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A mathematical framework for multiphase poromechanics in multiple porosity media

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The reviewers' comments have been listed in order below in *italic* text, followed by responses in normal text. All the revised parts have been highlighted in the revised manuscript. Thank you again for your comments and suggestions, which have greatly improved the manuscript.

Reviewer 1

Q 1.1 *I have read carefully the answers to the question and was particularly puzzled by the one to Remark 2.2 and following. "In the revised manuscript, we have stated clearly that our framework is developed for porous media that is saturated with two immiscible fluids (the capillarity is not significant), which are commonly known as the wetting phase fluid and the non-wetting phase fluid. For example, in a oil-water system, water is the wetting phase fluid, while the oil is the non-wetting phase fluid. Similarly, in a gas-water system, gas is the non-wetting phase fluid. This framework apparently can be downgraded to the case of saturated single phase flow. It is critical to recognize that this situation is clearly different from that of unsaturated porous media, as the suction stress* $s = p_a - p_w$ *is usually significant in the latter case and can also affect the constitutive law..." This simply adds confusion. The authors should consult the Lewis and Schrefler (1998), the only book to my best knowledge which treats both* ***partially saturated flow*** *and* ***three phase flow*** *in an oil reservoir. There is no difference between the two, both are* ***treated in the same manner with the same governing equations****, containing* ***capillary pressures****. It is* ***always*** *multi (two) phase flow in a porous medium. Water and air are still immiscible, only air can be dissolved in water (Henry's law), but this is a different situation. The only case when in presence of two fluids the pressure of one fluid can be taken as the reference (mean neutral) pressure (i.e.,* $\bar{p} = p_w$*) is when the second fluid is present in form of bubbles in the first fluid and does not touch the solid phase, see Figure 1.5 in Zienkiewicz et al. (1999). The authors may assume that capillarity is neglected but should get the physics right.*

Reply: Thank you for your insightful comment. We totally agree with you that the **partially saturated flow** and **three phase flow** have the same treatment on governing equations, and they both contain capillary pressures. This is verified by comparing our proposed framework when

$N = 1$ (single porosity) and 2.5.4.1 (p54) from Lewis and Schrefler (1998), a prestigious book in computational geomechanics.

However, there are still some unique aspects for partially saturated media, as pointed out by Zienkiewicz et al. (1999), "there are again several conflicting expressions in literature" (p40), one of which is the Bishop's parameter χ versus the degree of saturation S_w in the expression of mean neutral pressure $\bar{p}$, sometimes it is also known as the solid pressure p_s (Hassanizadeh and Gray, 1990). Lewis and Schrefler (1998) used volume averaging to show the validity of S_w, and the hybrid mixture theory proposed by Hassanizadeh and Gray (1990) also gives the same result under thermodynamic equilibrium condition or non-equilibrium condition but incompressible solid grain. Borja (2006) further extends the validity of S_w for compressible solid grain, as mentioned in his paper: "Continuum theory of thermodynamics is unable to support the validity of the Skempton stress. To the knowledge of the author, this is the first time that the effective stress tensor, which reflects the effects of solid grain compressibility and partial saturation, has emerged from continuum principles of thermodynamics". Recently, Cheng (2020) used micromechanical analysis to provide a model for unsaturated poroelasticity, in which a partition of S_w is also theoretically justified, see its (39). Nevertheless, the laboratory experimental evidence has supported the correctness of χ, especially for the shear strength of granular material. A best-fit relationship for χ obtained from experimental data is given in (4) of Khalili et al. (2004). In addition, Gray and Schrefler (2007) and Figure 1.5a in Zienkiewicz et al. (1999) also supports the use of χ. In contrast, for multiphase flow coupled with geomechanics in a reservoir, phase saturation is used (Yan et al., 2018), and to the best knowledge of the authors, the proposition of χ in three-phase flow does not exist.

We think it is also the laboratory experiment that motivates the advanced plasticity model (suction dependent) for partially saturated media, as described in 4.6 of Lewis and Schrefler (1998) and many other later influential papers such as Borja (2004); Loret and Khalili (2000). In contrast, for multiphase flow in a reservoir, the laboratory **mechanical** experiments are rare, and therefore, the capillary pressure is always ignored in the mechanical parameters (Camargo et al., 2021; Cusini et al., 2021; Kim and Moridis, 2013; Lewis and Ghafouri, 1997; Ren et al., 2018; White et al., 2019). In our opinion, there are two possible reasons. First, we could have many different capillary pressures among various phases, which are not easy to incorporate into a unified plasticity framework. Second, a complex plasticity model may not give a better match to the field production data. In fact, a benchmark problem for non-saturated flow in deforming porous media (gravity drainage of a sand column) given in 5.7 of Lewis and Schrefler (1998) just assumes linear elasticity and could still match with the experimental results (it by no means implies that the more advanced plasticity model is unnecessary, the more advanced plasticity model is more applicable to complex loading conditions).

Following your suggestion, we have modified our descriptions in the manuscript, in order to avoid any confusion from **the above two aspects**. We tend to use the terminology "multiphase" consistently throughout our manuscript. Besides, the gas dissolution in water is not considered in this work. More explanations (and limitations) about capillary pressure curve are also added on p8. Hope you are satisfied with those changes. If you have any further comments, please let us know. We are looking forward to your feedback.

Reviewer 2

Q 2.1 *The authors have addressed the review comments.*

Reply: Thank you for your approval and your time in reviewing our manuscript.

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