# Tutorial 1 - 3

### **Various Equations**

pure convection

Wave equation: 1D, 2D

Diffusion equation: 1D, 2D

Laplace equation: 1D, 2D

Inviscid burger equation: 1D, 2D

Viscous Burger equation: 2D

#### Laplace's Equation 1D and 2D

$$\frac{\partial^2 T}{\partial x^2} = 0$$

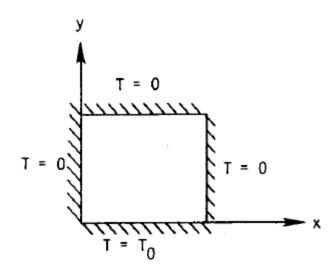
$$T = 350 \text{ K}$$

$$\longrightarrow \text{ X}$$

$$T = 300 \text{ K}$$

$$\nabla^{2}T = \frac{\partial^{2}T}{\partial x^{2}} + \frac{\partial^{2}T}{\partial y^{2}} = 0$$

$$0 \le x \le 1 \quad 0 \le y \le 1$$



#### 1D Wave Equation

$$\frac{\partial^2 T}{\partial t^2} = c^2 \frac{\partial^2 T}{\partial x^2}$$
 It is pure IV Problem C = wave speed

Ex1: c = 0.75 and in x direction periodic BC

*IC*: 
$$T(x,0) = \sin(6\pi x), 0 \le x \le 1$$
  
  $\Delta x = 0.01, \Delta t = 0.01$ 

X

Ex2: c = 0.65 and in x direction periodic BC

*IC*:  $T(x,0) = 2\sin(2\pi x - 0.4\pi)$ ,  $0 \le x \le 2$  $\Delta x = 0.01$ ,  $\Delta t = 0.01$ 

#### 1D Heat Equation

$$\frac{\partial T}{\partial t} = \alpha \left[ \frac{\partial^2 T}{\partial x^2} \right]$$

$$IC: \quad T(x,0) = \sin(\pi x), \quad 0 \le x \le 3$$

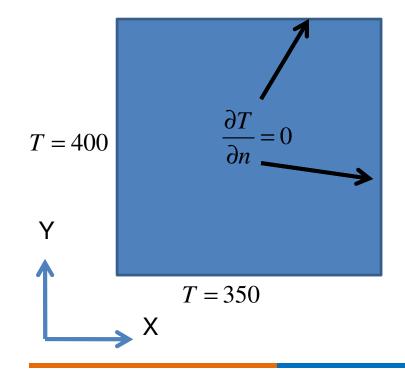
$$\Delta x = 0.01$$
,  $\Delta t = 0.01$   $\alpha = 0.02 \ m^2 / h$ 

$$T = 0$$



## 2D Heat Equation

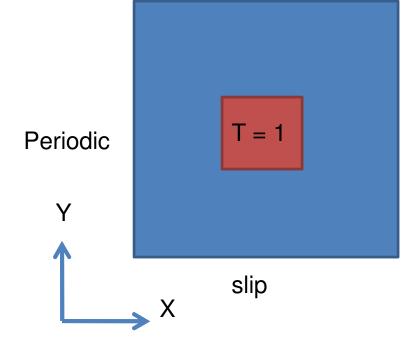
$$\frac{\partial T}{\partial t} = \alpha \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right]$$



*IC*: 
$$T(x, y, 0) = 0$$
  
 $\Delta x = 0.01$ ,  $\Delta t = 0.01$   
 $\alpha = 0.05$ 

# 2D Convection Diffusion Equation

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \mu \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right]$$



Periodic

$$0 \le x \le 2$$
$$0 \le y \le 2$$
$$\mu = 0.05$$

$$\Delta x = 0.01, \ \Delta t = 0.01$$

#### **Pure Convection**

$$\frac{\partial T}{\partial t} + \frac{\partial (\phi T)}{\partial x} = 0$$

$$u = (1, 0, 0)$$

$$T = 1$$

u = zeroGradient

T = zeroGradient

$$T = 0 \ and \ u = (1 \ 0 \ 0)$$



