$$-\frac{d^2T}{dz^2} + v\frac{dT}{dz} = Q(z), \qquad 0<z<1$$

$$Q(z) = \begin{cases} 0, & 0 \le z < a \\ Q_0 \sin\left(\frac{(z-a)\pi}{b-a}\right), & a \le z \le b \\ 0, & b < z \le 1 \end{cases}$$

$$T_0 = T(0)$$

Boundary condition: $T(1)$:

$$-\frac{dT}{dz} = \alpha(v)(T(1) - T_{out}), \qquad \alpha(v) = \sqrt{\frac{v^2}{4} + a_c^2} - \frac{v}{2}$$

$$\frac{d^2T}{dz^2} = \frac{T_{n-1} - 2T_n + T_{n+1}}{h^2} + O(h^2), \qquad \frac{dT}{dz} = \frac{T_{n+1} - T_{n-1}}{2h} + O(h^2)$$

$$-\frac{T_{j-1} - 2T_j + T_{j+1}}{h^2} + v\frac{T_{j+1} - T_{j-1}}{2h} = Q_0 \sin\left(\frac{(jh-a)\pi}{b-a}\right) \cdot \mathbb{1}_{[a,b]}(jh)$$

$$-\frac{T_{j-1}}{h^2} + \frac{2T_j}{h^2} - \frac{T_{j+1}}{h^2} + \frac{vT_{j+1}}{2h} - \frac{vT_{j-1}}{2h} = \left(-\frac{1}{h^2} - \frac{v}{2h}\right)T_{j-1} + \frac{2}{h^2}T_j + \left(\frac{v}{2h} - \frac{1}{h^2}\right)T_{j+1}$$

(*)

For $j = N \cdot h$:

$$-\frac{dT}{dz} = \alpha(v)(T(1) - T_{out}) \Rightarrow -\left(\frac{3T_j - 4T_{j-1} + T_{j-2}}{2h}\right) = \alpha(v)(T(1) - T_{out})$$

$$\Rightarrow -\frac{3T_j}{2h} + \frac{4T_{j-1}}{2h} - \frac{T_{j-2}}{2h} = \alpha(v)T_j - \alpha(v)T_{out}$$

$$\Rightarrow \left(-\frac{3}{2h} - \alpha(v)\right)T_j + \frac{4T_{j-1}}{2h} - \frac{T_{j-2}}{2h} = -\alpha(v)T_{out}$$

$\Rightarrow$ insert in (*) $\Rightarrow$ $(T_j \to T_{j+1})$

$$\left\{ T_{j+1} = \frac{1}{\left(-\frac{3}{2h} - \alpha(v)\right)}\left(-\alpha(v)T_{out} - \frac{4T_j}{2h} + \frac{T_{j-1}}{2h}\right) \right\}$$

$$\Rightarrow \left(-\frac{1}{h^2} - \frac{v}{2h}\right)T_{j-1} + \frac{2}{h^2}T_j + \left(\frac{v}{2h} - \frac{1}{h^2}\right) \cdot \frac{1}{\left(\frac{3}{2h} + \alpha(v)\right)}\left(\alpha(v)T_{out} + \frac{4T_j}{2h} - \frac{T_{j-1}}{2h}\right)$$

$$= \left(-\frac{1}{h^2} - \frac{v}{2h}\right) T_{j-1} + \left(\frac{2}{h^2}\right) T_j + \underbrace{\frac{\left(\frac{v}{2h} - \frac{1}{h^2}\right)}{\left(\frac{3}{2h} + \alpha(v)\right)}}_{\vartheta} \left(\alpha(v) T_{out} + \frac{4 T_j}{2h} - \frac{T_{j-1}}{2h}\right)$$

$$= \left(-\frac{1}{h^2} - \frac{v}{2h} - \frac{\vartheta}{2h}\right) T_{j-1} + \left(\frac{2}{h^2} + \vartheta \frac{4}{2h}\right) T_j + \vartheta \alpha(v) T_{out}$$

Therefore: The last raw in the equation system becomes the following:

$$\left(-\frac{1}{h^2} - \frac{v}{2h} - \frac{\vartheta}{2h}\right) T_{j-1} + \left(\frac{2}{h^2} + \frac{4}{2h}\vartheta\right) T_j = Q_0 \sin\left(\frac{(j \cdot h - a)\pi}{(b-a)}\right) \cdot \mathbb{1}_{[a,b]}(jh) - \vartheta \alpha(v) T_{out}$$

where $\alpha(v) = \sqrt{\frac{v^2}{4} + \alpha_0^2} - \frac{v}{2}$