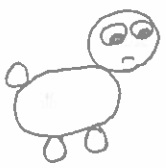
EXPECTIMAX

① Some games involve a random component, like dice-rolling or drawing cards from a deck. Consider blackjack as an example. In casinos, the dealer operates according to a prespecified algorithm, so they don't really count as a "player". Instead, we can tabulate the probabilities of the dealer's final hand:

dealer's final hand	probability
21	12%
20	18%
19	13%
18	14%
17	15%
Bust	28%

the dealer always hits on 16, so can't end up with a final hand ≤16.



② And from this, we can compute the expected payout of our final hand

$(1 \cdot .88) + (0 \cdot .12) + (-1 \cdot 0)$

our final hand	probability of win	probability of draw	probability of loss	expected payout (utility)
21	.88	.12	0	.88
20	.7	.18	.12	.58
19	.57	.13	.3	.27
18	.43	.14	.43	0
17	.28	.15	.57	−.29
<17	.28	0	.72	−.44
Bust	0	0	1	−1

EXPECTIMAX

(3) Given these utilities:

our final hand	utility
21	.88
20	.58
19	.27
18	0
17	-.29
<17	-.44
Bust	-1

we can factor out the dealer from our considerations and treat this as a one-player game, where we want to maximize our utility.

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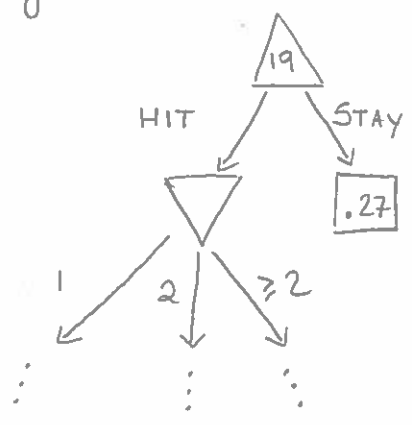
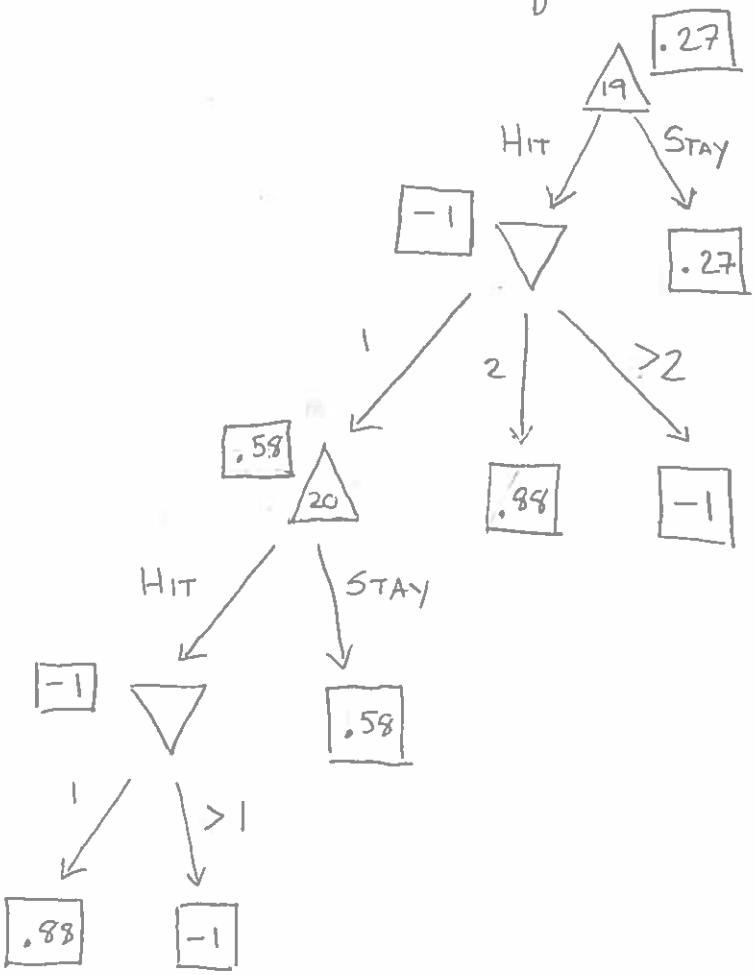
(4) But it's not really a one-player game. It's a two player game, between us and fate.

Suppose we already have a hand valued at 19. We have two available actions: HIT or STAY. If we STAY, then we get a utility of .27. If we HIT, then Fate takes its turn, choosing a card from the deck.



EXPECTIMAX

⑤ We can draw the complete minimax search tree, as usual.



