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A mathematical framework for multiphase poromechanics in multiple porosity media

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The reviewers' comments have been listed in order below in *italic* text, followed by responses in normal text. All the revised parts have been highlighted in the revised manuscript. Please review them according to the instructions in the following orange annotation box. Thank you again for your comments and suggestions, which have greatly improved the manuscript.

Dr. YAN has helped a lot in the major revision process for the current manuscript. Qi and Zihao are grateful to him for enlightening discussions pertaining to the reviewers' comments. He also helped design the new added numerical example. In the original submission, Dr. YAN has proofread the preliminary draft of this manuscript and gave many useful suggestions, despite his name did not appear in the original author list. Therefore, Dr. YAN is the corresponding author in the revised submission. Hope you can approve this request.

Reviewer 1

Q 1.1 *The authors present a mathematical model for elastoplastic anisotropic multiple porosity media and their solutions. As far as the solutions of the governing equations are concerned, the Laplace transform and numerical Laplace inversion methods are used for the uncoupled flow simulation while for a coupled flow and geomechanics situation the consolidation of a rectangular domain subjected to a strip load is solved by means of the finite element method.*

Reply: Thank you for a good and brief summary of our work.

Q 1.2 *First of all, it is not easy to understand what is the novelty of the paper. The authors use mixture theory which is well known to be written on the macroscale, where also thermodynamics is imposed. For double porosity media also the microscale is certainly of importance and should be taken into account. Mixture theory provides a macroscopic description (level of observation) of the system where conservation laws are introduced in analogy with the balance laws of single bodies. In the standard continuum mechanics approach, rational thermodynamics is employed at the macroscale to obtain closure relations. No connection is made with the microscopic reality. Interfacial properties are absent from both conservation laws and constitutive equations. No thermodynamical consistency*

exists with the microscale. On the other hand, averaging theories seem much more suitable for describing the problem. For instance, the thermodynamically constrained averaging theory (TCAT) framework (Gray and Miller, 2014; Gray et al., 2013) provides porous media models that are thermodynamically consistent across scales and would hence give much more insight in the problem.

Reply: Thank you for your critical comment. We agree that our framework is devoid of interfacial properties, which is also discussed in Q 1.6. Based on Prof. Borja and Prof. Coussy's work on double porosity media (Borja and Koliji, 2009; Choo et al., 2016; Coussy, 2003; Zhang et al., 2019; Zhao and Borja, 2021), we have learned that the mixture theory could be applied to the microscale if the microscale is homogenized over the whole domain, which is illustrated in Section 2 of Borja and Koliji (2009). As a result, we can have the mass balance equation for the fluid in the microscale (analogous to "species based"), as shown in 1.5.3 of Coussy (2003). For the balance of linear momentum and balance of energy, it is also formulated on each scale at the beginning, but finally we add them together to get the balance equations on the whole mixture (analogous to "entity based") to identify energy-conjugate pairs and possible constitutive relations (Coussy, 2003). Therefore, from our own opinion, we believe the mixture theory can handle the microscale in a significantly more simplified way than the TCAT approach, while the derivation could be less lengthy than the comprehensive TCAT approach. This might be also true for the case of N constituents (e.g., one solid constituent and $N-1$ fluid constituents) by using the mixture theory (Bowen, 2014). In addition, for multiple porosity media, the volume averaging from the smallest scale to the largest scale might be difficult because the traditional TCAT approach usually contains three length scales (micro, macro, mega) (Gray et al., 2013). Figures 1 and 2 from Mehrabian and Abousleiman (2015) display exactly the multiscale nature of shale and biological tissues. That's the reason why we choose mixture theory as the basic principle to investigate multiphase poromechanics in multiple porosity media. Nevertheless, we have referred the important contributions of the TCAT approach to the environmental problems (Gray and Miller, 2014; Gray et al., 2013) in our revised introduction. Thank you for providing such important references. Sections 1 and 5

As for the novelty of this paper, it has been summarized in the following bullet form. Hope you are satisfied with our explanation we can get your approval.

