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Hypothesis testing is a formal statistical method used to determine whether there is enough evidence in a sample to support a specific claim about a population. It functions like a trial where a claim is tested against a standard of "beyond a reasonable doubt" using probability.

1. The Two Hypotheses

Every test involves two mutually exclusive statements:

- **Null Hypothesis (H_0):** The default assumption that there is no effect, no difference, or no relationship.
- **Alternative Hypothesis (H_1 or H_a):** The research claim you want to prove, suggesting a real effect or difference exists.

2. Standard Procedure (The 5 Steps)

Ask anything

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2. Standard Procedure (The 5 Steps)

1. **State the Hypotheses:** Define both H_0 and H_1 clearly.
2. **Choose Significance Level (α):** Set a threshold for "rarity" before seeing the data. The standard is **0.05 (5%)**, meaning you accept a 5% risk of being wrong if you reject the null.
3. **Perform a Statistical Test:** Select a test based on your data (e.g., t-test for small samples, Z-test for large samples, or Chi-square for categories) and calculate a **test statistic**.
4. **Find the p-value:** Calculate the probability of seeing your sample results if the null hypothesis were true. A smaller p-value indicates stronger evidence against H_0.
5. **Make a Decision:**
    1. If **p-value** $< \alpha$: Reject the null

5. **Make a Decision:**
   1. If **p-value** $\leq \alpha$: Reject the null hypothesis (statistically significant).
   2. If **p-value** $> \alpha$: Fail to reject the null hypothesis (not enough evidence).

3. Possible Errors

Because testing relies on samples rather than entire populations, two types of errors can occur:

- **Type I Error (α):** A "false positive"—rejecting a true null hypothesis (e.g., claiming a drug works when it does not).
- **Type II Error (β):** A "false negative"—failing to reject a false null hypothesis (e.g., failing to see that a drug actually works).

4. Directional vs. Non-directional Tests

- **Type II Error (β):** A "false negative"—failing to reject a false null hypothesis (e.g., failing to see that a drug actually works).

4. Directional vs. Non-directional Tests

- **One-Tailed Test:** Used when you are only interested if a parameter is *greater than* or *less than* a value (e.g., "Will this new drug increase lifespan?").
- **Two-Tailed Test:** Used when you are looking for *any* difference, either larger or smaller (e.g., "Is the average height of this group different from the norm?").

What are some real-world examples of hypothesis testing?

1. The Two Hypotheses

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2. Standard Procedure (The 5 Steps)

1. **State the Hypotheses:** Define both H_0 and H_1 clearly.
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Scenario: A researcher claims the average weight of men is greater than 100 kg with a standard deviation (σ) of 15 kg. A random sample of 30 men shows an average weight ($\bar{x}$) of 112.5 kg. Test this claim at a 5% significance level ($\alpha = 0.05$).

- **Step 1: State Hypotheses**
  - $H_0 : \mu = 100$
  - $H_a : \mu > 100$ (Right-tailed test)
- **Step 2: Identify Critical Value**
  - For a 95% confidence level ($\alpha = 0.05$), the critical Z-value from the normal table is **1.645**.
- **Step 3: Calculate Test Statistic (Z)**

$$Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{112.5 - 100}{15/\sqrt{30}} \approx 4.56$$

- **Step 4: Decision**
  - Since $4.56 > 1.645$, the test statistic falls in the rejection region

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Scenario: A manufacturer claims their tire's average life is more than 20,000 km. A sample of 16 tires has a mean ($\bar{x}$) of 22,000 km and a sample standard deviation (s) of 5,000 km. Test at $\alpha = 0.05$.

- **Step 1: State Hypotheses**
  - $H_0 : \mu = 20,000$
  - $H_a : \mu > 20,000$ (Right-tailed test)
- **Step 2: Identify Critical Value**
  - Degrees of Freedom $(df) = n - 1 = 15$.
  - From the T-table for $df = 15$ and $\alpha = 0.05$, the critical t-value is **1.753**.
- **Step 3: Calculate Test Statistic (t)**

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{22,000 - 20,000}{5,000/\sqrt{16}} = \frac{2,000}{1,250} = 1.6$$

- **Step 4: Decision**
  - $1.60 < 1.753$. The test statistic does not fall in the rejection region.