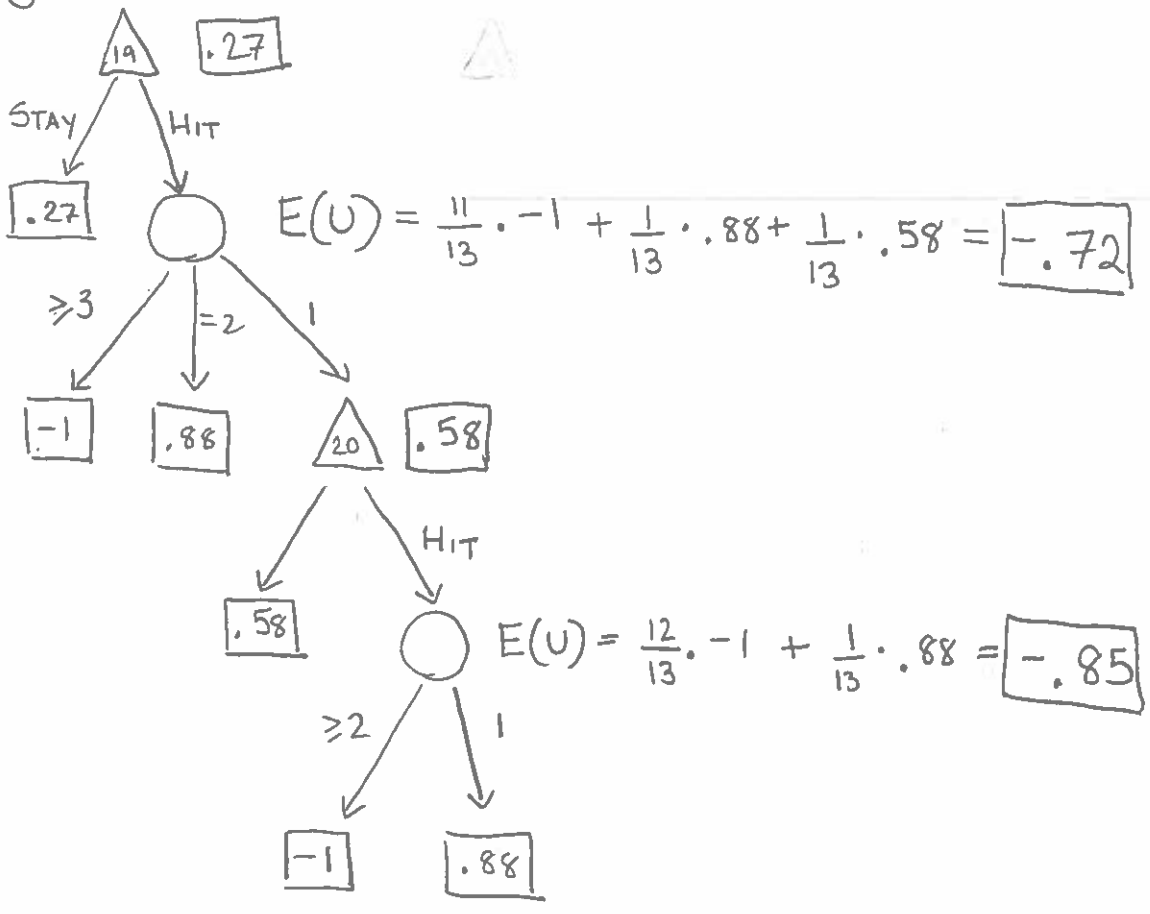
⑥ But this wouldn't be a very interesting game, if Fate played perfectly. We could never HIT if our hand total was 12 or more, because FATE would always choose a 10.

Expectimax

⑦ Fortunately, Fate doesn't play perfectly. Rather than minimizing our utility, it randomly chooses its next move.

This variant of minimax, in which our opponent plays randomly, is called expectimax.

