

HPC 563: Classical Simulation

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March 31, 2008

Chapter 1

Setup

1.1 General Approach

- GNU autotools is used to maintain this project:
autoconf version 2.61 automake version 1.10
- Getopt is used to parse command-line parameters.
A help message can be displayed by specifying "-?" or "-h" to the command-line of the executable.
- The GSL library was chosen to provide some mathematical functions (i.e., `GSL_MAX` and `GSL_MIN`) and to control the floating point environment of the application.
- The cunit library was used to implement the unit tests.
- The application was implemented with OpenMP support. The problem domain lends itself to multi-threaded programming, especially expensive operations such as the matrix-vector multiply and the initialisation of the matrix. OpenMP was not tested on the IITAC cluster though, because the configure script didn't recognise any support for OpenMP. So, OpenMP was tested with gcc 4.2.3.

1.2 Configuration

Each feature of the application is configurable. That means that the libraries required for certain features are checked when the configure script is executed and an appropriate message is presented to the user. Some features, such as MPI, GSL, and unit tests have inter-dependencies which are enforced.

- The assignment ships with a configure script generated by autotools. The following options are supported:
 - **–enable-test**: enables testing using cunit. The unit tests require GSL to be configured as well. An error message is produced, if the GSL library is not configured or not available.
 - **–enable-gcov**: enables the coverage analysis using gcov and lcov. Call `make lcov` to generate the HTML report.
 - **–enable-gsl**: enables the GSL library.
 - **–enable-mpi**: enables the parallel execution of the application using MPI. If MPI is enabled the unit tests are disabled, because the tests cannot be executed in a parallel fashion.
 - **–enable-openmp**: enables OpenMP in order to support multi-threaded execution.
 - **–enable-debug**: enables debug messages and asserts in some of the functions.
 - **–enable-report**: Enable the report generation which is written in latex.
 - **–enable-plot**: Enable the plot generation

1.3 Make

The project is set up in such a way that all sources can be built with a single make command in the root project directory. Additionally, it is also possible to cd into a c module or the doc directory to build those individually.

- Installing the application was not considered.
- To build the project with MPI support do:

1. No gcov, no tests, no MPI

```
./configure  
make
```

Compile the sources

2. No gcov, no tests, MPI enabled

```
./configure --enable-mpi  
make
```

Compile the sources for parallel execution

3. No gcov, no tests, MPI and OpenMP enabled

```
./configure --enable-mpi --enable-openmp  
make
```

Compile the sources for hybrid execution, i.e., that means that the courser tasks are distributed among several remote nodes, while the finer level is parallelised on a single machine with threads.

4. No gcov

```
./configure --enable-test  
make check
```

This command will compile everything and run the cunit tests.

5. Enable gcov and testing

```
./configure --enable-test --enable-gcov  
make lcov
```

This command will compile everything, run the cunit tests, and generate a snazzy HTML coverage report using lcov ¹.

6. Enable the report generation

```
./configure --enable-report --enable-plot  
make
```

This command will compile the c, the gnuplot graphs, and the latex sources.

1.4 Assignment Layout

The assignment is structured in the following way:

- src (the sources)
- test (unit tests)
- doc (doxygen generated code-documentation)
 - eval (gnuplot script and surface plots)
 - report (the report)

A Doxygen configuration file is provided to generate the code documentation in HTML. doxygen support is integrated into the makefiles. Run: make doxygen-doc

¹<http://ltp.sourceforge.net/coverage/lcov.php>

- doc
 - doxygen
 - html

The generated doxygen report details the inter-relationships between the implemented modules and the source files.

The lcov coverage report is provided in the coverage folder. Though, only the infrastructural modules are tested, such as vector and matrix operations.

Chapter 2

Approach

This chapter outlines the general approach taken for the development of the parallel conjugate gradient solver.

2.1 Configuration of the Code

The C code is implemented in such a way that the features, i.e., MPI, OpenMP, debug, and GSL, can be configured individually. As a result the code is portable across a number of platforms and does not depend on a specific feature to be available. The disadvantage of this approach is that the code becomes a bit harder to maintain, because the features are enclosed in `ifdef` statements.

2.2 Data Structures

The two main data structures, vector and matrix, are implemented as C `structs` with supporting routines, such as allocation, free, add, scale, dot product, multiply, etc. The vector structure provides a pointer to a double primitive type and the length of the vector. The matrix consists of a matrix type (symmetric band SB, and generic band GB), the matrix storage format (compressed diagonal storage CDS), a pointer to diagonal vectors, the number of diagonals, and an index which determines the relative positions of the specified diagonals. Based on these characteristics different matrix-vector multiply routines apply (see Section 2.4.2).