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- **Conclusion:** Fail to reject the null hypothesis. There is not enough significant evidence at the 5% level to prove the mean life exceeds 20,000 km.

Comparison Table for Decision Making

Feature	Z-Test	T-Test
Sample Size (n)	Large ($n \geq 30$)	Small ($n < 30$)
Standard Deviation	Population σ is known	Population σ is unknown
Distribution	Standard Normal (Z)	Student's t

For more practice, you can find detailed

Scenario: A marketing firm claims 50% of first-time borrowers take small loans. A bank samples 100 borrowers and finds 53 have small loans. Test if the percentage differs from 50% at $\alpha = 0.05$.

- **Step 1: Hypotheses**
  - $H_0 : p = 0.50$
  - $H_a : p \neq 0.50$ (Two-tailed)
- **Step 2: Test Statistic (Z)**

$$p' = \frac{53}{100} = 0.53$$

$$Z = \frac{p' - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{0.53 - 0.50}{\sqrt{\frac{0.50 \times 0.50}{100}}} = \frac{0.03}{0.05} = 0.60$$

- **Step 3: Decision**
  - For $\alpha = 0.05$, critical values are ± 1.96.
  - Since $|0.60| < 1.96$, we **fail to reject H_0**.
- **Conclusion:** There is no significant evidence that the proportion of small loans differs from

Scenario: A researcher compares newborn weights between two hospitals. Hospital A has 20 babies with a mean of 7.5 lbs ($s = 0.8$). Hospital B has 20 babies with a mean of 7.1 lbs ($s = 1.2$). Test for a difference at $\alpha = 0.05$.

- **Step 1: Hypotheses**
  - $H_0 : \mu_1 = \mu_2$
  - $H_a : \mu_1 \neq \mu_2$
- **Step 2: Test Statistic (t)**

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{7.5 - 7.1}{\sqrt{\frac{0.8^2}{20} + \frac{1.2^2}{20}}} \approx 1.77$$

- **Step 3: Decision**
  - Degrees of Freedom (df) = $20 + 20 - 2 = 38$.
  - Critical t-value ($df = 38, \alpha = 0.05$) is **2.024**.
  - Since $1.77 < 2.024$, we **fail to reject H_0**.

Scenario: A study examines if gender and voting preference are independent. A sample of 1,000 voters is taken. Calculated $\chi^2 = 16.2$ with 2 degrees of freedom at $\alpha = 0.05$.

- **Step 1: Hypotheses**
  - H_0: Gender and voting preference are independent.
  - H_a: They are not independent.
- **Step 2: Identify Critical Value**
  - For $df = 2$ and $\alpha = 0.05$, the critical χ^2 value is **5.99**.
- **Step 3: Decision**
  - Since $16.2 > 5.99$, we **reject H_0**.
- **Conclusion:** Gender and voting preference are significantly related.

Summary of Critical Values (at $\alpha = 0.05$)

- For $df = 2$ and $\alpha = 0.05$, the critical χ^2 value is **5.99**.

- **Step 3: Decision**
  - Since $16.2 > 5.99$, we **reject** $\boldsymbol{H_0}$.
- **Conclusion:** Gender and voting preference are significantly related.

Summary of Critical Values (at $\alpha = 0.05$)

Test Type	Critical Value(s)
Z-test (One-tailed)	$+1.645$ or -1.645
Z-test (Two-tailed)	± 1.96
Chi-Square ($df = 1$)	3.84
Chi-Square ($df = 2$)	5.99

Tricky Element: Finding the probability of failing to reject a false null hypothesis when given a specific alternative value.

Question: A machine is designed to produce rods with a mean diameter of 10 mm and $\sigma = 0.2$ mm. You test $H_0 : \mu = 10$ vs $H_a : \mu > 10$ using a sample of $n = 25$ and $\alpha = 0.05$. If the true mean has actually shifted to 10.1 mm, find the **Power of the Test**.