- We have used the mixture theory to derive a complete framework for porous media exhibiting multiple porosity scales that are saturated with one or two types of fluids (zero to weak capillarity for two immiscible fluids) for the first time.
- We have identified several emerging energy-conjugate pairs relevant for constitutive modeling.
- The impacts of micro-fracture permeability and plasticity model are highlighted for the strip load problem, which has not been reported comprehensively in the previous publications.
- A new 3D example that concerns time-dependent solutions related to quadruple-porosity is presented and the effect of capillary force is discussed. This is inspired from Q 2.6 of the second reviewer.

Q 1.3 *In their development the authors have introduced some strong assumptions. All these limits of their approach should hence be discussed and justified. For example, the internal structure does not change over time.*

Reply: Thank you for your important comment. Due to lack of the experimental evidence in the measurement of pore fraction evolution for the most general condition (multiphase flow and elastoplastic deformation), we assume a constant ψ_i. In Zhao and Borja (2021), the authors Section 3

mentioned that "Due to the small-strain assumption, the plasticity model employed in this work does not include changes in the internal structure of the material in the form of the evolution of pore fractions ψ^M and ψ^m". In Choo et al. (2016), a similar assumption was also made and it is concluded that the variation of ψ_i is an open question in the finite deformation range. Nevertheless, we argue that under the condition of single phase flow and elastic small deformation, variation of ψ_i (*i.e.*, $\mathrm{d}\psi_i/\mathrm{d}t$) can be obtained following a special routine, despite some assumptions in this routine are not verified or validated yet. For your interest, the details are given in the Appendix of this response letter. We do not include these derivations in the revised manuscript as this evolution law is not universal. Hope you are satisfied with our explanation.

Q 1.4 *Please justify the assumption of multiple interacting continua for the mass transfer terms.*

Reply: Thank you for your important comment. The MINC assumption originates from Pruess and Narasimhan (1985), and it is now widely used in the reservoir simulations. For example, Figure 3 of Yan et al. (2018a) and Figure 8a of Wang et al. (2017) display the connectivity list of the MINC model coupled with hydraulic fracture. Because of the relatively low permeability of the matrix sub-elements (the permeability of the matrix is much lower than that of fractures), the connection between the neighboring matrix sub-blocks can be neglected, and the matrix only behaves as a sink/source term of the fracture sub-block. Hope you are satisfied with our explanation.

Section 3 and Figure 1

Q 1.5 *Please justify the assumption of dual porosity single permeability for the first example.*

Reply: Thank you for your important comment. This assumption is made in Warren and Root (1963) for the first time, suggested by its Eqs. (6)(9). We have justified this assumption by comparing the results with those from a dual porosity dual permeability model using FVM (Finite Volume Method). Under the setting of the first example, as long as the permeability of the matrix continuum is small enough, the results are almost identical. In addition, Laplace transform and numerical Laplace inversion methods could be more efficient than FVM in terms of the computational time. From the connectivity list shown in Figure 5 of Jiang and Younis (2016), we can see that there is no connection between adjacent matrix element (e.g., m1 and m2, m1 and m3, etc), which also suggests the correctness of the dual porosity single permeability model. Hope you are satisfied with our explanation.

Section 4.1.1

Q 1.6 *Definition of capillary pressure is questionable in the light of Gray et al. (2013).*

Reply: Thank you for your criticism. We agreed that this definition of capillary pressure may not be fully comprehensive from the TCAT description. A correct form of p_c requires proper accounting for the properties of interfaces (Gray et al., 2013). By using the mixture theory (Bowen, 2014), we can only identify energy-conjugate pairs from the balance of energy (Borja, 2006), and the interface properties are not fully accounted. As a result, only the heuristic relation could be used such as the Brooks-Corey model and van Genuchten model, which have been supported by substantial experimental evidences. This heuristic relation is also used in shale gas reservoir with a complex fracture network (Jiang and Younis, 2015). A sketch of this p_c is shown in Figure 7 of Jiang and Younis (2015) and our new added 3D example. Furthermore, a specific form of the abstract constitutive equation $-\gamma^{wn} J_w^{wn} = p^c\left(s^w, \epsilon^{wn}\right)$ from TCAT (Gray and Schrefler, 2007) is still very challenging in numerical simulations.