2.3 Domain Decomposition

The finite difference scheme for Poisson's equation is decomposed into equally-sized rows among the available nodes. If the square dimension of the finite difference scheme is not divisible by the number of configured nodes then the dimension is adjusted and appended to the matrix and the respective vectors in the linear system.

The 1D decomposition does not require MPI communication. Instead, each node is responsible for setting up the respective data structures in the appropriate way.

2.4 MPI

2.4.1 Dot Product

The dot product is relatively easy to calculate in a cluster environment. Each node computes the partial dot product of the local vector view. Once the local sum is complete, the `MPI_Allreduce()` routine is executed which sums the local parts into a global result. This result is visible to all nodes. However, the communication just involves sending the partial dot product value to the MPI environment.

2.4.2 Matrix-Vector Multiply

While the serial code takes advantage of the symmetry of the matrix, the parallel code sets up a 5-banded matrix on each node. Both matrix-vector multiplications are solved with different functions that take advantage of the given structure. In fact the 5-banded matrix-vector multiplication is implemented as a generic band matrix-vector multiply. Consequently, the conjugate gradient solver can be used for any 2 dimensional finite difference problem. The solver interprets the structure of the matrix and delegates to the appropriate multiplication routine. However, the solver does not test for the well-posedness of the problem.

MPI communication only takes place to retrieve the global u-vector before calculating the matrix-vector product. For this the `MPI_Allgather()` method is called, which results in any node having a global u-vector.

2.5 OpenMP

Much of the effort in implementing the parallel conjugate gradient solver went into parallelising the coarse tasks among configured nodes in a cluster with MPI. However, there still are operations on a finer scale that do not take advantage of multi-processor nodes. Therefore, the GNU profiler (gprof) was used in order to learn more about where the application is spending most of its time. This should indicate which routines can be further improved using multi-threaded execution. OpenMP, which is part of the gcc 4.2 and newer, provides a neat way of declaring which parts of the code should be parallelised using `pragma` directives.

The following steps were executed in the root directory of the project to retrieve a flat execution profile of the application:

1. `./configure CFLAGS'-pg' --enable-gsl=`
2. `make clean all`
3. `./src/main/pdepoiss_solv -d 0.01`
4. `gprof ./src/main/pdepoiss_solv gmon.out`

The flat profile looks like this:

Each sample counts as 0.01 seconds.

%	cumulative	self		self	total	
time	seconds	seconds	calls	s/call	s/call	name
77.02	650.58	650.58	124003	0.01	0.01	dotProduct
15.44	781.01	130.43	62002	0.00	0.00	dcdssbmV
3.38	809.59	28.58	124003	0.00	0.00	daxpy
1.78	824.64	15.06	62001	0.00	0.00	add
1.76	839.53	14.89	62001	0.00	0.00	scale

This profile shows that two methods, `dotProduct()` and `dcdssbmV()`, are responsible for more than 90% of the runtime of the application.

In order to improve the performance, those two methods were declared to run in parallel execution contexts. Additionally, other routines that are easy to parallelise and constitute potential huge iteration counts were also considered. Those include `zero()` which zeros a vector, `daxpy()` to add a scaled vector to another one, `add()` to add two vectors, `scale()` to scale a vector, and those that initialise the matrix and vectors.

The initialisation of the data structures further support a nested threading model, which can be controlled with the `OMP_NESTED` environment variable.

The results of the efficiency and the speedup are presented in the following table.

Chapter 3

Execution

The conjugate gradient solver for Poisson's equation accepts a number of command-line arguments which are parsed with `getopt`. The executable resides in the `src/main` folder and can be called with

`pdepoiss_solv`

- `-s` : Space dimension (Note: specify grid dimensions or the delta value)
- `-t` : Time dimension (Note: specify grid dimensions or the delta value)
- `-d` : Delta value (Note: specify grid dimensions or the delta value)
- `-r` : Error threshold for the conjugate gradient method
- `-f` : File name for the result surface
- `-1` : Lower bound of the domain in the x dimension
- `-2` : Upper bound of the domain in the x dimension
- `-3` : Lower bound of the domain in the y dimension
- `-4` : Upper bound of the domain in the y dimension
- `-?/-h` : The help message of the application

The space and time dimensions are required to be equal. This is ensured by the application itself. The default value for both is 26. If only one dimension is specified on the command-line then both are set to the highest value, i.e., 26 if the specified value is smaller than 26 or otherwise the specified value. The delta value will be calculated based on this setting.

