# Programming Module

# Session-8

# **Solver 1:**
# Implementation of the transport equation model into cavitatingFoam: ”cavitatingTransportFoam”

# Solver1: cavitatingTransportFoam

**About cavitatingFoam**

Barotropic equation of state: $$\frac{D\rho_m}{Dt} = \Psi \frac{DP}{Dt}$$

Continuity equation: $$\frac{\partial \rho_m}{\partial t} + \nabla \bullet (\rho_m U) = 0$$

Momentum equation: $$\frac{\partial \rho_m U}{\partial t} + \nabla \bullet (\rho_m U U) = -\nabla P + \nabla \bullet \left[ (\mu_{eff} (\nabla U + (\nabla U)^T) \right]$$

Vapor mass faction: $$\gamma = \frac{\rho_m - \rho_{l,sat}}{\rho_{v,sat} - \rho_{l,sat}}$$

$\rho_m$, $\rho_{l,sat}$ and $\rho_{v,sat}$ are density of the mixture, liquid and vapor densities at saturation pressure

# Solver1: cavitatingTransportFoam

**About interPhaseChangeFoam**

Continuity equation: $\frac{\partial \rho_m}{\partial t} + \nabla \bullet (\rho_m U) = 0$

Momentum equation: $\frac{\partial \rho_m U}{\partial t} + \nabla \bullet (\rho_m U U) = -\nabla P + \nabla \bullet \left[ (\mu_{eff} (\nabla U + (\nabla U)^T) \right]$

Transport equation:

$$\frac{\partial (\alpha \rho_l)}{\partial t} + \nabla \bullet (\alpha \rho_l \mathbf{U}) + \nabla \bullet \left[ \alpha \mathbf{U}_c (1-\alpha) \right] = R_c - R_e$$

The mixture density and viscosity:

$$\rho_m = (1-\alpha)\rho_v + \alpha \rho_l$$

$$\mu_m = (1-\alpha)\mu_v + \alpha \mu_l$$

# Solver1: cavitatingTransportFoam

**Kunz Model**

Transport equation:

$$\frac{\partial(\alpha\rho_l)}{\partial t} + \nabla \bullet (\alpha\rho_l \mathbf{U}) + \nabla \bullet \left[\alpha \mathbf{U}_c (1-\alpha)\right] = R_c - R_e$$

Evaporation source term

$$R_e = Cv \frac{\alpha\rho_v}{t_\infty (0.5\rho_l \mathbf{U}_\infty{}^2)} \min[0, P - P_v]$$

Condensation source term

$$R_c = Cc \frac{(1-\alpha)\alpha^2 \rho_v}{t_\infty}$$

$U_\infty$: *mean stream velocity*
$P_v$: *vapor pressure*
$C_c$: *empirical condensation constants*
$C_v$: *empirical vaporization constants*

## **Solver 2:**

## Implementation of ODE solvers

# Solver1: ODE Solvers

**ODETest**

$$\frac{dy_1}{dx} = -y_2, \qquad \frac{dy_2}{dx} = y_1 - \frac{y_2}{x},$$

$$\frac{dy_3}{dx} = y_2 - \frac{2y_3}{x}, \qquad \frac{dy_4}{dx} = y_3 - \frac{3y_4}{x}$$

Then we calculate the jacobian for x and y:

$$\frac{-dy_2}{dx} = 0, \qquad \frac{d(y_1 - \frac{y_2}{x})}{dx} = \frac{y_2}{x^2}, \qquad \frac{d(y_2 - \frac{2y_3}{x})}{dx} = \frac{2y_3}{x^2}, \qquad \frac{d(y_3 - \frac{3y_4}{x})}{dx} = \frac{3y_4}{x^2}$$

$$\frac{-dy_2}{dy_1} = 0, \qquad \frac{-dy_2}{dy_2} = -1, \qquad \frac{-dy_2}{dy_3} = 0, \qquad \frac{-dy_2}{dy_4} = 0$$

$$\frac{d(y_1 - \frac{y_2}{x})}{dy_1} = 1, \qquad \frac{d(y_1 - \frac{y_2}{x})}{dy_2} = -\frac{1}{x}, \qquad \frac{d(y_1 - \frac{y_2}{x})}{dy_3} = 0, \qquad \frac{d(y_1 - \frac{y_2}{x})}{dy_4} = 0$$

$$\frac{d(y_2 - \frac{2y_3}{x})}{dy_1} = 0, \qquad \frac{d(y_2 - \frac{2y_3}{x})}{dy_2} = 1, \qquad \frac{d(y_2 - \frac{2y_3}{x})}{dy_3} = -\frac{2}{x}, \qquad \frac{d(y_2 - \frac{2y_3}{x})}{dy_4} = 0$$

$$\frac{d(y_3 - \frac{3y_4}{x})}{dy_1} = 0, \qquad \frac{d(y_3 - \frac{3y_4}{x})}{dy_2} = 0, \qquad \frac{d(y_3 - \frac{3y_4}{x})}{dy_3} 1, \qquad \frac{d(y_3 - \frac{3y_4}{x})}{dy_4} = -\frac{3}{x}$$