Eq. (35) and Section 4.3

Reviewer 2

Q 2.1 *This paper aims to provide a continuum formulation for hydromechanical modeling of heterogeneous porous media involving multiple porosity and anisotropic permeability. Constitutive laws are also claimed to have been developed for elastoplasticity and anisotropy. A depletion problem is studied that represents uncoupled flow in a saturated fractured reservoir (dual porosity case); in which their governing differential equations are solved using the Laplace transform followed by their numerical inverse transforms. Subsequently, a coupled plane strain consolidation problem is studied within a saturated dual porosity framework. Overall, the paper is well presented and the governing equations for multiple porosity are derived rather carefully, although they seem to be superposition of equations from single porosity theory. There are some main concerns that the authors need to consider in their revision. Therefore, the paper requires a major revision based on the comments outlined in the following.*

Reply: Thank you for a comprehensive summary! Our theory can be downgraded to the single porosity theory, while you can see from our revised theoretical part, the balance laws contain some terms that would not appear in the single porosity theory. In the following contents, we have tried our best to reply to each of your comments carefully. Hope you are satisfied with our explanations.

Q 2.2 *The main criticism relates to the manner in which unsaturated porous media are treated in this study. It is stated (line 211, page 9) that: "in most reservoir simulations (kilometer scale) the capillary pressure is always ignored." For unsaturated problems, it is well-known that capillary pressure plays a major role on their hydromechanical behavior. Ignoring such effects in this study undermines the generality of derived continuum formulations. Therefore, the authors need to carefully define their position whether their formulation is valid for some specific applications or it is generally applicable to unsaturated porous media involving multiple porosity. I think you need to carefully define whether the problem is saturated or unsaturated. If you also consider unsaturated cases define what the role of non-wetting fluid would be on the hydro-mechanical behavior of the multiple porosity systems? Hydro-mechanical behavior regarding the change in the saturation degree caused by both suction and net stress is essentially ignored.*

Reply: Thanks for your critical comment. In the revised manuscript, we have stated clearly that our framework is developed for porous media that is **saturated** with two immiscible fluids (the capillarity is not significant), which are commonly known as the wetting phase fluid and the non-wetting phase fluid. For example, in a oil-water system, water is the wetting phase fluid, while the oil is the non-wetting phase fluid. Similarly, in a gas-water system, gas is the non-wetting phase fluid. This framework apparently can be downgraded to the case of saturated single phase flow. It is critical to recognize that this situation is clearly different from that of unsaturated porous media, as the suction stress[1] $s = p_a - p_w$ is usually significant in the latter case and can also affect the constitutive law. This conclusion is already supported by plenty of published work. For instance, Loret and Khalili (2000) considered the suction-dependent preconsolidation stress $\bar{p}_c$ through the multiplicative function and the additive function. In Borja (2004); Borja et al. (2013); Ip and Borja (2022); Ip et al. (2021); Song and Borja (2014); Song and Silling (2020); Song et al. (2017), a different form of $\bar{p}_c$ was adopted $\bar{p}_c = a(s) \times p_c^{b(s)}$, where p_c is the saturated preconsolidation stress, *i.e.*, the value which $\bar{p}_c$ tends to in the limit of full saturation. Researchers from Wuhan University also developed their own constitutive model for unsaturated soil (Hu et al., 2015, 2016, 2018), in which a bonding factor ζ was defined as a function of saturation and void ratio. The $\bar{p}_c$ now depends on ζ,

Abstract, Sections 1 and 3 (highlight)

[1]The terminologies "suction stress" and "capillary pressure" have the same definition (Dangla and Pereira, 2014), while the former is always used in the soil mechanics and hydrology, the latter is always used in the petroleum engineering.

and for the hydraulic model, a void ratio-dependent water retention curve model was presented that could consider the hysteresis effect between the drainage and imbibition processes.