- **Step 1: Find Critical Value ($\bar{x}_c$)**
  - For $\alpha = 0.05$, $Z_{0.05} = 1.645$.
  - $\bar{x}_c = \mu_0 + Z_\alpha \cdot \left(\frac{\sigma}{\sqrt{n}}\right) = 10 + 1.645 \cdot \left(\frac{0.2}{5}\right)$

    .
- **Step 2: Calculate β (Type II Error)**
  - $\beta = P(\text{fail to reject } H_0 \mid \mu = 10.1) = P(\bar{X} < 10$

    .
  - Standardize using the *new* mean:

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- **Step 2: Calculate β (Type II Error)**
  - $\beta = P(\text{fail to reject } H_0 \mid \mu = 10.1) = P(\bar{X} < 10.0658 \mid \mu = 10.1)$.
  - Standardize using the *new* mean: $Z = \dfrac{10.0658 - 10.1}{0.2/5} = \dfrac{-0.0342}{0.04} = -0.855$.

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- **Step 2: Calculate β (Type II Error)**
  - $\beta = P(\text{fail to reject } H_0 \mid \mu = 10.1) = P(\bar{X} < 10$
  .
  - Standardize using the *new* mean:
  $$Z = \frac{10.0658 - 10.1}{0.2/5} = \frac{-0.0342}{0.04} = -0.855$$
  .
  - From Z-table, $P(Z < -0.855) \approx 0.1963$.
- **Step 3: Power of Test**
  - $\text{Power} = 1 - \beta = 1 - 0.1963 = \mathbf{0.8037}$ (or 80.37%).

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2. Decision Making via *p*-value (Numerical/NAT)

Tricky Element: Calculating the exact p-value from a test statistic and comparing it to α.

2. Decision Making via p-value (Numerical/NAT)

Tricky Element: Calculating the exact p-value from a test statistic and comparing it to α.

Question: A data scientist is testing if a new recommendation algorithm improves the Click-Through Rate (CTR) from the baseline of 0.12. In a sample of 400 users, the CTR is 0.14. Calculate the p-value for a right-tailed test and decide if the result is significant at $\alpha = 0.05$.

- **Step 1: Calculate Z-statistic for Proportion**
  - $p_0 = 0.12, \hat{p} = 0.14, n = 400.$
  - $Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.14 - 0.12}{\sqrt{\frac{0.12 \times 0.88}{400}}} = \frac{0.02}{0.01625} \approx$

    .
- **Step 2: Find p-value**
  - $p\text{-value} = P(Z > 1.23) = 1 - P(Z < 1.23) =$

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Question: A data scientist is testing if a new recommendation algorithm improves the Click-Through Rate (CTR) from the baseline of 0.12. In a sample of 400 users, the CTR is 0.14. Calculate the p-value for a right-tailed test and decide if the result is significant at $\alpha = 0.05$.

- **Step 1: Calculate Z-statistic for Proportion**
  - $p_0 = 0.12, \hat{p} = 0.14, n = 400.$
  - $Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.14 - 0.12}{\sqrt{\frac{0.12\times0.88}{400}}} = \frac{0.02}{0.01625} \approx 1$

    .
- **Step 2: Find p-value**
  - $p\text{-value} = P(Z > 1.23) = 1 - P(Z < 1.23) = 1$

    .
- **Decision:** Since $0.1093 > 0.05$, **fail to reject H_0**. The improvement is not statistically significant.

Ask anything

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Question: Suppose you are conducting a hypothesis test. Which of the following statements are **correct** for a fixed significance level α?

1. Increasing the sample size (n) will always decrease the probability of a Type II error (β).
2. If α is decreased, β will automatically decrease. (False: They have an inverse relationship).
3. The power of the test ($1-\beta$) increases as the true population mean moves further away from the null hypothesis mean.
4. A p-value of 0.03 is significant at both 1% and 5% levels. (False: Only at 5%).

Answers: 1 and 3 are correct.

Core GATE DA 2026 Focus Areas

Topic	Critical Insight

1. Poisson Distribution: The Moment Trick

Question: The second moment of a Poisson-distributed random variable X is 2. Find the mean (λ) of the random variable.

- **Step 1: Use the Second Moment Formula**
  The second moment is given by
  $E[X^2] = Var(X) + (E[X])^2$.
- **Step 2: Apply Poisson Properties**
  For a Poisson distribution, Mean = Variance = λ.
  $E[X^2] = \lambda + \lambda^2$
- **Step 3: Solve for $\boldsymbol{\lambda}$**
  $2 = \lambda + \lambda^2 \implies \lambda^2 + \lambda - 2 = 0$
  Factoring the quadratic: $(\lambda + 2)(\lambda - 1) = 0$.
  Since $\lambda > 0$, the mean is $\boldsymbol{\lambda = 1}$.