# Solver1: ODE Solvers

**myODETest**

$$\frac{d^2y}{dx^2} = y, y(0) = 0, y'(0) = 1$$

Then change our ODE to first order ODEs

$$\frac{dy}{dx} = z, \frac{dz}{dx} = y, y(0) = 0, z(0) = 1$$

Jacobian for x:

$$\frac{dz}{dx} = 0, \frac{dy}{dx} = 0$$

Jacobian for y:

$$\frac{dz}{dy} = 0, \frac{dz}{dz} = 1, \frac{dy}{dy} = 1, \frac{dy}{dz} = 0,$$

# **Solver 3:**
# Implementation of Electro-Magnetic Solver

# Solver 3: Electro-Magnetic Solver

**Governing equations**

**Maxwell's equations:** $\nabla \times E = 0$

where E is the electric field strength. $\nabla \cdot B = 0$

where B is the magnetic flux density. $\nabla \times H = J$

where H is the magnetic field strength and J is current density

**Charge continuity:** $\nabla \cdot J = 0$

**Ohm's law:** $J = \sigma E$ where $\sigma$ is the electric conductivity

**Constitutive law:** $B = \mu_0 H$

where $\mu_0$ is the magnetic permeability of vacuum

# Solver 3: Electro-Magnetic Solver

**Governing equations**

Combining above all equations and Coulomb gauge condition ($\nabla \cdot A = 0$) leads to

Poissons's equation for the magneto static fields: $\nabla^2 A = \mu_0 \sigma (\nabla \phi)$

Laplace's equation for the electric potential: $\nabla \cdot [\sigma (\nabla \phi)] = 0$

# **Solver 4:**
# Implementation of chtMultiRegionFoam for incompressible fluid

## Solver 5:

Modification of stressedFoam for the implementation of material non-linearity

# Solver 5: stressedFoam with non-linearity

**Momentum balance**

$$\frac{\partial^2(\rho\mathbf{u})}{\partial t^2} - \nabla \cdot \boldsymbol{\sigma} = \mathbf{0}$$

**Constitutive relation (linear elasticity)** $\boldsymbol{\sigma} = 2\mu\varepsilon + \lambda tr(\varepsilon)\mathbf{I}$

**Stain-displacement relation** $\varepsilon = \frac{1}{2}[\nabla\mathbf{u} + (\nabla\mathbf{u})^T]$

$\mathbf{u}$ is the solid displacement vector

$\rho$ is the density

$\boldsymbol{\sigma}$ is the stress tensor

$\mathbf{I}$ is the unit tensor

$\mu$ and $\lambda$ are material properties called Lamé parameters

# Solver 5: stressedFoam with non-linearity

Combining all above equations, the governing equation for linear elastic solid body is written with the only unknown variable u.

$$\frac{\partial^2(\rho\mathbf{u})}{\partial t^2} - \nabla \cdot \left[\mu\nabla\mathbf{u} + \underline{\mu(\nabla\mathbf{u})^T + \lambda\mathbf{I}tr(\nabla\mathbf{u})}\right] = \mathbf{0}$$

Further rearrangement to improve the convergence

$$\nabla \cdot \boldsymbol{\sigma} = \underbrace{\nabla \cdot (\mu\nabla\mathbf{u})}_{implicit} + \underbrace{\nabla \cdot [\mu(\nabla\mathbf{u})^T + \lambda\mathbf{I}tr(\nabla\mathbf{u})]}_{explicit}$$

$$= \underbrace{\nabla \cdot [(2\mu+\lambda)\nabla\mathbf{u}]}_{implicit} + \underbrace{\nabla \cdot [\mu(\nabla\mathbf{u})^T + \lambda\mathbf{I}tr(\nabla\mathbf{u}) - (\mu+\lambda)\nabla\mathbf{u}]}_{explicit}$$

# Solver 5: stressedFoam with non-linearity

**The traction Displacement boundary**

Fixed traction force (T), which can be expressed as: $\mathbf{T} = \boldsymbol{\sigma} \cdot \mathbf{n}$

Where **n** is the surface normal on the boundary.

Since the displacement **u** is the computing variable , the traction force is converted into displacement on the boundary.

$$\mathbf{T} = \boldsymbol{\sigma} \cdot \mathbf{n} = \left[\mu\nabla\mathbf{u} + \mu(\nabla\mathbf{u})^T + \lambda\mathbf{I}tr(\nabla\mathbf{u})\right] \cdot \mathbf{n}$$

$$= [\underbrace{(2\mu + \lambda)\nabla\mathbf{u}}_{implicit} + \underbrace{\mu(\nabla\mathbf{u})^T + \lambda\mathbf{I}tr(\nabla\mathbf{u}) - (\mu + \lambda)\nabla\mathbf{u}}_{explicit}] \cdot \mathbf{n}$$

$$(\nabla\mathbf{u}) \cdot \mathbf{n} = \frac{\mathbf{T} - [\mu(\nabla\mathbf{u})^T + \lambda\mathbf{I}tr(\nabla\mathbf{u}) - (\mu + \lambda)\nabla\mathbf{u}] \cdot \mathbf{n}}{2\mu + \lambda}$$

# Solver 5: stressedFoam with non-linearity

**Implementation of material non-linearity like a plasticity in a solid body**

Add plastic feature $[2\mu(d\varepsilon_p) + \lambda \mathbf{I} tr\,(d\varepsilon_p)]$

$d\varepsilon_p$ stands for the incremental plastic strain tensor

$$\nabla \cdot \left\{ \mu \nabla(d\mathbf{u}) + \mu [\nabla(d\mathbf{u})]^T + \lambda \mathbf{I} tr[\nabla(d\mathbf{u})] - \underbrace{[2\mu(d\varepsilon_p) + \lambda \mathbf{I} tr\,(d\varepsilon_p)]}_{plasticity} \right\} = \mathbf{0}$$

$$\nabla \cdot [(2\mu + \lambda)\nabla(d\mathbf{u})] =$$

$$-\nabla \cdot \{\mu[\nabla(d\mathbf{u})]^T + \lambda \mathbf{I} tr[\nabla(d\mathbf{u})] - (\mu + \lambda)\nabla(d\mathbf{u})\} +$$

$$\nabla \cdot [2\mu(d\varepsilon_p) + \lambda \mathbf{I} tr\,(d\varepsilon_p)]$$

# Thank you