

**L3 – MIASHS 2021 2022**  
**Parcours Informatique**  
**UE MIC0602T**

**TD 1 - Expressions régulières et Grammaires**  
**Solutions**

**Pierre-Jean Charrel – Sophie Ebersold**

1.

(a)

$$V_N = \{S, T\}$$

$$V_T = \{a, b\}$$

$$R = \begin{cases} S & \rightarrow TaTaTaT \\ T & \rightarrow bT \mid b \mid \varepsilon \end{cases}$$

(b)

$$V_N = \{S, T\}$$

$$V_T = \{a, b\}$$

$$R1 = \begin{cases} S & \rightarrow aT \mid bS \mid \varepsilon \\ T & \rightarrow bS \mid \varepsilon \end{cases}$$

(c)

$$V_N = \{S\}$$

$$V_T = \{a, b\}$$

$$R = \begin{cases} S & \rightarrow aSa \mid bSb \mid a \end{cases}$$

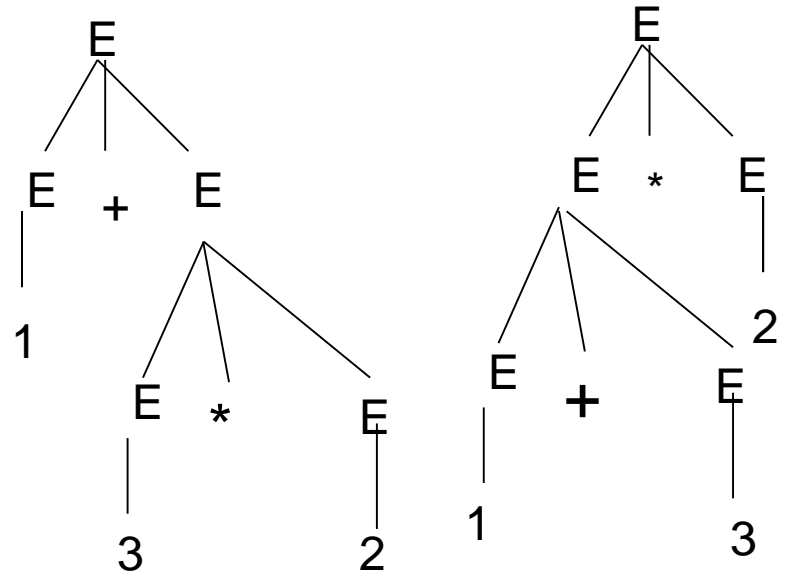
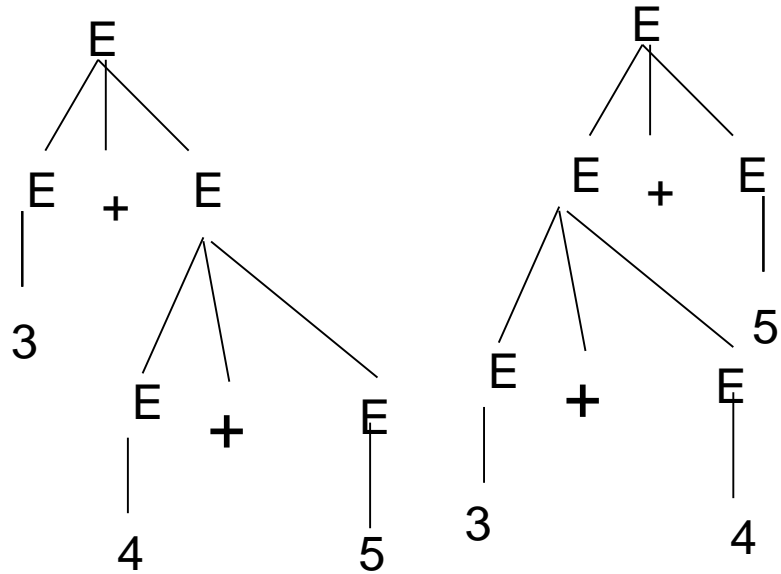
(a)  $V_N = \{S, T\}$   $V_T = \{ (, ), a, b, ", " \}$  Racine T il peut exister des expressions non parenthésées.

