Programming Module

Session-8

Implementation of a non-Newtonian model

Non-Newtonian Model	Mathematical Eq.	Coefficient
`powerLaw`	$\eta = K\dot{\gamma}^{n-1}$	K: Consistency index $\dot{\gamma}$: Shear rate n: power law index
`CrossPowerLaw`	$\eta = \frac{\eta_0 - \eta_{inf}}{1+(m\dot{\gamma})^n} + \eta_{inf}$	m: time constant η_0: lower bound viscosity η_{inf}: upper bound viscosity $\dot{\gamma}$: Shear rate
`HerschelBulkley`	$\eta = \tau_y + (k\dot{\gamma})^{n-1}$	τ_y: yield stress k: time constant $\dot{\gamma}$: Shear strain rate
`BirdCarreau`	$\eta = \eta_{inf} + (\eta_0 - \eta_{inf})(1 + (k\dot{\gamma})^{n-1})$	k: time constant η_0: lower bound viscosity η_{inf}: upper bound viscosity $\dot{\gamma}$: Shear rate

Mathematical formulation of non-Newtonian models in OpenFOAM

Implementation of a non-Newtonian model : Casson model

$$\sqrt{\eta} = \sqrt{\frac{\tau_y}{\dot{\gamma}}} + \sqrt{m}$$

η is the viscosity (m^2/s)

τ_y is yield stress (m^2/s^2)

$\dot{\gamma}$ is shear strain rate

m is the consistency index (m^2/s)

Implementation of a Temperature Dependent Viscosity Model

Strain Rate in OpenFOAM

$$Strain\ Rate = \sqrt{2 *\ symm(gard(U)) : symm(gard(U))}$$

$$symm\big(grad(U)\big) = \ grad(U) + \ grad(U).T$$

The dependency of viscosity obeys the following equations:

$$\mu = k(\frac{\partial u_i}{\partial x_i})^{n-1} \qquad k = k_0 - m_k(T - T_0)$$

which $\boldsymbol{k_0}$ and $\boldsymbol{m_k}$ are initial value of viscosity and temperature dependency coefficient.

Modification in dynamicInkJetFvMesh to generate moving waves

Three different wave types are included in the MovingWave class

linear wave: $\eta(x,t) = \mathrm{a} * \cos(\kappa \mathrm{x} - \omega \mathrm{t})$

second order Stocks wave: $\eta(x,t) = a cos\Theta + 0.5(\kappa a)\ cos2\Theta$

third order Stocks wave:

$$\eta(x,t) = a cos\Theta + 0.5(\kappa a) cos2\Theta + {}^{3}/_{8}\,(\kappa a)^2 cos3\Theta$$

$$\Theta = (\kappa x - \omega t)$$

η :the surface elevation of a deep water wave,

x : the horizontal coordinate;

t : time;

a : the first-order wave amplitude;

k : the angular wavenumber, $k = 2\pi / \lambda$ with λ being the wavelength;

ω: the angular frequency, $\omega = 2\pi / \tau$ where τ is the period time.

Thank you