

# STAT 5443: Computational Statistics

## Homework 1 – Due October 13 , 2021

Consider the coal-mining disaster data, available in the *R scripts and examples* folder in Blackboard. As we did in class, assume that there is a change point in the series after  $\theta$  observations. Suppose  $\theta$  has a discrete uniform distribution on the set  $\{1, 2, \dots, 111\}$ . If  $X_j$  is the number of accidents in year  $j$ , for  $j = 1, \dots, 111$ , the model is

$$\begin{aligned}X_1, \dots, X_\theta &\sim \mathcal{Poi}(\lambda_1), \\X_{\theta+1}, \dots, X_{111} &\sim \mathcal{Poi}(\lambda_2).\end{aligned}$$

Assume that  $\lambda_2 = \alpha\lambda_1$ . Use the prior defined as follows.

$$\begin{aligned}\lambda_1|a &\sim \mathcal{Gam}(3, a), \\a &\sim \mathcal{Gam}(10, 10), \\\log \alpha &\sim \mathcal{Unif}(\log 1/8, \log 2).\end{aligned}$$

1. Using a SIR approach, estimate the posterior mean (Bayes estimate) of  $\theta$ , providing a histogram of the posterior distribution and a 90% Credible Interval.
2. Provide similar information for  $\lambda_1$  and for  $\lambda_2$ .
3. Make a scatterplot of  $\lambda_1$  against  $\lambda_2$  for the initial SIR sample, highlighting the points resampled at the second stage of SIR.
4. Report your initial sample size and resampling sampling size, the number of unique points and the highest observed frequency in the final sample. Report also a measure of the effective sampling size.
5. Discuss your results.