(b)

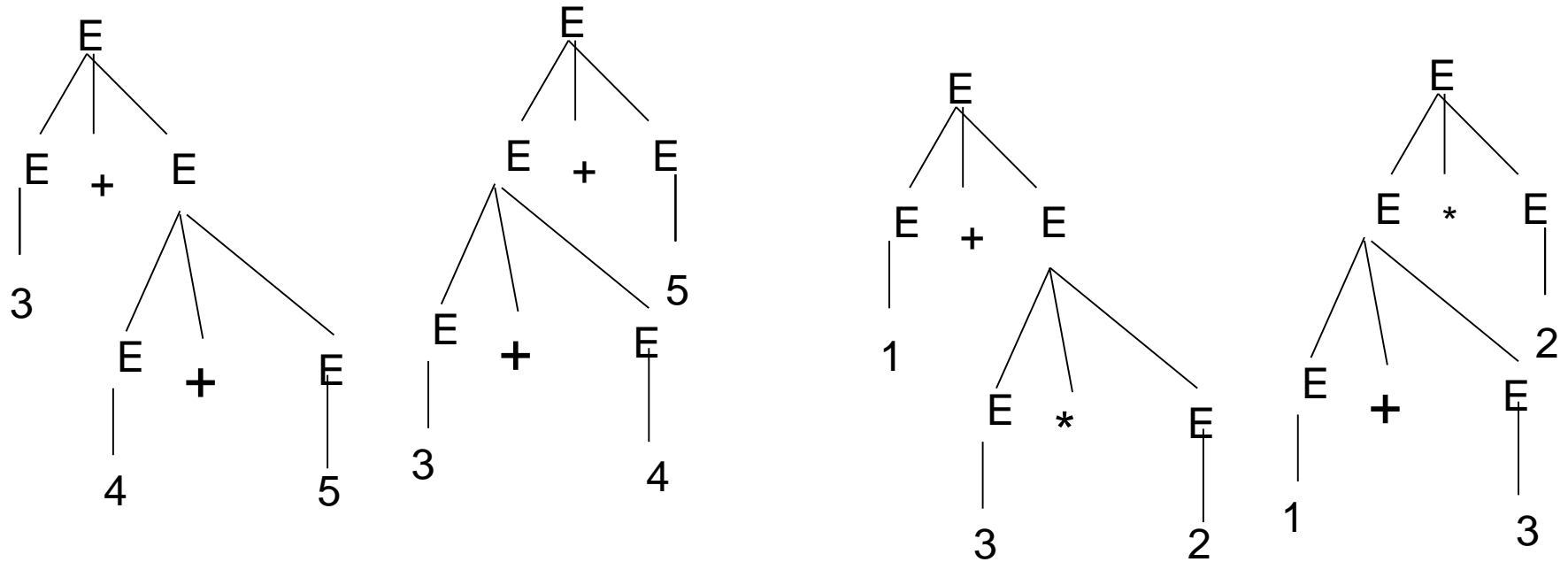
The image shows two parse trees for the expression  $(b, (a, b))$ . The left tree has root  $T$ , which expands to  $S$ , then to  $(T)$ , then to  $T$ , comma, and  $S$ , finally to  $a$ , comma, and  $b$ . The right tree has root  $T$ , which expands to  $S$ , then to  $(T)$ , then to  $T$ , comma, and  $S$ , finally to  $b$ , comma, and  $(T)$ , which further expands to  $T$ , comma, and  $S$ , finally to  $a$ , comma, and  $b$ .

$$\begin{array}{ll} S & \rightarrow a \mid b \mid (T) \\ T & \rightarrow S T_1 \\ T_1 & \rightarrow , S T_1 \mid \varepsilon \end{array}$$

3.



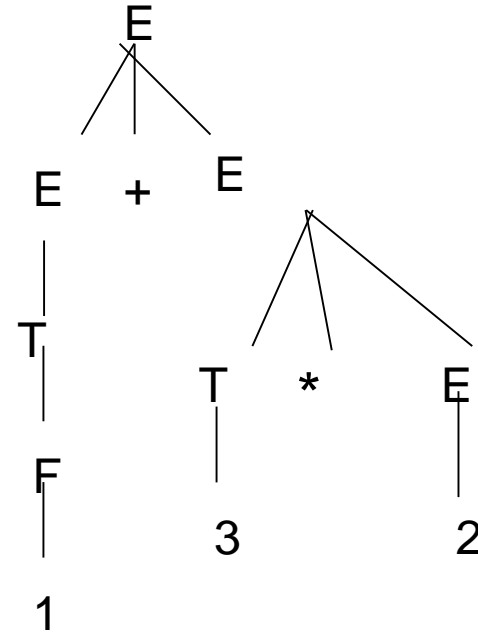
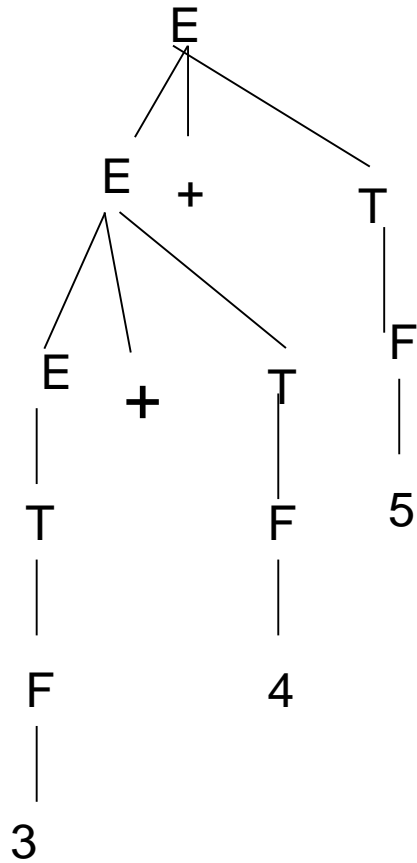
3.