Alternatively, the delta value can be specified upon which the space and time dimensions are derived. Fixing the delta value to some sensible values make sense to ensure that floating point operations are performed more accurately. It has been observed that fixing the dimensions to some value (i.e., 53) yields NaNs as a result. I assume this has something to do with floating point operations of very small numbers (such as 4.7308317893592655e-157). The following gdb debugger session identifies the location and the cause of the SIGFPE floating point exception:

```
> gdb
(gdb) set env GSL_IEEE_MODE=double-precision
(gdb) handle SIGFPE stop nopass
(gdb) file ./src/main/pdepoiss_solv
(gdb) r -s 53

Program received signal SIGFPE, Arithmetic exception.
0x08049ef2 in dcdssbm (mat=0xbf98862c, u=0xbf9885b0, v=0xbf9885a8)
    at mult.c:47
47          v->data[i] = mat->diags[0].data[i] * u->data[i];
(gdb) p v->data[i]
$1 = 4.7308317893592655e-157
(gdb) p u->data[i]
$2 = 4.7308317893592655e-157
(gdb) p mat->diags[0].data[i]
$3 = 4
```

The error occurred in the serial code of the matrix-vector multiply function `dcdssbm()`.

These obvious errors go undetected, unless GSL is configured. GSL allows to control the floating point environment of C/C++ applications. Executing the application with

```
GSL_IEEE_MODE="double-precision," \
  mask-denormalized,mask-underflow" \
  ./src/main/pdepoiss_solv -s 53
```

ignores any errors related to denormalised and under-flowing small numbers, but it traps overflows, division by zero, and invalid operations¹.

The domain of the function space can be specified as well. The default values are set to the square region provided in the assignment description.

The file name of the gnuplot data for the surface plot can be specified which is set to `result.dat` as default name.

¹<http://www.gnu.org/software/gsl/manual/>

3.1 Environment Variables

As mentioned in the previous section GSL allows some environment variables to be set for the application to control the floating point environment. The same is true for OpenMP. The following list details the ones used throughout the development of the application and to retrieve the results.

- **GSL_IEEE_MODE**: Sets control variables values for the floating point environment. The variables used are *double-precision*, *mask-denormalized*, and *mask-underflow*.
- **OMP_NUM_THREADS**: Controls the maximum number of threads available to the application. Different values were used ranging from 3-8.
- **OMP_NESTED**: Indicates to the OpenMP environment whether nested threads are allowed. Some parts of the code support thread nesting. These parts can be optimised with setting this variable to true.

To summarise, the following command-line was used to run the application:

```
GSL_IEEE_MODE="double-precision,mask-underflow,mask-denormalized"; \  
OMP_NUM_THREADS="8";OMP_NESTED="true" \  
./src/main/pdepoiss_solv -d 0.01
```

Chapter 4

Results

This chapter presents the solution to the parallel conjugate gradient method to solve Poisson's equation

$$\nabla^2 u = -f \quad (4.1)$$

with $u \equiv u(x, y)$ on the square region

$$ABCD, A = (-0.5, -2), B = (2, -2), C = (2, 0.5), D = (-0.5, 0.5), \quad (4.2)$$

where the source density s given by

$$f(x, y) = 4 \cos(x + y) \sin(x - y) \quad (4.3)$$

The exact solution and boundary conditions are given by

$$u = \cos(x + y) \sin(x - y) \quad (4.4)$$

which results in the following graph

Two simulation runs were performed – one in serial mode and the other one in parallel mode using the Message Passing Interface (MPI) with 16 configured nodes. Both runs are configured with $\delta = 0.01$ which yields 251 grid points. Figure 4.2 presents both graphs with the gradient on the surface illustrating the error Equation 4.5 in the conjugate gradient method compared to the analytical solution Equation 4.4.

$$\epsilon = |u(x, y) - v_{i,j}| \quad (4.5)$$

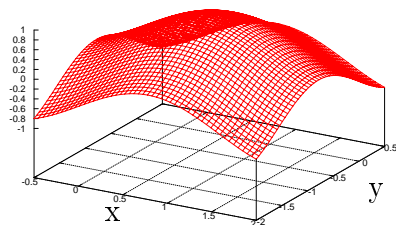
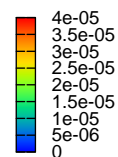
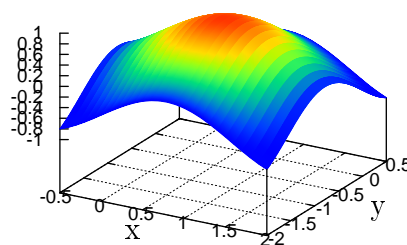
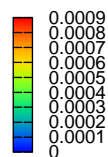
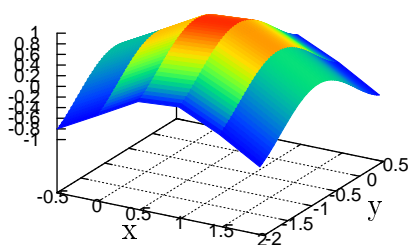


Figure 4.1: Exact Solution of Poisson's Equation

Serial

Parallel



ϵ

Figure 4.2: Approximate Solution of Poisson's Equation $\nabla^2 u = -f$