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2. Exponential Distribution:

2. Exponential Distribution: Memoryless Property

Question: The time to repair a machine follows an exponential distribution with a mean of 2 hours. If a repair has already taken 8 hours, what is the probability that the total repair time exceeds 11 hours?

- **Step 1: Identify the Parameter**
  Mean $E[X] = 1/\lambda = 2 \implies \lambda = 0.5$.
- **Step 2: Apply the Memoryless Property**
  The memoryless property states
  $P(X > s + t \mid X > s) = P(X > t)$.
  Here, $s = 8$ and $s + t = 11$, so $t = 3$.
  $P(X > 11 \mid X > 8) = P(X > 3)$
- **Step 3: Calculate the Result**
  For exponential distribution,
  $P(X > x) = e^{-\lambda x}$.
  $P(X > 3) = e^{-0.5 \times 3} = e^{-1.5} \approx \mathbf{0.2231}$.

3. Normal Distribution: Standardizing & Area

Question: If X is a normal variable with mean $\mu = 30$ and $\sigma = 5$, find $P(26 \leq X \leq 34)$. You are given that the area under the standard normal curve for $Z = 0.8$ is 0.2881 (from 0 to Z).

- **Step 1: Standardize the Bounds**
  Calculate Z for $x = 26$ and $x = 34$.
  $$Z_1 = \frac{26 - 30}{5} = -0.8, \quad Z_2 = \frac{34 - 30}{5} = 0.8$$
- **Step 2: Find the Probability**
  $P(-0.8 \leq Z \leq 0.8)$ is the area from -0.8 to 0.8.
  Because the normal curve is symmetric, this is $2 \times P(0 \leq Z \leq 0.8)$.
  $2 \times 0.2881 = \mathbf{0.5762}$.

---

4. Binomial Distribution: Finding

4. Binomial Distribution: Finding Parameters

Question: A random variable X follows a Binomial distribution with expectation $E[X] = 7$ and variance $Var(X) = 6$. Find the probability of success p.

- **Step 1: Use Mean and Variance Formulas**
  $E[X] = np = 7$
  $Var(X) = np(1 - p) = 6$
- **Step 2: Substitute np into the Variance Equation**
  $7(1 - p) = 6$
  $1 - p = \frac{6}{7}$
- **Step 3: Solve for p**
  $p = 1 - \frac{6}{7} = \mathbf{\frac{1}{7}}.$

Quick GATE DA 2026 Reference Table

Question: A server's downtime follows an exponential distribution with mean $\mu = 10$ minutes. If the server has already been down for 5 minutes, what is the expected **total** downtime?

- **The Trap:** Many students calculate $E[X|X > 5]$ as simply 10 or 15 without applying the memoryless property correctly.
- **The Logic:** By the memoryless property, the *additional* time $Y = X - 5$ given $X > 5$ still follows the original distribution $Exp(\lambda)$.
- **Calculation:**
  - $E[X - 5|X > 5] = E[X] = 10.$
  - Total Downtime = Time already spent + Expected additional time = $5 + 10 = \mathbf{15}$ minutes.

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2. Hypothesis Testing: The "Power"

Challenge

Question: You are testing $H_0 : \mu = 50$ vs $H_a : \mu = 55$ for a normal population with $\sigma = 10$ and $n = 100$. If you set your critical value $\bar{x}_c = 52$, what is the **Power of the Test**?

- **Step 1: Understand Power.** Power is the probability of rejecting H_0 when H_a is actually true.
  - $\text{Power} = P(\bar{X} > 52 \mid \mu = 55)$.
- **Step 2: Standardize under H_a.**
  - Standard Error (SE) = $\sigma/\sqrt{n} = 10/\sqrt{100} = 1$.
  - $Z = \dfrac{52 - 55}{1} = -3.0$.
- **Step 3: Find Area.**
  - $P(Z > -3.0) = P(Z < 3.0) \approx \mathbf{0.9987}$.
- **Tricky Part:** Note how a small change in $\bar{x}$...