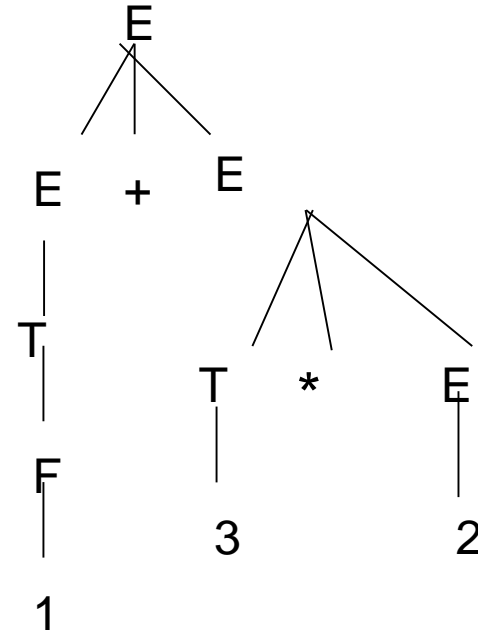
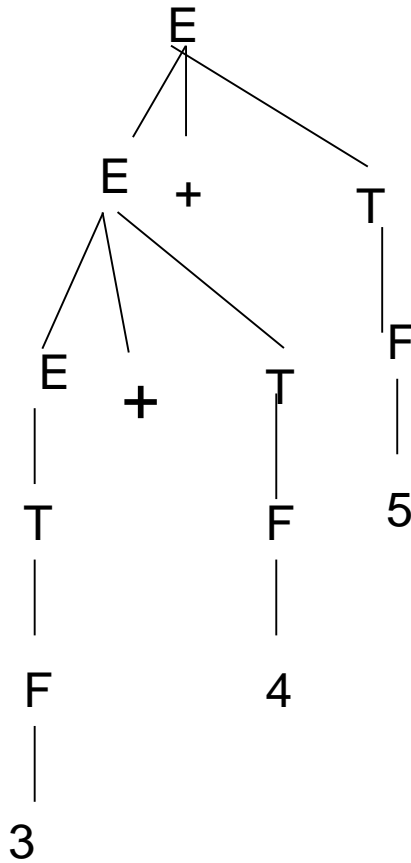
Degré d'ambiguïté = 2 : 2 arbres de dérivation possibles  
C'est la sémantique des opérateurs qui permet de choisir  
l'arbre.

Elle n'est pas décrite dans les règles de la grammaire

#### 4. (a) Degré d'ambiguïté = 1 : 1 arbre de dérivation possible



#### 4. (a) Degré d'ambiguïté = 1 : 1 arbre de dérivation possible



(b)

