Departamento de Ciencias Exactas
Area de Analisis Funcional
EDO

Actividad de aprendizaje S13

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Ecuaciones Diferenciales Ordinarias

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Departamento de Ciencias Exactas - ESPE

1. Para la ecuación $x^3y'' + 4x^2y' + 3y = 0$, $x_0 = 0$ es un punto singular regular? Justifique

$$x^3y'' + 4x^2y' + 3y = 0$$

$$y'' + \frac{4x^2}{x^3}y' + \frac{3y}{x^3} = 0$$

$$y'' + \frac{4}{x}y' + \frac{3y}{x^3} = 0$$

$$\lim_{x\to 0} x\left(\frac{4}{x}\right) = \lim_{x\to 0} 4 = 4 \quad \exists$$

$$\lim_{x\to 0} x^2\left(\frac{3}{x^3}\right) = \lim_{x\to 0} \frac{3}{x} = \infty \quad \nexists$$

$\rightarrow$ $x=0$ es un punto singular irregular //

Determine la solución de $xy'' + y' - 4y = 0$ en el punto $x_0 = 0$.

$$xy'' + y' - 4y = 0$$

$$y = \sum_{n=0}^{\infty} C_n (x - x_0)^{n+r} \qquad x_0 = 0$$

$$\rightarrow \quad y = \sum_{n=0}^{\infty} C_n x^{n+r}$$

$$y' = \sum_{n=0}^{\infty} (n+r) C_n x^{n+r-1}$$

$$y'' = \sum_{n=0}^{\infty} (n+r-1)(n+r) C_n x^{n+r-2}$$

Reemplazando

$$x \sum_{n=0}^{\infty} (n+r-1)(n+r) C_n x^{n+r-2} + \sum_{n=0}^{\infty} (n+r) C_n x^{n+r-1} - 4 \sum_{n=0}^{\infty} C_n x^{n+r} = 0$$

$$\sum_{n=0}^{\infty} (n+r-1)(n+r) C_n x^{n+r-1} + \sum_{n=0}^{\infty} (n+r) C_n x^{n+r-1} - 4 \sum_{n=0} C_n x^{n+r} = 0$$

$$x^r \left[\sum_{n=0}^{\infty} (n+r-1)(n+r) C_n x^{n-1} + \sum_{n=0}^{\infty} (n+r) C_n x^{n-1} - 4 \sum_{n=0}^{\infty} C_n x^n \right] = 0$$

$$\sum_{n=0}^{\infty} \left[(n+r-1)(n+r) + (n+r) \right] C_n x^{n-1} - 4 \sum_{n=0}^{\infty} C_n x^n = 0$$

$$(n+r)(n+r-1+1) \Rightarrow (n+r)^2$$

$$\sum_{n=0}^{\infty} (n+r)^2 C_n x^{n-1} - 4 \sum_{n=0}^{\infty} C_n x^n = 0$$

$$r^2 C_0 x^{-1} + \sum_{n=1}^{\infty} (n+r)^2 C_n x^{n-1} - 4 \sum_{n=0} C_n x^n = 0 \qquad k = n$$

$$k = n-1 \rightarrow n = k+1 \rightarrow \text{si } n = 1 \rightarrow k = 0$$

$$r^2 C_0 x^{-1} + \sum_{k=0}^{\infty} (k+1+r)^2 C_{k+1} x^k - 4 \sum_{k=0}^{\infty} C_k x^k = 0$$

$$r^2 C_0 x^{-1} = 0$$

$$r^2 = 0 \rightarrow r_1 = r_2 = 0$$

Si $r = 0$

$$\sum_{k=0}^{\infty} (k+1)^2 C_{k+1} x^k - 4 \sum_{k=0}^{\infty} C_k x^k = 0$$

$$\sum_{k=0}^{\infty} \left[(k+1)^2 C_{k+1} - 4 C_k \right] x^k = 0$$

$$(k+1)^2 C_{k+1} - 4C_k = 0$$

$$C_{k+1} = \frac{4C_k}{(k+1)^2}$$

Si $k=0$

$$C_1 = \frac{4C_0}{1} = 4C_0$$

Si $k=1$

$$C_2 = \frac{4C_1}{4} = C_1 = 4C_0$$

Si $k=2$

$$C_3 = \frac{4C_2}{9} = \frac{4}{9}(4C_0) = \frac{16}{9}C_0$$

Si $k=3$

$$C_4 = \frac{4C_3}{16} = \frac{1}{4}\left(\frac{16}{9}C_0\right) = \frac{4}{9}C_0$$

