Algorithm for Computing SVD (A = U*S*V')

Given

A ~ m x n matrix

Desired

U ~ m x m orthogonal matrix

S ~ m x n rectangular diagonal matrix

V ~ n x n orthogonal matrix

formulas

A'*A = V*(S'*S)*V' [1]

A*V = U*S [2]

A*A' = U*(S*S')*U' [3]

Steps

1/ compute A'*A

2/ the diagonal entries (i.e. singular values) of S are the square roots of the eigen values of A'*A in a descending order, and its off-diagonal entries are all zeros

3/ the columns of V are the normalized eigen vectors associated with the singular values in S (from [1])

4/ the first k columns of U correspond to the first k non-zero singular values in S and satisfy the equation $A*v_i = s_i*u_i$ (from [2])

5/ construct the remaining columns of U such that, they are the solutions of the homogeneous system of equations $(A*A')*u_i = 0$ (from [3])