$E \rightarrow T E'$

$E' \rightarrow + T E' \mid \varepsilon$

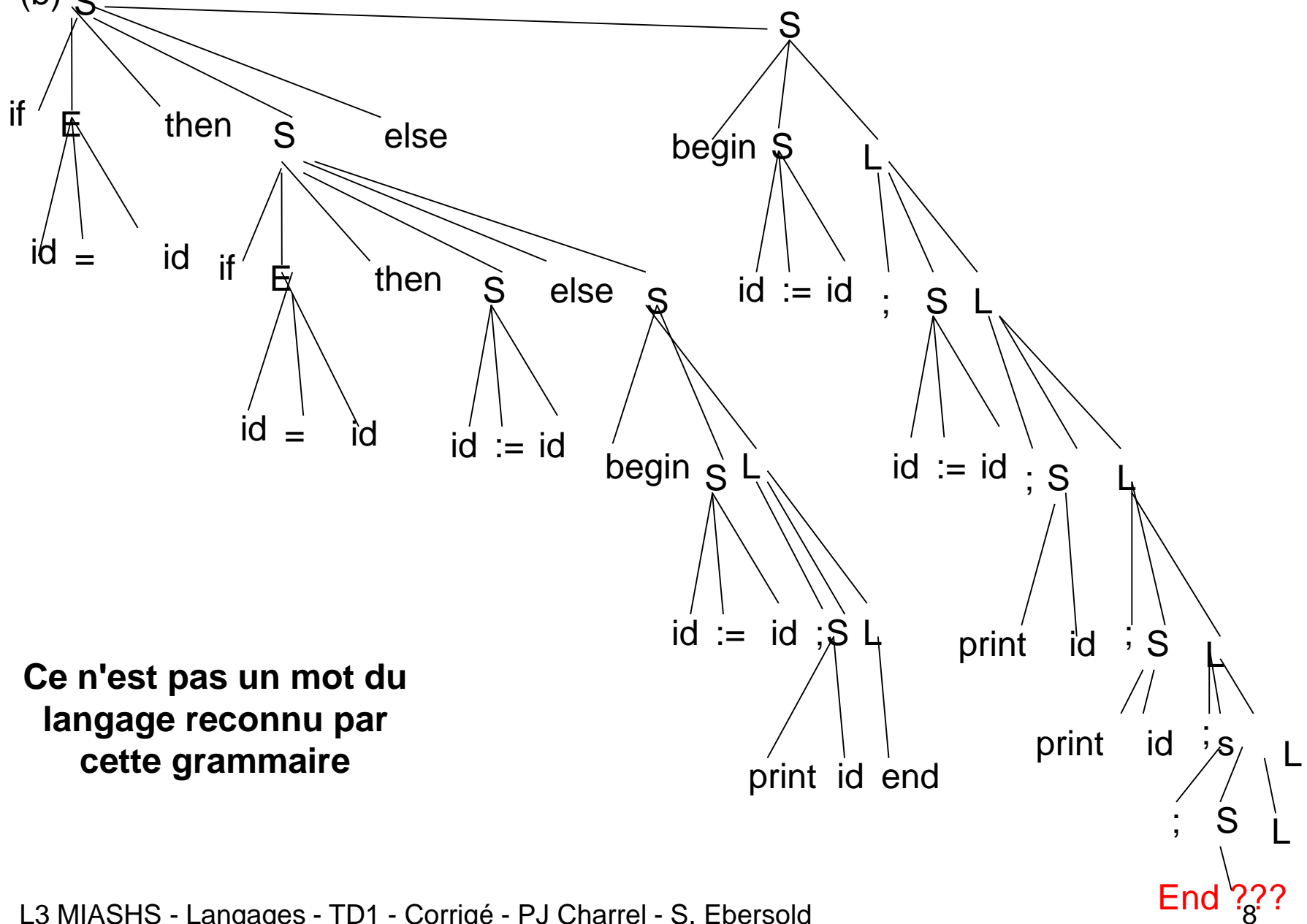
$T \rightarrow F T'$

$T' \rightarrow * F T' \mid \varepsilon$

$F \rightarrow \text{nombre}$

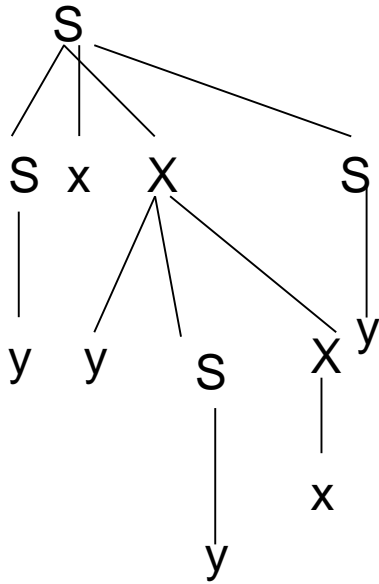
5. (a)  $V_N = \{S, L, E\}$   $V_T = \{\text{if, then, else, =, :=, begin, end, print, id, ;}\}$   $S$

(b) ~~S~~

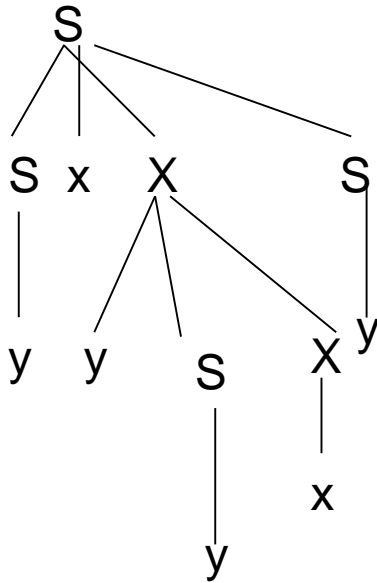




6. (a)



## 6. (a)



(b)