Question: Two independent processes, A and B, follow Poisson distributions. Process A has an average of 2 arrivals/hour, and B has 3 arrivals/hour. What is the probability that in a given hour, the **total** arrivals from both processes is exactly 1?

- **The Logic:** The sum of two independent Poisson variables $X \sim \text{Pois}(\lambda_A)$ and $Y \sim \text{Pois}(\lambda_B)$ is also Poisson: $Z = (X + Y) \sim \text{Pois}(\lambda_A + \lambda_B)$.
- **Calculation:**
  - $\lambda_{total} = 2 + 3 = 5$.
  - $P(Z = 1) = \frac{e^{-5} \cdot 5^1}{1!} = 5 \cdot e^{-5} \approx \mathbf{0.0337}$.

---

4. Uniform Distribution: Probability of Range

4. Uniform Distribution: Probability of Range

Question: X is uniformly distributed over $[0, 10]$. Let $Y = X^2$. Find $P(Y < 25)$.

- **Step 1: Relate Y to X.**
  - $P(Y < 25) = P(X^2 < 25) = P(-5 < X < 5)$
    .
- **Step 2: Account for the Support.**
  - Since X is only defined on $[0, 10]$, the interval becomes $[0, 5)$.
- **Step 3: Calculate PDF-based Probability.**
  - $f(x) = 1/10$ for $0 \leq x \leq 10$.
  - $P(0 \leq X < 5) = \int_0^5 \frac{1}{10}\, dx = \frac{5-0}{10} = \mathbf{0.5}$
    .

GATE DA 2026 Strategy Tip

1. Z-Score Calculation (Normal Distribution)

Question: A student scores **85** on an exam where the mean is **70** and the standard deviation is **10**. What is the student's Z-score?

- **Formula:** $Z = \frac{x - \mu}{\sigma}$
- **Calculation:** $Z = \frac{85 - 70}{10} = \frac{15}{10} = \mathbf{1.5}$
- **Interpretation:** The score is 1.5 standard deviations above the mean.

---

2. Standard Error of the Mean

Question: A population has a standard deviation (σ) of **20**. If you take a random sample of size $n = 100$, what is the standard error of the sample mean?

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Question: A population has a standard deviation (σ) of **20**. If you take a random sample of size $n = 100$, what is the standard error of the sample mean?

- **Formula:** $SE = \frac{\sigma}{\sqrt{n}}$
- **Calculation:**

$$SE = \frac{20}{\sqrt{100}} = \frac{20}{10} = \mathbf{2.0}$$

---

3. Poisson Probability (Direct)

Question: On average, a website receives **3** hits per minute. What is the probability that it receives exactly **0** hits in a given minute?

- **Formula:** $P(X = k) = \frac{e^{-\lambda} \cdot \lambda^k}{k!}$

3. Poisson Probability (Direct)

Question: On average, a website receives **3** hits per minute. What is the probability that it receives exactly **0** hits in a given minute?

- **Formula:** $P(X = k) = \dfrac{e^{-\lambda} \cdot \lambda^k}{k!}$
- **Parameters:** $\lambda = 3, k = 0.$
- **Calculation:**

$$P(X = 0) = \frac{e^{-3} \cdot 3^0}{0!} = e^{-3} \approx \mathbf{0.0498}$$

---

4. Confidence Interval (Z-interval)

Question: A sample of **100** items has a mean weight of **50 kg**. The population standard deviation is **5 kg**. Find the **95% Confidence Interval** for the population mean. (Use $Z_{0.95} = 1.96$)

- **Formula:** $CI = \bar{x} \pm Z \cdot \left(\frac{\sigma}{\sqrt{n}}\right)$
- **Calculation:**

$$Margin of Error = 1.96 \cdot \left(\frac{5}{\sqrt{100}}\right) = 1.96 \cdot 0.5$$

- **Result:** $50 \pm 0.98 = [\mathbf{49.02, 50.98}]$

---

5. Binomial Mean and Variance

Question: If a fair coin is tossed **100** times, find the mean and variance of the number of heads.

- **Parameters:** $n = 100, p = 0.5.$
- **Mean (np):** $100 \times 0.5 = \mathbf{50}$
- **Variance (npq):** $100 \times 0.5 \times 0.5 = \mathbf{25}$

Quick Formula Sheet for 2026

5. Binomial Mean and Variance

Question: If a fair coin is tossed **100** times, find the mean and variance of the number of heads.