In the former case adopted in our manuscript, the so-called capillary pressure is insignificant or it is even ignored in some applications, *i.e.*, $p_{nw} = p_w$, which is supported by Ren et al. (2018) and the work from LLNL (Camargo et al., 2021; Cusini et al., 2021; White et al., 2019). This is a frequent assumption in many reservoir engineering applications (Durlofsky, 2005; Karimi-Fard and Durlofsky, 2016). In addition, as mentioned in Choo et al. (2016), due to the absence of firm experimental evidence, incorporating the effect of multi-scale suctions (because we have a multiple porosity media) to elastoplasticity is still an open and challenging question. In Kim and Moridis (2013), the Drucker-Prager and Mohr-Coulomb failure models are independent of saturation/capillary pressure. In Wei and Zhang (2010); Yan et al. (2018a), the authors also did not take the effect of saturation on elastic properties (Young's modulus E and Poisson's ratio ν) into consideration. Considering these factors, we thus assume the material parameters are independent of saturation and capillary pressure in our constitutive equations. In the future, if we can incorporate the suction-dependent constitutive law into our framework, our derived continuum formulation will surely be more general and attract wider attentions.

Q 2.3 *Effects of suction, capillary pressure and sportive water on the hydromechanical behavior of unsaturated porous media and the associated constitutive laws are significant. None of these aspects are discussed nor incorporated in the governing equations. Even though a reference is made to the capillary pressure on page 9, it is not expanded in any form.*

Reply: Thank you for your valuable comment. You are correct that the effect of suction stress must be considered in the unsaturated poromechanics, which has also been mentioned in the response to Q 2.2. For the governing equations, as derived by prominent researchers in our field (Borja, 2004, 2006; Borja and Koliji, 2009; Khalili, 2008; Khalili et al., 2008; Loret and Khalili, 2000), the balance of mass and balance of linear momentum equations are applicable to both the case of two immiscible fluids (small s) and the case of unsaturated porous media. The difference lies in the constitutive equations (Loret and Khalili, 2000). For the capillary pressure, it will be expanded into our formulation when we take p_{nw} and S_w as the primary unknowns, and if we need to take the derivative of p_w with respect to S_w, the capillary pressure curve will be used. Similarly, for the equation $S_r = S_r(s)$, it will enter into the numerical simulation through $\dot{S}_r = S_r'(s)\,(\dot{p}_a - \dot{p}_w)$. Section 3

Q 2.4 *In Section 2, please describe what you mean by "wetting phase and non-wetting phase fluids" considering the assumption that the pores are fully occupied by two immiscible fluids.*

Reply: Thank you for your insightful comment. The definitions of wetting phase and non-wetting phase fluids are related to the interfacial tension and capillarity. For more details, please refer to the following chapter named "Reservoir rock properties" from Satter and Iqbal (2016) and the book of Aziz and Settari (Aziz and Settari, 2002). In Section 2.2.2 (page 8) of this book, the conservation equation for multiphase, multicomponent flow (rigid porous media, single porosity) is given, and a typical example is known as the black-oil model, which is described in Section 2.2.2.1. In this black-oil model, water is the wetting phase, oil has an intermediate wettability, and gas is the non-wetting phase. Water and oil are assumed to be immiscible. For our current framework, we briefly considered the basic multiphase flow while the porous media is deformable and it contains multiple levels of pore structure complexity, namely, the multiple porosity media. We hope our framework could be extended to the case of multicomponent flow in the future publications. Section 1

Q 2.5 *The constitutive law presented for solid deformation in Section 3.1 involves the mathematical relations for a non-associated Drucker-Prager model with no hardening mechanisms (with respect to suction or saturation degree). Therefore, the point of representing this model in the paper is missing making its presentation virtually unnecessary.*

Reply: Thank you for your criticism. As we have explained in the response to Q 2.2, the material parameters are independent of saturation and capillary pressure. Nevertheless, we agreed that the presentation of this plasticity model is virtually unnecessary. As a result, by referring to the style in Choo et al. (2016), we only highlight the energy-conjugate pairs identified from the balance of energy, and the specific constitutive model is only mentioned in numerical simulations. We hope that you will be satisfied with this change.

