Alegbraic Topology Problem Bank

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Homework 1

Hatcher 0.2

Construct an explicit deformation retraction of $\mathbb{R}^n - \{0\}$ onto S^{n-1}.

Hatcher 0.3

(a) Show that the composition of homotopy equivalences $X \to Y$ and $Y \to Z$ is a homotopy equivalence $X \to Z$. Deduce that homotopy equivalence is an equivalence relation.

(b) Show that the relation of homotopy among maps $X \to Y$ is an equivalence relation.

(c) Show that a map homotopic to a homotopy equivalence is a homotopy equivalence.

Hatcher 0.6

Hatcher 0.10

Show that a space X is contractible if and only if every map $f : X \to Y$, for arbitrary Y, is nullhomotopic. Similarly, show X is contractible if and only if every map $f : Y \to X$ is nullhomotopic.

Hatcher 0.11

Show that $f : X \to Y$ is a homotopy equivalence if there exist maps $g, h : Y \to X$ such that $fg \cong \mathrm{id}$ and $hf \cong \mathrm{id}$. More generally, show that f is a homotopy equivalence if fg and hf are homotopy equivalences.

Hatcher 0.16

Hatcher 0.17

Hatcher 0.20

Show that the subspace $X \subset \mathbb{R}^3$ formed by a Klein bottle intersecting itself in a circle is homotopy equivalent to $S^1 \vee S^1 \vee S^1$.