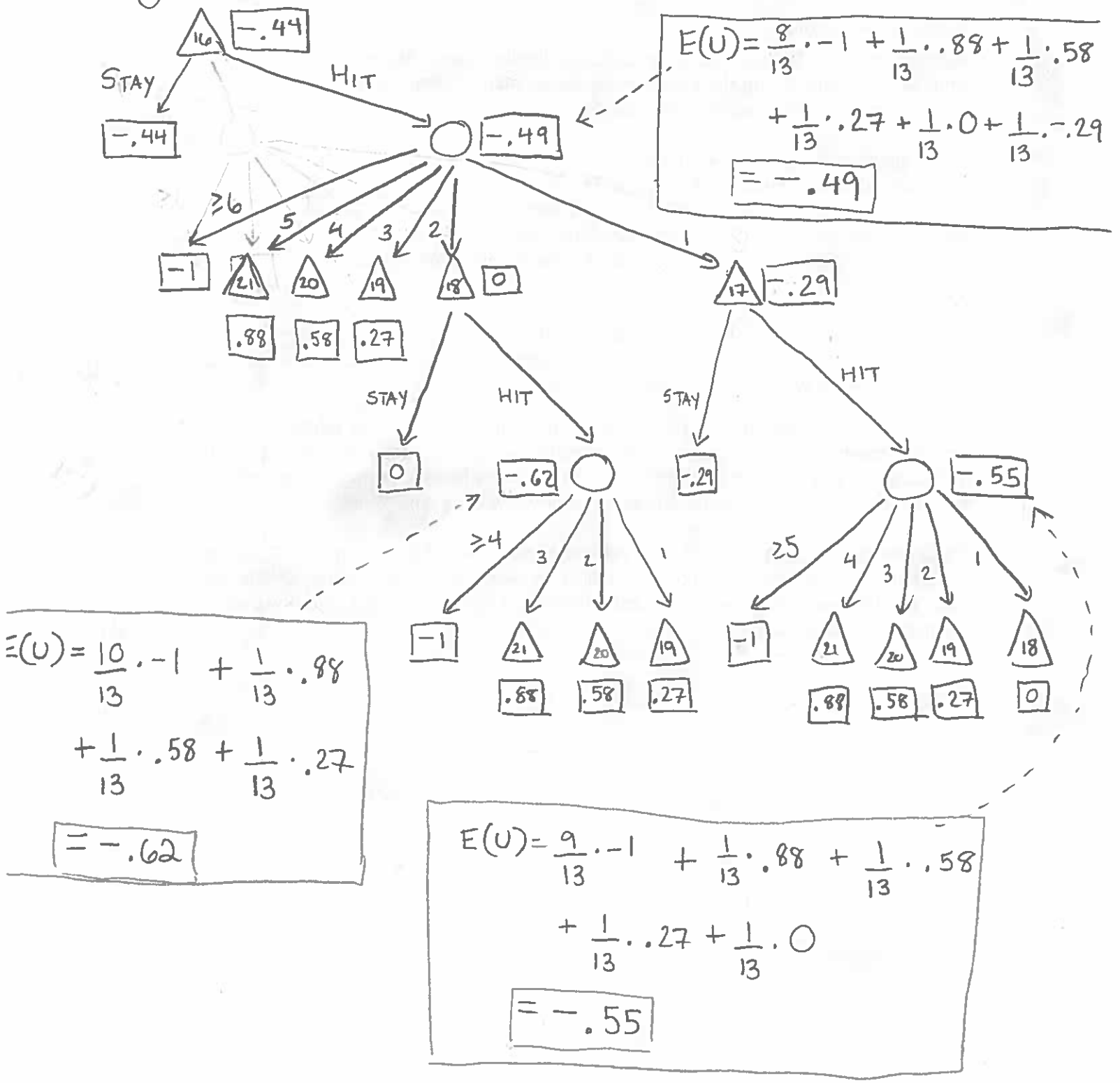
⑧ The change to minimax is small. Instead of our opponent choosing the minimum utility of its successors, it computes the expected utility of its successors according to a probability distribution over its actions.



$E(U) = \frac{11}{13} \cdot -1 + \frac{1}{13} \cdot .88 + \frac{1}{13} \cdot .58 = \boxed{-.72}$

$E(U) = \frac{12}{13} \cdot -1 + \frac{1}{13} \cdot .88 = \boxed{-.85}$

⑨ So if we have 19, then the expected utility if we STAY is .27. The expected utility if we HIT is −.72. Clearly we should STAY. What if we have 16?



$$E(U) = \frac{8}{13}\cdot -1 + \frac{1}{13}\cdot .88 + \frac{1}{13}\cdot .58 + \frac{1}{13}\cdot .27 + \frac{1}{13}\cdot 0 + \frac{1}{13}\cdot -.29 = -.49$$

$$E(U) = \frac{10}{13}\cdot -1 + \frac{1}{13}\cdot .88 + \frac{1}{13}\cdot .58 + \frac{1}{13}\cdot .27 = -.62$$

$$E(U) = \frac{9}{13}\cdot -1 + \frac{1}{13}\cdot .88 + \frac{1}{13}\cdot .58 + \frac{1}{13}\cdot .27 + \frac{1}{13}\cdot 0 = -.55$$

It's a close call, but we should STAY.