Sections 3 and 4.2.1

Q 2.6 *The second main criticism is about the presented examples. The first example involves a single phase uncoupled flow in a saturated double porosity medium. The second one is a coupled saturated porous medium with a network of embedded micro-fractures. Therefore, both examples are special cases of saturated porous media with dual porosity. Therefore, an example illustrating the application of the derived formulations to an unsaturated case with multiple porosity (say for instance triple-porosity or quadruple-porosity) is necessary. In particular, the time-dependent solutions related to multiple-porosity are rare in literature, but there are few for dual-porosity materials. Furthermore, presenting such example would illustrate how you practically lump the physical properties of a lower-level model to obtain the ones corresponding to a higher-level porosity.*

Reply: Yes, both of the initial two examples fall within the category of saturated double porosity media. Following your suggestion and the limitation of our framework in Q 2.2, we add a third example in the revised manuscript, which concerns the gas and water production from a deformable 3D reservoir model with quadruple-porosity. All the results are time-dependent, which can be visualized from the change of saturation contour and cumulative production curves. Furthermore, the effect of capillarity is also illustrated (water will be quickly imbibed from the natural fracture continuum to the remaining matrix continua, which promotes the gas production and inhibits the water production). We hope that you will be satisfied with this new 3D example.

Section 4.3

Q 2.7 *The authors mention in the abstract that "we extend ... in the previous study to those for the elastoplastic anisotropic multiple porosity media". Which study you mean by "the previous study"?*

Reply: Thank you for your comment. We should state it more clearly. The previous study concerns mainly the work of Prof. Borja that applies the mixture theory formulation to the two-phase or three-phase porous media. Please refer to Borja (2004, 2006) for more details.

Q 2.8 *If your study explores saturated cases only, you may need to explicitly mention it in the abstract.*

Reply: Thank you for your comment. We have explicitly mentioned the scope of application of our framework in the abstract.

Q 2.9 *The authors state (line 67, page 3) that: "Constitutive laws are then developed that are able to account for anisotropy and elastoplasticity". I do not see any constitutive law developed in this study. By the development, do you mean the representation of Drucker-Prager model? Also, the authors only consider anisotropic permeability in the formulation while the mechanical model does not consider fabric or stress anisotropy. To avoid confusion, please be more precise when refereeing to anisotropy.*

Reply: Thank you for your comment. Yes, we consider anisotropic permeability alone, so we will be more careful when refereeing to anisotropy. As for the development of the constitutive law, we are not creating/deriving any constitutive laws. In fact, we want to "present" some constitutive models that can be incorporated into our study. Inspired by Q 2.5, we highlight the energy-conjugate pairs identified from the balance of energy in the revised manuscript. Section 3

Q 2.10 *Please comment on how a complete set of mechanical property coefficients for multiple porosity systems can be derived in practice.*

Reply: Thank you for your comment. In the new added example, a complete set of mechanical property coefficients is already given in Table 1. The Young's modulus and Poisson's ratio could be prescribed for the bulk material, or it can be obtained through an upscaling of each sub-element (Kim et al., 2012; Yan et al., 2018b). Through the geological exploration, we can determine the natural fracture spacing, aperture, and intrinsic porosity (due to some fillings, the intrinsic porosity could be less than 1), which are related to the permeability ($\boldsymbol{k}_{fr}$) and volume fraction of the natural fracture continuum (ψ_1). By further calculating the remaining ψ_i from f_i and intrinsic porosity, while the shape factor can be evaluated according to Eqs. (14a)-(14f) in Pruess and Narasimhan (1985), we now have a complete set of mechanical parameters for numerical simulation. Sections 4.2.1 and 4.3