$S \rightarrow y S1$

$S1 \rightarrow x X S S1 \quad | \quad \varepsilon$

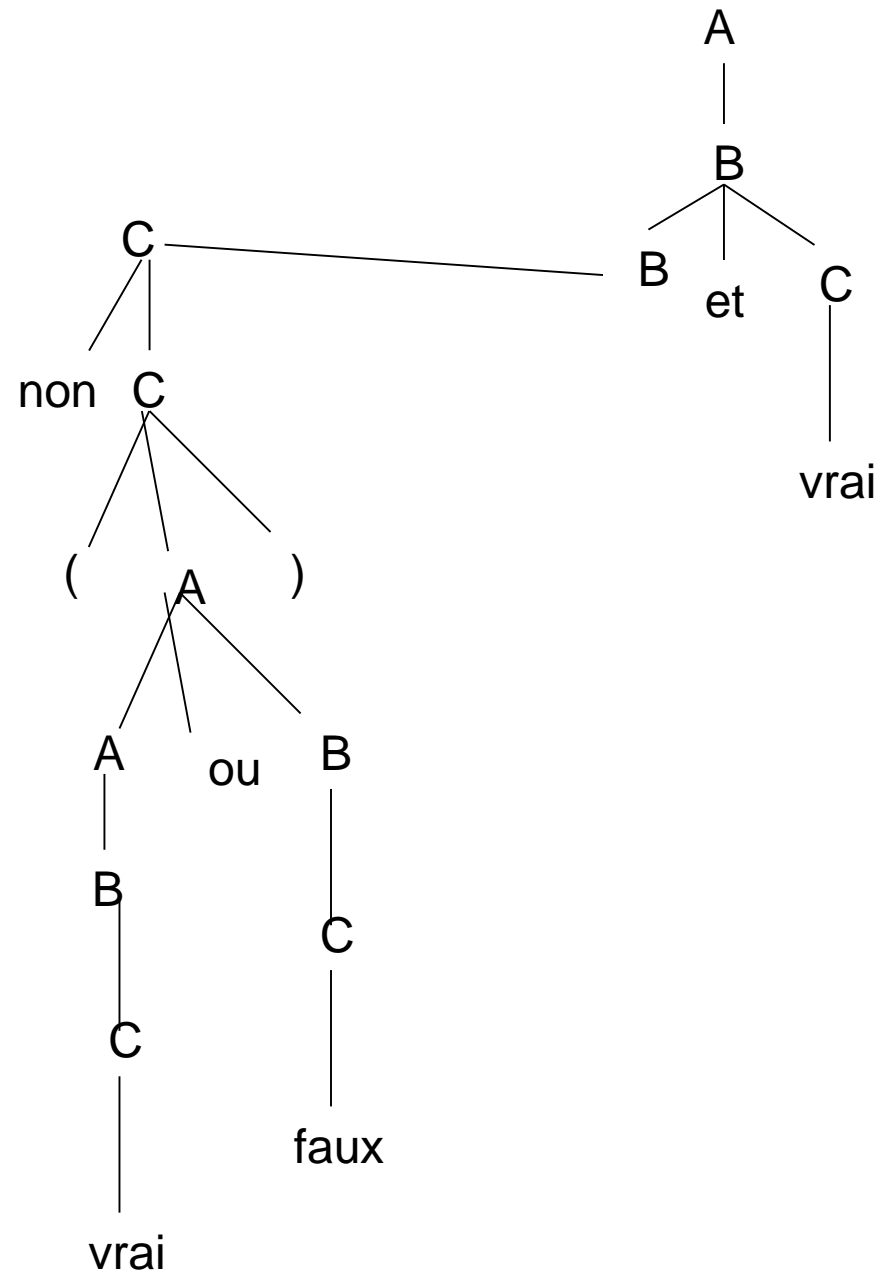
$X \rightarrow y S X \quad | \quad x \quad | \quad y \quad | \quad \varepsilon$