$$y(x) = C_0 + 4C_0 x + 4C_0 x^2 + \frac{16}{9}C_0 x^3 + \frac{4}{9}C_0 x^4 + \dots$$

$$y(x) = C_0\left(1 + 4x + 4x^2 + \frac{16}{9}x^3 + \frac{4}{9}x^4 + \dots\right) //$$

$$x^2y'' + xy' + (x^2 - 4/9)y = 0$$

$$y = \sum_{n=0}^{\infty} C_n x^{n+r}$$

$$y' = \sum_{n=0}^{\infty} (n+r)C_n x^{n+r-1}$$

$$y'' = \sum_{n=0}^{\infty} (n+r)(n+r-1)C_n x^{n+r-2} //$$

Reemplazando

$$x^2\sum_{n=0}^{\infty}(n+r)(n+r-1)C_n x^{n+r-2} + x\sum_{n=0}^{\infty}(n+r)C_n x^{n+r-1} + (x^2-4/9)\sum_{n=0}^{\infty}C_n x^{n+r} = 0$$

$$\sum_{n=0}^{\infty}(n+r)(n+r-1)C_n x^{n+r} + \sum_{n=0}^{\infty}(n+r)C_n x^{n+r} + \sum_{n=0}^{\infty}C_n x^{n+r+2} - 4/9\sum_{n=0}^{\infty}C_n x^{n+r} = 0$$

$$r(r-1)C_0x^r + rC_0x^r - 4/9\,C_0x^r + (r+1)(r)C_1x^{r+1} + (r+1)C_1x^{r+1} - 4/9\,C_1x^{r+1}$$

$$+\sum_{n=2}^{\infty}(n+r)(n+r-1)C_n x^{n+r} + \sum_{n=2}^{\infty}(n+r)C_n x^{n+r} + \sum_{n=0}^{\infty}C_n x^{n+r+2} - 4/9\sum_{n=2}^{\infty}C_n x^{n+r} = 0$$

$k = n-2$

$$C_0x^r\left[r(r-1) + r - 4/9\right] + C_1x^{r+1}\left[r(r+1) + r+1 - 4/9\right] + \sum_{k=0}^{\infty}(k+r+2)(k+r+1)C_{k+2}x^{k+r+2}$$

$$+\sum_{k=0}^{\infty}(k+r+2)C_{k+2}x^{k+r+2} + \sum_{k=0}^{\infty}C_k x^{k+r+2} - 4/9\sum_{k=0}^{\infty}C_{k+2}x^{k+r+2} = 0$$