Q 2.11 *In the second example, dilation angle is 20 degrees, it is suggested to consider a case with zero dilation angle and observe its effects on the performance of micro-fractures and overall behavior of the system.*

Reply: Following your suggestion, we have added the analysis to investigate the impacts of dilation angle ψ and inter-porosity exchange parameter (*i.e.*, leakage parameter). As you can see from the new results, a zero dilation angle will make the whole porous media less stiff, leading to a higher excess pressure accumulation in the two continua because of the greater plastic compaction (Zhao and Borja, 2020). This change of p (due to change of ψ) in the porous matrix continuum is even more significant than that in the micro-fracture network continuum. For the overall behavior of the system, we focus on the settlement at the center of the footing. The result shows that a zero dilation angle gives a larger settlement than the case of $\psi = 20°$. This finding is analogous to that in Borja et al. (1998); Zhao and Borja (2020) when the authors changed the preconsolidation stress. Section 4.2.2

Q 2.12 *Please also illustrate the inter-porosity exchange effects on drainage of fluid and the evolution of pore pressures and effective stresses. Please investigate how the inter-porosity fluid exchange affects the early-time response (e.g., undrained bulk modulus) of the studied examples?*

Reply: Thank you for your comment. We have added the corresponding analysis in our revised manuscript. The difference can be clearly observed from the new added figure. A smaller leakage parameter will lead to a greater pressure decline hysteresis. The dashed orange curve illustrates a weak "double-shell" characteristic, similar to Figure 10b in Zhang et al. (2021). The dashed blue curve decreases faster than the solid blue curve in the beginning, but finally, due to a slower inter-porosity fluid exchange, we will expect a longer time for the completion of the consolidation. For the NFR example, the inter-porosity exchange effect is embedded into the parameter λ, which is shown in Figure 3. Other qualitative results can also be obtained from Zhang et al. (2021) through a 1D consolidation example of a double porosity layer. For the early-time response such as the undrained bulk modulus K_{u} and other undrained quantities (Cheng, 2016), they should be independent of leakage parameter as the deformation depends on the mean pressure. The leakage parameter will not affect the mean pressure at $t = 0^{+}$, which can be verified from the 1D example Sections 4.1.2 and 4.2.2

in Zhang et al. (2021) and 3D sphere example in Zhang et al. (2019). For the case of poroelasticity, when fluid content variations (Cheng, 2016) $\zeta_1 = \zeta_2 = 0$ (undrained condition when Δt is very small), it can be shown that the K_u is the same as that in the single porosity situation.

Q 2.13 *Please replace "in this thesis" mentioned in line 423 with in this paper or in this study.*

Reply: Thank you for your careful check. We have fixed this issue. This sentence is copied from Ph.D. thesis of Qi and we might have forgotten to change this word.

Evolution law of the pore fraction (single phase poroelasticity)

We start from the effective stress law: Appendix of Q 1.3

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}' - \sum_{i=1}^{N} \psi_i p_i \mathbf{1}\,, \tag{1}$$

and by making using of a trivial decomposition $\boldsymbol{\sigma} = \sum_{i=1}^{N} \psi_i \boldsymbol{\sigma}$ and an assumed decomposition $\boldsymbol{\sigma}' = \sum_{i=1}^{N} \psi_i \boldsymbol{\sigma}'_i$, we can rewrite the above equation as:

$$\sum_{i=1}^{N} \psi_i \boldsymbol{\sigma} = \sum_{i=1}^{N} \psi_i \boldsymbol{\sigma}'_i - \sum_{i=1}^{N} \psi_i p_i \mathbf{1}\,. \tag{2}$$

Here, the new quantities $\boldsymbol{\sigma}'_i$ $(i = 1, 2, \ldots, N)$ represent N new superimposed single porosity systems, which are generated from the original N-porosity material. For instance, the phase diagram of the j^{th} single porosity system is obtained by combining the j^{th} pore (void) space with a portion of the solid material. In the phase diagram, the j^{th} pore (void) space has a height of ϕ_j, and the corresponding portion of the solid material has a height of $\phi_s \psi_j$. As a result, the height of the j^{th} new single porosity system is $\phi_j + \phi_s \psi_j = \phi \psi_j + \phi_s \psi_j = \psi_j$, and all the N new superimposed single porosity systems automatically satisfy $\sum_{j=1}^{N} \psi_j = 1$, which is consistent with the original phase diagram of the multiple porosity media.

