Programming Module

Session-9-10

Solver 1:

Implementation of the transport equation model into cavitatingFoam: ”cavitatingTransportFoam”

Solver1: cavitatingTransportFoam

About cavitatingFoam

Barotropic equation of state: $$\frac{D\rho_m}{Dt} = \Psi \frac{DP}{Dt}$$

Continuity equation: $$\frac{\partial \rho_m}{\partial t} + \nabla \bullet (\rho_m U) = 0$$

Momentum equation: $$\frac{\partial \rho_m U}{\partial t} + \nabla \bullet (\rho_m U U) = -\nabla P + \nabla \bullet \left[(\mu_{eff} (\nabla U + (\nabla U)^T) \right]$$

Vapor mass faction: $$\gamma = \frac{\rho_m - \rho_{l,sat}}{\rho_{v,sat} - \rho_{l,sat}}$$

ρ_m, $\rho_{l,sat}$ and $\rho_{v,sat}$ are density of the mixture, liquid and vapor densities at saturation pressure

Solver1: cavitatingTransportFoam

About interPhaseChangeFoam

Continuity equation: $$\frac{\partial \rho_m}{\partial t} + \nabla \bullet (\rho_m U) = 0$$

Momentum equation: $$\frac{\partial \rho_m U}{\partial t} + \nabla \bullet (\rho_m U U) = -\nabla P + \nabla \bullet \left[(\mu_{eff} (\nabla U + (\nabla U)^T) \right]$$

Transport equation:

$$\frac{\partial (\alpha \rho_l)}{\partial t} + \nabla \bullet (\alpha \rho_l \mathbf{U}) + \nabla \bullet \left[\alpha \mathbf{U}_c (1-\alpha) \right] = R_c - R_e$$

The mixture density and viscosity:

$$\rho_m = (1-\alpha)\rho_v + \alpha \rho_l$$

$$\mu_m = (1-\alpha)\mu_v + \alpha \mu_l$$

Solver1: cavitatingTransportFoam

Kunz Model

Transport equation:

$$\frac{\partial(\alpha\rho_l)}{\partial t} + \nabla \bullet (\alpha\rho_l \mathbf{U}) + \nabla \bullet \left[\alpha \mathbf{U}_c (1-\alpha)\right] = R_c - R_e$$

Evaporation source term

$$R_e = Cv \frac{\alpha\rho_v}{t_\infty (0.5 \rho_l \mathbf{U}_\infty{}^2)} \min[0, P - P_v]$$

Condensation source term

$$R_c = Cc \frac{(1-\alpha)\alpha^2 \rho_v}{t_\infty}$$

U_∞: *mean stream velocity*
P_v: *vapor pressure*
C_c: *empirical condensation constants*
C_v: *empirical vaporization constants*

Solver 2:
Implementation of ODE solvers

Solver1: ODE Solvers

ODETest

$$\frac{dy_1}{dx} = -y_2, \qquad \frac{dy_2}{dx} = y_1 - \frac{y_2}{x},$$

$$\frac{dy_3}{dx} = y_2 - \frac{2y_3}{x}, \qquad \frac{dy_4}{dx} = y_3 - \frac{3y_4}{x}$$

Then we calculate the jacobian for x and y:

$$\frac{-dy_2}{dx} = 0, \qquad \frac{d(y_1 - \frac{y_2}{x})}{dx} = \frac{y_2}{x^2}, \qquad \frac{d(y_2 - \frac{2y_3}{x})}{dx} = \frac{2y_3}{x^2}, \qquad \frac{d(y_3 - \frac{3y_4}{x})}{dx} = \frac{3y_4}{x^2}$$

$$\frac{-dy_2}{dy_1} = 0, \qquad \frac{-dy_2}{dy_2} = -1, \qquad \frac{-dy_2}{dy_3} = 0, \qquad \frac{-dy_2}{dy_4} = 0$$

$$\frac{d(y_1 - \frac{y_2}{x})}{dy_1} = 1, \qquad \frac{d(y_1 - \frac{y_2}{x})}{dy_2} = -\frac{1}{x}, \qquad \frac{d(y_1 - \frac{y_2}{x})}{dy_3} = 0, \qquad \frac{d(y_1 - \frac{y_2}{x})}{dy_4} = 0$$

$$\frac{d(y_2 - \frac{2y_3}{x})}{dy_1} = 0, \qquad \frac{d(y_2 - \frac{2y_3}{x})}{dy_2} = 1, \qquad \frac{d(y_2 - \frac{2y_3}{x})}{dy_3} = -\frac{2}{x}, \qquad \frac{d(y_2 - \frac{2y_3}{x})}{dy_4} = 0$$

$$\frac{d(y_3 - \frac{3y_4}{x})}{dy_1} = 0, \qquad \frac{d(y_3 - \frac{3y_4}{x})}{dy_2} = 0, \qquad \frac{d(y_3 - \frac{3y_4}{x})}{dy_3} 1, \qquad \frac{d(y_3 - \frac{3y_4}{x})}{dy_4} = -\frac{3}{x}$$

Solver1: ODE Solvers

myODETest

$$\frac{d^2y}{dx^2} = y, y(0) = 0, y'(0) = 1$$

Then change our ODE to first order ODEs

$$\frac{dy}{dx} = z, \frac{dz}{dx} = y, y(0) = 0, z(0) = 1$$

Jacobian for x:

$$\frac{dz}{dx} = 0, \frac{dy}{dx} = 0$$

Jacobian for y:

$$\frac{dz}{dy} = 0, \frac{dz}{dz} = 1, \frac{dy}{dy} = 1, \frac{dy}{dz} = 0,$$

Solver 3:

Implementation of Electro-Magnetic Solver

Solver 3: Electro-Magnetic Solver

Governing equations

Maxwell's equations: $\nabla \times E = 0$

where E is the electric field strength. $\nabla \cdot B = 0$

where B is the magnetic flux density. $\nabla \times H = J$

where H is the magnetic field strength and J is current density

Charge continuity: $\nabla \cdot J = 0$

Ohm's law: $J = \sigma E$ where σ is the electric conductivity

Constitutive law: $B = \mu_0 H$

where μ_0 is the magnetic permeability of vacuum

Solver 3: Electro-Magnetic Solver

Governing equations

Combining above all equations and Coulomb gauge condition ($\nabla \cdot A = 0$) leads to

Poissons's equation for the magneto static fields: $\nabla \cdot [\sigma(\nabla \phi)] = 0$

Laplace's equation for the electric potential: $\nabla^2 A = \mu_0 \sigma(\nabla \phi)$

Solver 3: Electro-Magnetic Solver

Solver 3: Electro-Magnetic Solver

Thank you