$\to r(r-1)+r-4/9=0$

$r^2-r+r-4/9=0$

$r^2=4/9 \to r=\pm 2/3$

$\to r_1=2/3 \quad , \quad r_2=-2/3$

$r_1-r_2=4/3$ soluciones independientes

$(k+r+2)(k+r+1)C_{k+2}+(k+r+2)C_{k+2}+C_k-4/9\,C_{k+2}=0$

$\to r=2/3$

$(k+2/3+2)(k+2/3+1)C_{k+2}+(k+2/3+2)C_{k+2}+C_k-4/9\,C_{k+2}=0$

$\left[(k+8/3)(k+5/3)+(k+8/3)-4/9\right]C_{k+2}+C_k=0$

$$C_{k+2}=-\frac{C_k}{k^2+\frac{15k}{3}+20/3}=-\frac{3C_k}{3k^2+16k+20}$$

$\to r=-2/3$

$(k-2/3+2)(k-2/3+1)C_{k+2}+(k-2/3+2)C_{k+2}+C_k-4/9\,C_{k+2}=0$

$\left[(k+4/3)(k+1/3)+(k+4/3)-4/9\right]C_{k+2}+C_k=0$

$$C_{k+2}=-\frac{C_k}{k^2+8/3\,k+4/3}=-\frac{3C_k}{3k^2+8k+4}$$

$r=2/3$

$$C_{k+2}=-\frac{3C_k}{3k^2+16k+20}$$

$k=0$

$$C_2=-\frac{3C_0}{20}$$

$k=1$

$$C_3=-\frac{3C_1}{39}$$

$k=2$

$$C_4=-\frac{3C_2}{64}=\frac{9}{1280}C_0$$

$k=3$

$$C_5=-\frac{3C_3}{95}=\frac{9}{39(95)}C_1$$

$r=-2/3$

$$C_{k+2}=-\frac{3C_k}{3k^2+8k+4}$$

$k=0$

$$C_2=-\frac{3}{4}C_0$$

$k=1$

$$C_3=-\frac{3}{15}C_1=-\frac{1}{5}C_1$$

$k=2$

$$C_4=-\frac{3}{32}C_2=\frac{9}{128}C_0$$

$k=3$

$$C_5=-\frac{3C_3}{55}=\frac{9}{275}C_1$$

$$y(x) = x^{2/3} \sum_{n=0}^{\infty} c_n x^n + x^{-2/3} \sum_{n=0}^{\infty} c_n x^n$$

$$= x^{2/3} \left(c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + \dots \right)$$

$$+ x^{-2/3} \left(c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + \dots \right)$$

$$= x^{2/3} \left(c_0 + c_1 x - \frac{3}{20} c_0 x^2 - \frac{3}{39} c_1 x^3 + \frac{9}{1280} c_0 x^4 + \frac{9}{3705} c_1 x^5 \right)$$

$$+ x^{-2/3} \left(c_0 + c_1 x - \frac{3}{4} c_0 x^2 - \frac{1}{5} c_1 x^3 + \frac{9}{128} c_0 x^4 + \frac{9}{275} c_1 x^5 + \dots \right)$$

$$= x^{2/3} \left[c_0 \left(1 - \frac{3}{20} x^2 + \frac{9}{1280} x^4 + \dots \right) + c_1 \left(x - \frac{3}{39} x^3 + \frac{9}{3705} x^5 + \dots \right) \right]$$

$$+ x^{-2/3} \left[c_0 \left(1 - \frac{3}{4} x^2 + \frac{9}{128} x^4 + \dots \right) + c_1 \left(x - \frac{1}{5} x^3 + \frac{9}{275} x^5 + \dots \right) \right]$$

4.- Aplique la definicion de la transformada de Laplace para:

4.1.- $f(t) = e^{-2t-5}$

Acomodamos $\rightarrow e^{-2t} e^{-5} \rightarrow e^{-5} \mathcal{L}[e^{-2t}]$

por definición $\mathcal{L}[e^{at}] = \frac{1}{s-a}$ $= e^{-5} \mathcal{L}[e^{-2t}] =$

$e^{-5} \frac{1}{s+2} \rightarrow \frac{1}{e^{5}} \frac{1}{s+2} = \frac{1}{e^{5}(s+2)}$ //

4.2.- $f(t) = te^{4t}$

$$\mathcal{L}[te^{4t}] = \int_0^\infty e^{-st} \cdot te^{4t}\, dt$$

$$= \int_0^\infty te^{-st+4t}\, dt \rightarrow \lim_{b\to\infty} \int_0^b te^{(4-s)t}\, dt$$

$u = t \qquad dv = e^{(4-s)t} dt$

$du = dt \qquad v = \frac{e^{(4-s)t}}{4-s}$

$$= \lim_{b\to\infty} \left[\frac{te^{(4-s)t}}{4-s} - \int_0^b \frac{e^{(4-s)t}}{4-s} dt \right]$$

$$= \lim_{b\to\infty} \left[\frac{te^{(4-s)t}}{4-s} - \frac{e^{(4-s)t}}{(4-s)^2} \right] \Big|_0^b$$

$$= \lim_{b\to 0} \left[\cancel{\frac{be^{(4-s)b}}{4-s}}^{\to 0} - \cancel{\frac{e^{(4-s)b}}{(4-s)^2}}^{\to 0} - \left(0 - \frac{1}{(4-s)^2}\right) \right]$$

$$\mathcal{L}[te^{4t}] = \frac{1}{(4-s)^2} \quad \forall \; s > 4$$

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4,35 $f(t) = t\,\text{sen}(t)$

$$\mathcal{L}[t\,\text{sen}(t)] = \int_0^\infty e^{-st}\, t\,\text{sen}(t)\, dt$$

$$= \lim_{b\to\infty} \int_0^b e^{-st}\, t\,\text{sen}t\, dt$$

Integración por partes

$$\frac{t\,(-se^{-st}\,\text{sen}(t) - e^{-st}\cos(t))}{s^2+1} - \int \frac{-se^{-st}\,\text{sen}(t) - e^{-st}\cos(t)}{s^2+1}\, dt$$

$$\frac{s}{s^2+1}\int e^{-st}\,\text{sen}(t)\, dt - \frac{1}{s^2+1}\int e^{-st}\cos(t)\, dt$$