$X \rightarrow y X1 \quad | \quad x \quad | \quad \varepsilon$

$X1 \rightarrow S X \quad | \quad \varepsilon$

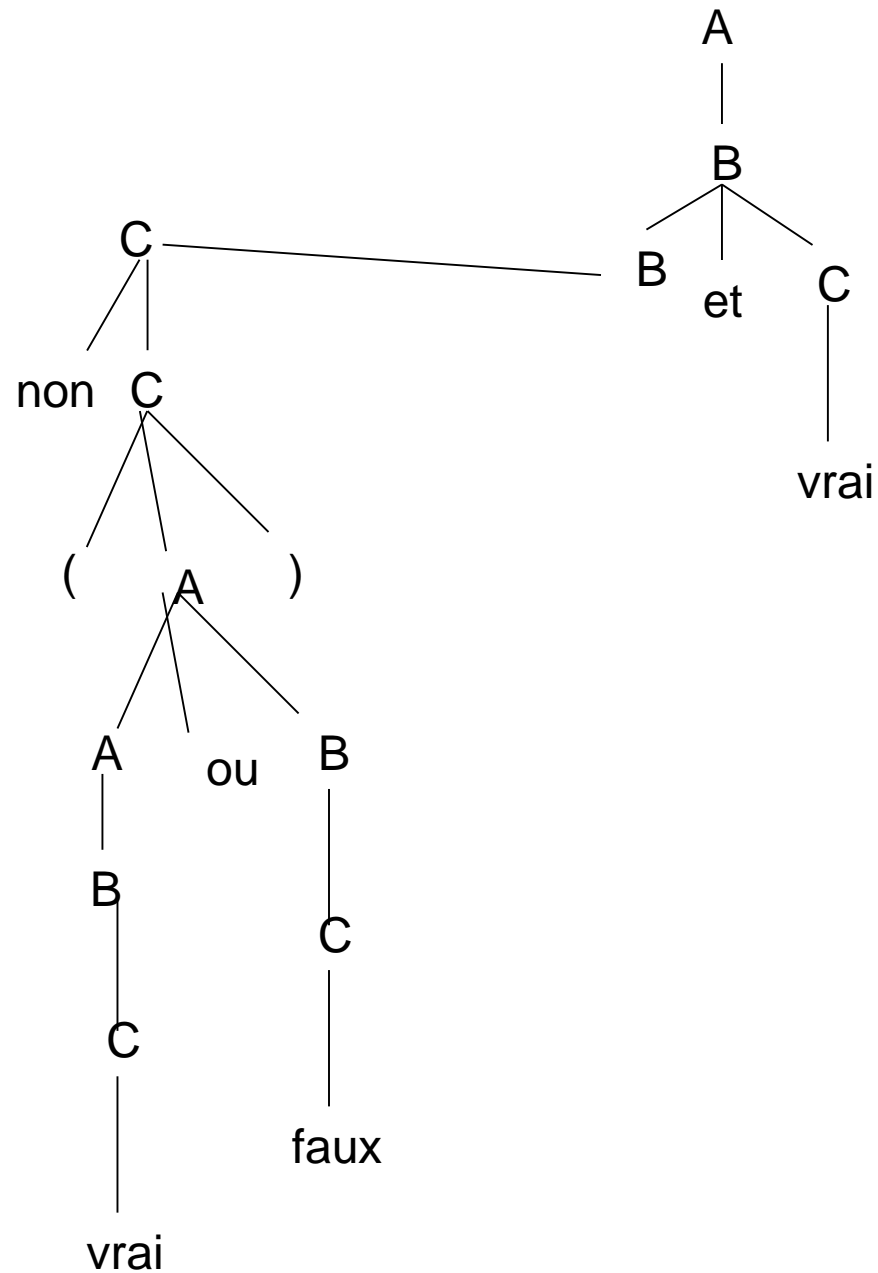
7.

(a)



7.

(a)



(b)

$A \rightarrow B A1$

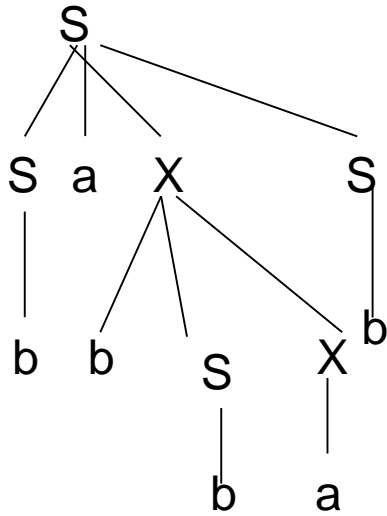
$A1 \rightarrow \text{ou } B A1 \mid \varepsilon$

$B \rightarrow C B1$

$B1 \rightarrow \text{et } C B1 \mid \varepsilon$

Pas de factorisation

8. (a)



(b) Elimination de la récursivité

$S \rightarrow b S1$	1
$S1 \rightarrow a X S S1 \mid \varepsilon$	2 3
$X \rightarrow b S X \mid a \mid b \mid \varepsilon$	4 5 6 7

(b) Elimination de la récursivité

$S \rightarrow b S1$	1
$S1 \rightarrow a X S S1 \mid \varepsilon$	2 3
$X \rightarrow b S X \mid a \mid b \mid \varepsilon$	4 5 6 7

(c) Factorisation : X est une source - 4 et 6

$X \rightarrow b X1 \mid a \mid \varepsilon$	4 5 6
$X1 \rightarrow S X \mid \varepsilon$	7 8

(d) Grammaire propre – intégrer les résultats des  $\varepsilon$  productions dans la grammaire

$S \rightarrow b S1 \mid b$	à cause de la règle 3
$S1 \rightarrow a X S S1 \mid a S S1 \mid a X S \mid a S$	( $S1 \rightarrow \varepsilon$ et $X \rightarrow \varepsilon$ )
$X \rightarrow b X1 \mid a \mid b$	( $X1 \rightarrow \varepsilon$ )
$X1 \rightarrow S X \mid S$	( $X \rightarrow \varepsilon$ )