- **Parameters:** $n = 100, p = 0.5.$
- **Mean (np):** $100 \times 0.5 = \mathbf{50}$
- **Variance (npq):** $100 \times 0.5 \times 0.5 = \mathbf{25}$

Quick Formula Sheet for 2026

Distribution	Mean	Variance
Uniform (a, b)	$(a + b)/2$	$(b - a)^2/12$
Exponential (λ)	$1/\lambda$	$1/\lambda^2$
Bernoulli (p)	p	$p(1 - p)$

For more direct practice, check the Statology

(CDF).

1. Discrete Case: Cumulative Mass Function (CMF)

Question: A discrete random variable X has the following probability mass function (PMF):
$P(X = 1) = 0.2$, $P(X = 2) = 0.5$, and
$P(X = 3) = 0.3$.
Find the value of the CDF, $F(x)$, at $x = 2.5$.

- **Logic:** The CDF $F(x)$ is defined as $P(X \leq x)$. For a discrete variable, it is the sum of all probabilities for values less than or equal to x.
- **Calculation:**
  $F(2.5) = P(X \leq 2.5) = P(X = 1) + P(X = 2)$
  $F(2.5) = 0.2 + 0.5 = \mathbf{0.7}$
- **Note:** Even though 2.5 is not a "point" in the distribution, the CDF is defined for all real numbers and stays constant (flat) between jumps.

2. Continuous Case: Finding the Constant (k)

Question: A continuous random variable X has a PDF $f(x) = kx^2$ for $0 \le x \le 3$, and 0 otherwise. Find the constant k and the CDF $F(x)$.

- **Step 1: Find k (Total Area = 1)**

$$\int_0^3 kx^2\,dx = 1 \implies k\left[\frac{x^3}{3}\right]_0^3 = 1$$

$$k\left(\frac{27}{3}\right) = 1 \implies 9k = 1 \implies \mathbf{k = 1/9}$$

- **Step 2: Find the CDF $F(x)$**

$$F(x) = \int_0^x f(t)\,dt = \int_0^x \frac{1}{9}t^2\,dt$$

$$F(x) = \frac{1}{9}\left[\frac{t^3}{3}\right]_0^x = \frac{\mathbf{x^3}}{\mathbf{27}} \text{ for } 0 \le x \le 3$$

---

3. Tricky Numerical: PDF from CDF

Question: The CDF of a continuous random variable is given by:
$F(x) = 0$ for $x < 0$
$F(x) = 1 - e^{-2x}$ for $x \geq 0$
Find the PDF $f(x)$ at $x = 0.5$.

- **Logic:** The PDF is the derivative of the CDF:
  $$f(x) = \frac{d}{dx}F(x).$$
- **Step 1: Differentiate $F(x)$**
  $$f(x) = \frac{d}{dx}(1 - e^{-2x}) = 0 - (-2)e^{-2x} = 2e^{-2x}$$
- **Step 2: Substitute $x = 0.5$**
  $$f(0.5) = 2e^{-2(0.5)} = 2e^{-1} \approx 2 \times 0.3678 = \mathbf{0.7356}$$

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4. Direct Calculation: $P(a < X < b)$ using CDF

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4. Direct Calculation: $P(a < X < b)$ using CDF

Question: Let X have a CDF $F(x) = \frac{x^2}{16}$ for $0 \le x \le 4$. Find $P(1 < X < 3)$.

- **Logic:** For any continuous distribution, $P(a < X < b) = F(b) - F(a)$.
- **Calculation:**
  $P(1 < X < 3) = F(3) - F(1)$
  $F(3) = \frac{3^2}{16} = \frac{9}{16}$
  $F(1) = \frac{1^2}{16} = \frac{1}{16}$
  $Result = \frac{9}{16} - \frac{1}{16} = \frac{8}{16} = \mathbf{0.5}$