Se integra por partes 2 veces $\int fg' = fg - \int f'g$

$f = \text{sen}(t)$ $g' = e^{-st}$

$f' = \cos(t)$ $g = \frac{e^{-st}}{s}$

$$\frac{-e^{-st}\,\text{sen}(t)}{s} - \int -\frac{e^{-st}\cos(t)}{s}\, dt$$

segunda vez $f = \cos(t)$ $g' = -\frac{e^{-st}}{s}$

$f' = -\sin(t)$ $g = \frac{e^{-st}}{s^2}$

$$-\frac{e^{-st}\,\text{sen}(t)}{s} - \frac{e^{-st}\cos(t)}{s^2} + \frac{1}{s^2}\int e^{-st}\,\text{sen}(t)\, dt$$

$$\int e^{-st}\sin(t)\, dt = \frac{-se^{-st}\sin(t) - e^{-st}\cos t}{s^2+1}$$

mismo proceso para $\int e^{-st}\cos(t)$

$$= \frac{e^{-st}\,\text{sen}(t) - se^{-st}\cos(t)}{s^2+1}$$

$$= \frac{t\,(-se^{-st}\,\text{sen}(t) - e^{-st}\cos(t))}{s^2+1} + \frac{s(-se^{-st}\,\text{sen}(t) - e^{-st}\cos(t))}{(s^2+1)^2} + \frac{e^{-st}\,\text{sen}(t) - se^{-st}\cos t}{(s^2+1)^2}$$

$$= \left(-\frac{t}{s^2+1}^{0} - \cancel{\frac{2s}{(s^2+1)^2}}^{0}\right) e^{-st}\cos^{1} t + \left(-\frac{st}{s^2+1} - \cancel{\frac{s^2-1}{(s^2+1)^2}}^{0}\right)\cancel{e^{-st}}\sin(t)\Bigg|_0^{b=\infty} \quad \begin{matrix}\infty\\ 0\end{matrix}$$

$$0 + \frac{2s}{(s^2+1)^2} \qquad \mathcal{L}[t\,\text{sen}(t)] = \frac{2s}{(s^2+1)^2}$$

Notas

4.4 $\begin{cases} 4 & 0 \le t < 2 \\ 0 & t \ge 2 \end{cases}$

$$\mathcal{L}[f(t)] = \int_0^2 e^{-st}(4)\,dt + \cancel{\int_2^\infty e^{-st}(0)\,dt}$$

sustituimos

$u = st \quad dt = -\frac{1}{s}du \quad \rightarrow -\frac{4}{s}\int e^u du \rightarrow -\frac{4e^u}{s}$ volvemos a las variables

$\frac{du}{dt} = -s$

$$-\frac{4e^{-st}}{s}\Big|_0^2 \rightarrow \frac{-4e^{-s(2)}}{s} + \frac{4e^{-s(0)}}{s}$$

$$= \frac{-4e^{-2s}}{s} + \frac{4}{s} \rightarrow 4\left(\frac{-e^{-2s}}{s} + \frac{1}{s}\right)$$

4.5 $\begin{cases} 2t+1 & 0 \le t < 1 \\ 0 & t \ge 1 \end{cases}$

$$\mathcal{L}[f(t)] = \int_0^1 e^{-st}(2t+1)\,dt + \cancel{\int_1^\infty e^{-st}(0)\,dt}$$

Integral por partes

$u = 2t+1 \quad v = -1/s\, e^{-st} \quad \int u\,dv = uv - \int v\,du$

$du = 2dt \quad dv = e^{-st}dt$

$$= (2t+1)\left(-\frac{1}{s}e^{-st}\right) - \int\left(-\frac{1}{s}e^{-st}\right)(2)\,dt$$

$$= -\frac{2t+1}{s}e^{-st} - \frac{2}{s^2}\int_0^\infty e^{-st}dt$$

$$= \left[-\frac{2t+1}{s}e^{-st} - \frac{2}{s^2}e^{-s}\right|_0^1 \rightarrow \left[-\frac{3}{s}e^{-s} - \frac{2}{s^2}e^{-s} - \left(-\frac{1}{s} - \frac{2}{s^2}\right)\right]$$

$$= \frac{1}{s}(1 - 3e^{-s}) + \frac{2}{s^2}(1 - e^{-s})$$