From Eq. (2), we make a strong assumption such that for each $i = 1, 2, \ldots, N$, the tensor multiplied by ψ_i simply vanishes. Through this assumption, the effective stress laws of those N new superimposed single porosity systems are recovered:

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}'_i - p_i \mathbf{1} \quad (i = 1, 2, \ldots, N)\,. \tag{3}$$

Equation (3) implies $N - 1$ equations. Therefore, for $j = 1, 2, \ldots, N-1$, we have:

$$\boldsymbol{\sigma}'_j - p_j \mathbf{1} = \boldsymbol{\sigma}'_{j+1} - p_{j+1} \mathbf{1}\,. \tag{4}$$

Taking the trace and applying the material time derivative with respect to the solid motion gives:

$$\frac{\mathrm{d}\sigma'_j}{\mathrm{d}t} - \frac{\mathrm{d}p_j}{\mathrm{d}t} = \frac{\mathrm{d}\sigma'_{j+1}}{\mathrm{d}t} - \frac{\mathrm{d}p_{j+1}}{\mathrm{d}t} \quad (j = 1, 2, \ldots, N-1)\,, \tag{5}$$

where $\sigma'_j = \mathrm{Tr}\left(\boldsymbol{\sigma}'_j\right)/3$ and $\sigma'_{j+1} = \mathrm{Tr}\left(\boldsymbol{\sigma}'_{j+1}\right)/3$. In above equation, the σ'_j and σ'_{j+1} are related to the volumetric strains (θ_j and θ_{j+1}) of the N new superimposed single porosity systems, and the change of volumetric strain can be represented by those pore fractions. That is to say:

$$\frac{\mathrm{d}\sigma'_i}{\mathrm{d}t} = K_i^e \frac{\mathrm{d}\theta_i}{\mathrm{d}t} = \frac{K_i^e}{\psi_i} \frac{\mathrm{d}\psi_i}{\mathrm{d}t}\,, \tag{6}$$

where θ_i is the volumetric strain of the i^{th} new single porosity system. By combing Eq. (5) with Eq. (6) and using the closure condition $\sum_{i=1}^{N} \mathrm{d}\psi_i/\mathrm{d}t = 0$, we can finally obtain the N linear equations with N unknowns $\mathrm{d}\psi_i/\mathrm{d}t$ $(i = 1, 2, \ldots, N)$:

$$\begin{pmatrix} \frac{K_1^e}{\psi_1} & -\frac{K_2^e}{\psi_2} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \frac{K_{N-1}^e}{\psi_{N-1}} & \frac{K_N^e}{\psi_N} \\ 1 & \cdots & 1 & 1 \end{pmatrix} \begin{pmatrix} \frac{\mathrm{d}\psi_1}{\mathrm{d}t} \\ \vdots \\ \frac{\mathrm{d}\psi_{N-1}}{\mathrm{d}t} \\ \frac{\mathrm{d}\psi_N}{\mathrm{d}t} \end{pmatrix} = \begin{pmatrix} \frac{\mathrm{d}p_1}{\mathrm{d}t} - \frac{\mathrm{d}p_2}{\mathrm{d}t} \\ \vdots \\ \frac{\mathrm{d}p_{N-1}}{\mathrm{d}t} - \frac{\mathrm{d}p_N}{\mathrm{d}t} \\ 0 \end{pmatrix} . \tag{7}$$

The solution of Eq. (7) is the proposed internal structure evolution law of the multiple porosity media.

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