Summary Table for 2026 Revision

Feature	Discrete (PMF/CMF)	Cont (PDF

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$$F(1) = \frac{1^2}{16} = \frac{1}{16}$$

$$Result = \frac{9}{16} - \frac{1}{16} = \frac{8}{16} = \mathbf{0.5}$$

Summary Table for 2026 Revision

Feature	Discrete (PMF/CMF)	Cont (PDF
Relationship	$F(x) = \sum_{x_i \le x} P(X = x_i)$	$F(x)$
Recovery	$P(X = x_i) = F(x_i) - F(x_{i-1})$	$f(x)$
Total Sum/Area	$\sum P(x) = 1$	$\int f$

For interactive visualizations of these distributions, you can explore the Probability Distributions Applet.

$$F(1) = \frac{1^2}{16} = \frac{1}{16}$$

$$Result = \frac{9}{16} - \frac{1}{16} = \frac{8}{16} = \mathbf{0.5}$$

Summary Table for 2026 Revision

Feature	MF/CMF)	Continuous (PDF/CDF)
Relationship	$P(X = x_i)$	$F(x) = \int_{-\infty}^{x} f(t)dt$
Recovery	$= F(x_i) - F(x_{i-1})$	$f(x) = \frac{d}{dx} F(x)$
Total Sum/Area	1	$\int f(x)dx = 1$

For interactive visualizations of these distributions, you can explore the Probability Distributions Applet.

Question: A data center has two types of servers. Type A servers (60% of stock) have a lifetime following an Exponential distribution with mean 100 days. Type B servers (40% of stock) have a mean lifetime of 150 days. If a server is picked at random, what is the **variance** of its lifetime?

- **The Trap:** Variance is NOT the weighted average of the variances. You must use the **Law of Total Variance**: $Var(X) = E[Var(X|I)] + Var(E[X|I])$.
- **Step 1: Find $E[X]$ (Law of Total Expectation)**
  - $E[X] = (0.6 \times 100) + (0.4 \times 150) = 60 + 60$ days.
- **Step 2: Find $E[X^2]$**
  - For Exponential, $E[X^2] = 2/\lambda^2 = 2(\mu^2)$.
  - Type A: $2(100^2) = 20,000$.
  - Type B: $2(150^2) = 45,000$.

- **Step 2: Find $E[X^2]$**
  - For Exponential, $E[X^2] = 2/\lambda^2 = 2(\mu^2)$.
  - Type A: $2(100^2) = 20,000$.
  - Type B: $2(150^2) = 45,000$.
  - $E[X^2] = (0.6 \times 20,000) + (0.4 \times 45,000) =$
    .
- **Step 3: Calculate Variance**
  - $Var(X) = E[X^2] - (E[X])^2 = 30,000 - (12$
    .

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2. Hypothesis Testing: The p-value NAT Challenge

Question: You are testing $H_0 : \mu = 50$ against $H_a : \mu \neq 50$. The sample size is $n = 1$, and the single observation X comes from a **Uniform distribution** $U[0, \theta]$. Under H_0, $\theta = 100$. If you observe $X = 95$, what is the p-value?

- **Tricky Element:** This is a non-normal distribution test.
- **Logic:** The p-value is the probability of seeing a result "at least as extreme" as 95 under H_0.
  - In $U[0, 100]$, "extreme" means values far from the center (50).
  - The distance of 95 from 50 is $|95 - 50| = 45$.
  - Values as extreme as 95 are $X \geq 95$ or $X \leq 5$ (since $|5 - 50| = 45$).
- **Calculation:**
  - $P(X \geq 95) = (100 - 95)/100 = 0.05$.
  - $P(X \leq 5) = (5 - 0)/100 = 0.05$.
  - $p\text{-value} = 0.05 + 0.05 = \mathbf{0.10}$.

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3. Joint Distributions: Probability of a

3. Joint Distributions: Probability of a Region

Question: Let X and Y be independent random variables, both uniformly distributed on $[0, 1]$. Find $P(X + Y < 0.5)$.

- **The Trap:** This requires geometric probability or double integration.
- **Logic:** Since $X, Y \sim U[0, 1]$, the joint PDF $f(x, y) = 1$ over a 1×1 square in the xy-plane.
- **Geometric Approach:** The region $X + Y < 0.5$ is a triangle with vertices at $(0, 0)$, $(0.5, 0)$, and $(0, 0.5)$.
- **Calculation:**
  - Area of Triangle = $1/2 \times \text{base} \times \text{height}$
  - $\text{Area} = 1/2 \times 0.5 \times 0.5 = 0.125$.
  - Since total area of the square is 1, $P = \mathbf{0.125}$ (or $1/8$).