On recrée des situations d'ambiguïté

## Factorisation

$S \rightarrow b S2$

$S2 \rightarrow S1 \mid \varepsilon$

$S1 \rightarrow a S3 \mid \varepsilon$

$S3 \rightarrow X S S1 \mid S S1 \mid X S \mid S$

$X \rightarrow b X2 \mid a$

$X1 \rightarrow X1 \mid \varepsilon$

$X1 \rightarrow S X2$

$X2 \rightarrow X1 \mid \varepsilon$

$S3 \rightarrow X S S4 \mid S S1 \mid S$

$S4 \rightarrow S1 \mid \varepsilon$

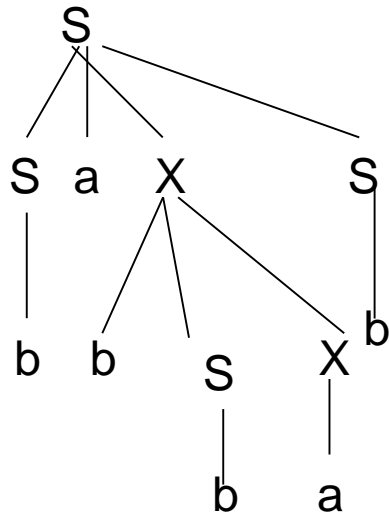
$S3 \rightarrow S S5 \mid X S S4$

$S5 \rightarrow S1 \mid \varepsilon$  La grammaire n'est pas propre

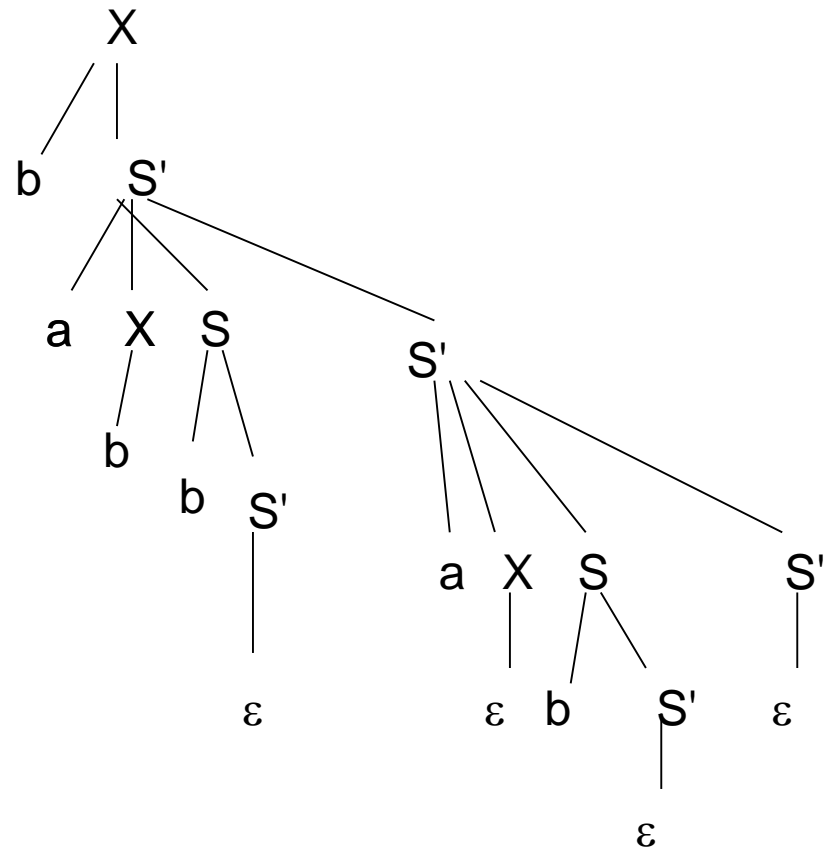
Il faut choisir entre propre et non ambiguë



8. (a)



(e)



