$$\frac{\partial^2 T}{\partial x^2} \approx \frac{T(x_{j+1}, y_i) - 2T(x_j, y_i) + T(x_{j-1}, y_i)}{h^2}$$

$$\frac{\partial^2 T}{\partial y^2} \approx \frac{T(x_j, y_{i+1}) - 2T(x_j, y_i) + T(x_j, y_{i-1})}{h^2}$$

For $\frac{\partial T}{\partial x}(0,y) = 0 \Rightarrow \frac{\partial T}{\partial x}(x_j, y_i) \approx \frac{-3T(x_j,y_i) + 4T(x_{j+1},y_i) - T(x_{j+2},y_i)}{2h}$

$$\Rightarrow \frac{-3T(x_0,y_i) + 4T(x_1,y_i) - T(x_2,y_i)}{2h} = 0 \Leftrightarrow -3T(x_0,y_i) + 4T(x_1,y_i) - T(x_2,y_i) = 0$$

$$\Rightarrow T(x_0,y_i) = \frac{4T(x_1,y_i) - T(x_2,y_i)}{3}$$

insert in $\Longrightarrow$ equation system

$$-\frac{T(x_2,y_i) - 2T(x_1,y_i) + \left(\frac{4T(x_1,y_i) - T(x_2,y_i)}{3}\right)}{h^2} = f$$

$$= -\frac{3T(x_2,y_i) - 6T(x_1,y_i) + 4T(x_1,y_i) - T(x_2,y_i)}{3h^2} = f$$

$$= -\frac{2T(x_2,y_i) - 2T(x_1,y_i)}{3h^2} = f$$

$$\frac{\partial T}{\partial x}(x_j, y_i) = \frac{3T(x_j, y_i) - 4T(x_{j-1}, y_i) + T(x_{j-2}, y_i)}{2h} = 0$$

$$\Rightarrow \quad 3T(x_N, y_i) - 4T(x_{N-1}, y_i) + T(x_{N-2}, y_i) = 0 \quad \Rightarrow T(x_N, y_i) = \frac{4T(x_{N-1}, y_i) - T(x}{3}$$

$$\frac{-\left(\frac{4T(x_{N-1}, y_i) - T(x_{N-2}, y_i)}{3}\right) + 2T(x_{N-1}, y_i) - T(x_{N-2}, y_i)}{h^2} = f$$

$$\frac{-4T(x_{N-1}, y_i) + T(x_{N-2}, y_i) + 6T(x_{N-1}, y_i) - 3T(x_{N-2}, y_i)}{3h^2} = f$$

$$= \frac{2T(x_{N-1}, y_i) - 2T(x_{N-2}, y_i)}{3h^2}$$

Same principle for $\frac{\partial T}{\partial y}(x_j, y_i)$