Question: The MGF of a random variable X is given by $M_X(t) = \exp(3t + 8t^2)$. Find the mean (μ) and variance (σ^2) of X.

- **Step 1: Identify the Distribution.**
  The MGF of a Normal Distribution $N(\mu, \sigma^2)$ is $e^{\mu t + \frac{1}{2}\sigma^2 t^2}$.
- **Step 2: Compare Terms.**
  - $\mu t = 3t \implies \mu = 3$.
  - $\frac{1}{2}\sigma^2 t^2 = 8t^2 \implies \sigma^2 = 16$.
- **Result:** Mean = **3**, Variance = **16**.

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2. MGF of a Linear Transformation

Question: Let X be a random variable with MGF $M_X(t)$. Define $Y = 2X + 5$. Express $M_Y(t)$ in terms of $M_X(t)$.

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Question: Let X be a random variable with MGF $M_X(t)$. Define $Y = 2X + 5$. Express $M_Y(t)$ in terms of $M_X(t)$.

- **Formula:** $M_{aX+b}(t) = e^{bt} \cdot M_X(at)$.
- **Application:**
  - Here $a = 2$ and $b = 5$.
  - $M_Y(t) = e^{5t} \cdot M_X(2t)$.

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3. Tricky Numerical: Series Expansion

Question: The MGF of a discrete random variable X is $M_X(t) = \frac{1}{4} + \frac{1}{2}e^t + \frac{1}{4}e^{2t}$. Find $P(X = 1)$.

- **Logic:** The MGF of a discrete variable is
$M_X(t) = \sum P(X = x_i)e^{tx_i}$

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3. Tricky Numerical: Series Expansion

Question: The MGF of a discrete random variable X is $M_X(t) = \frac{1}{4} + \frac{1}{2}e^t + \frac{1}{4}e^{2t}$. Find $P(X = 1)$.

- **Logic:** The MGF of a discrete variable is $M_X(t) = \sum P(X = x_i)e^{tx_i}$.
- **Expansion:**
  - $M_X(t) = (0.25)e^{t(0)} + (0.50)e^{t(1)} + (0.25)e^{t(2)}$
- **Compare Coefficients:**
  - The coefficient of e^{1t} is $P(X = 1)$.
- **Result:** $P(X = 1) = \mathbf{0.5}$.

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4. Advanced: Sum of Independent Variables

Ask anything

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4. Advanced: Sum of Independent Variables

Question: X and Y are independent Poisson random variables with $\lambda_1 = 2$ and $\lambda_2 = 3$. Find the MGF of $Z = X + Y$.

- **Logic:** For independent variables, $M_{X+Y}(t) = M_X(t) \cdot M_Y(t)$.
- **Poisson MGF Formula:** $M(t) = \exp(\lambda(e^t - 1))$.
- **Calculation:**
  - $M_Z(t) = \exp(2(e^t - 1)) \cdot \exp(3(e^t - 1))$
  - $M_Z(t) = \exp((2+3)(e^t - 1)) = \exp(\mathbf{5(e^t - 1)}$

    .
- **Conclusion:** Z follows a Poisson distribution with $\lambda = 5$.

5. Finding the Second Moment ($E[X^2]$)

Question: If $M_X(t) = (1 - 2t)^{-3}$, find $E[X^2]$. (This is a Gamma Distribution form).

- **Formula:** $E[X^n] = M_X^{(n)}(0)$ (The n-th derivative at $t = 0$).
- **First Derivative:**
  - $M'_X(t) = -3(1 - 2t)^{-4}(-2) = 6(1 - 2t)^{-4}$.
- **Second Derivative:**
  - $M''_X(t) = 6(-4)(1 - 2t)^{-5}(-2) = 48(1 - 2t)^{-5}$.
- **Evaluate at $t = 0$:**
  - $E[X^2] = 48(1 - 0)^{-5} = \mathbf{48}$.

2026 GATE DA Exam Tip:

If you see an MGF that looks like a fraction (e.g., $\frac{p}{}$), it's likely **Geometric**. If it's a power of