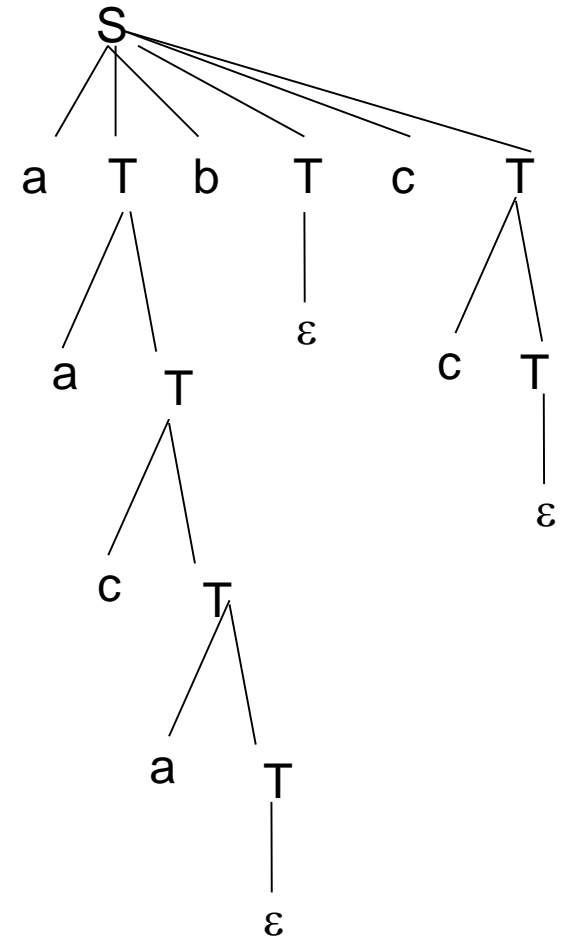
9.(a)

1 2	S →	a T b T c T	a T c T b T
3 4		b T a T c T	b T c T a T
5 6		c T a T b T	c T b T a T
7 8 9 10	T →	a T          b T	c T          ε

9.(a)

1 2	$S \rightarrow$	a T b T c T	a T c T b T
3 4		b T a T c T	b T c T a T
5 6		c T a T b T	c T b T a T
7 8 9 10	$T \rightarrow$	a T          b T	c T          $\epsilon$

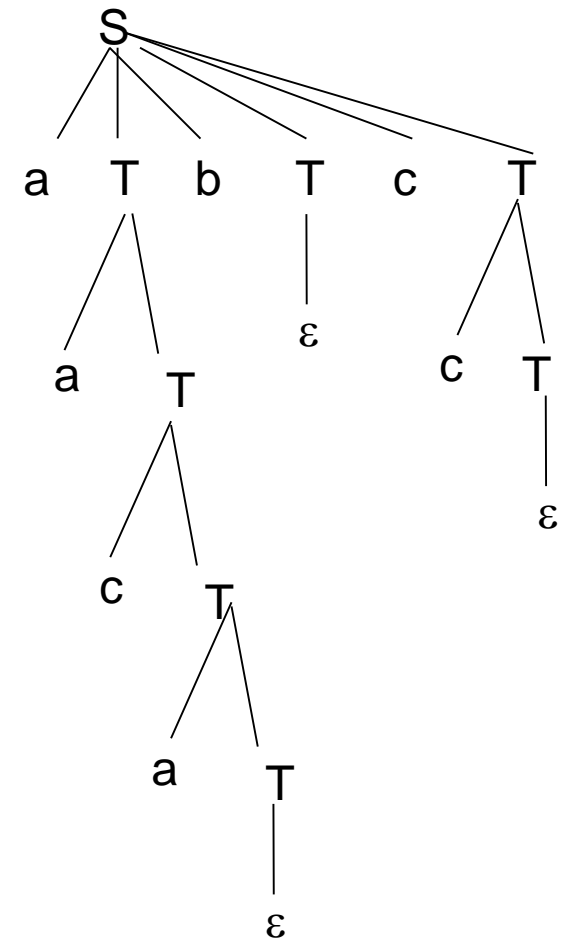
b)



9.(a)

1 2	$S \rightarrow$	$aTbTcT$	$  aTcTbT  $
3 4		$bTaTcT$	$  bTcTaT  $
5 6		$cTaTbT$	$  cTbTaT  $
7 8 9 10	$T \rightarrow$	$aT$	$  bT \quad   cT \quad   \varepsilon$

b)



(c) Grammaire factorisée, non ambiguë

$S$	$\rightarrow aT$	$S1$	$  bTaTcT  $
		$bTcTaT$	$  cTaTbT  $
		$cTbTaT$	
$S1$	$\rightarrow bT$	$cT$	$  cTbT$
$S$	$\rightarrow bT$	$S2$	$  aTS1   cTaTbT  $
		$cTbTaT$	
$S2$	$\rightarrow aT$	$cT$	$  cTaT$
$S$	$\rightarrow cT$	$S3$	$  bTS2   aTS1  $
$S3$	$\rightarrow aT$	$bT$	$  bTaT$
$S$	$\rightarrow aT$	$S1$	$  bT S2 \quad   cT S3$
$S1$	$\rightarrow bT$	$cT$	$  cTbT$
$S2$	$\rightarrow aT$	$cT$	$  cTaT$
$S3$	$\rightarrow aT$	$bT$	$  bTaT$
$T$	$\rightarrow aT$	$  bT \quad   cT \quad   \varepsilon$	