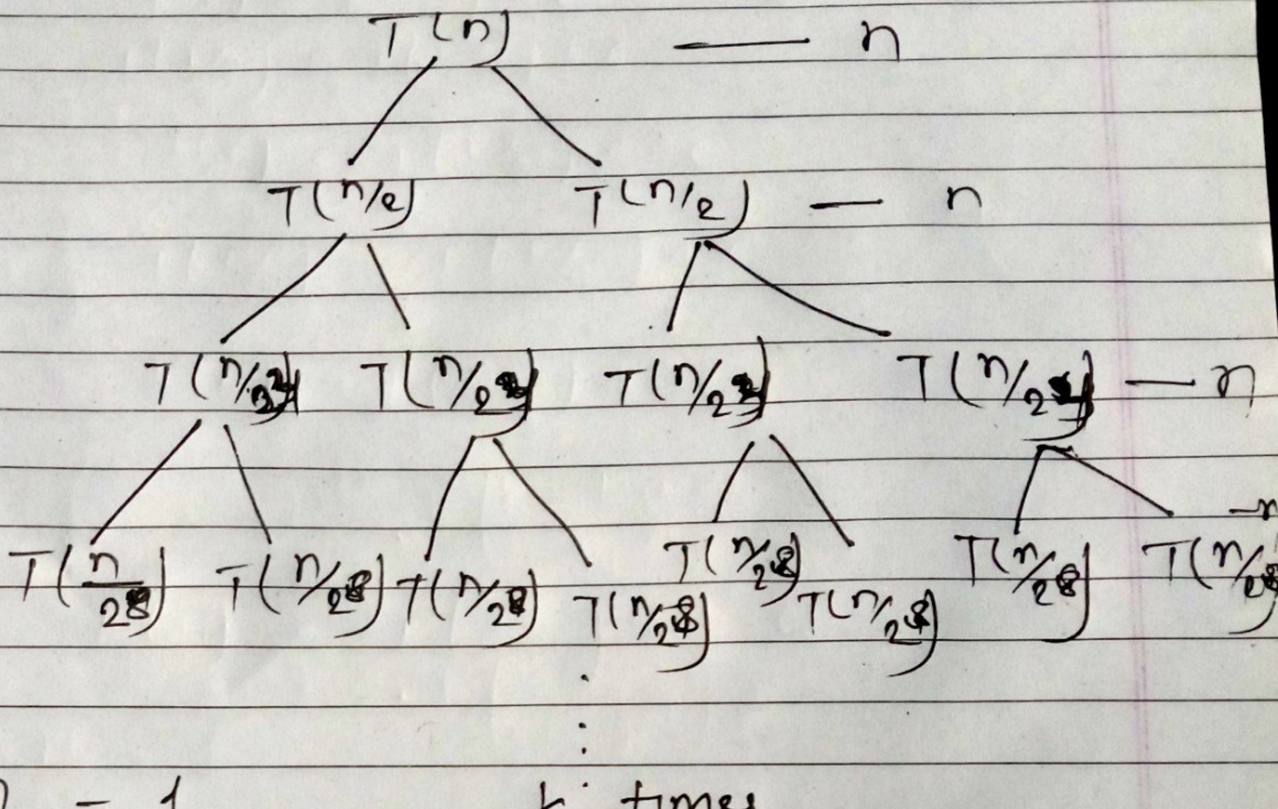
$T(n) = 2^{n-1} \cdot T(n-(n-1)) + (n-1)$

$= 2^{n-1} \cdot T(1) + (n-1)$

$= 2^{n-1} + n - 1$

$\therefore T(n) =$ Time complexity $= O(2^n)$

(b) $T(n) = 2T(n/2) + n$



$\frac{n}{2^k} = 1$

$n = 2^k$

$\log_2 n = \log_2 2^k$

$2k = \log_2 n$

$k = \log_2 n$

$\Rightarrow n \cdot k$

$= n \cdot \frac{\log_2 n}{2}$

Time Complexity $\Rightarrow O(n \cdot \log_2 n)$