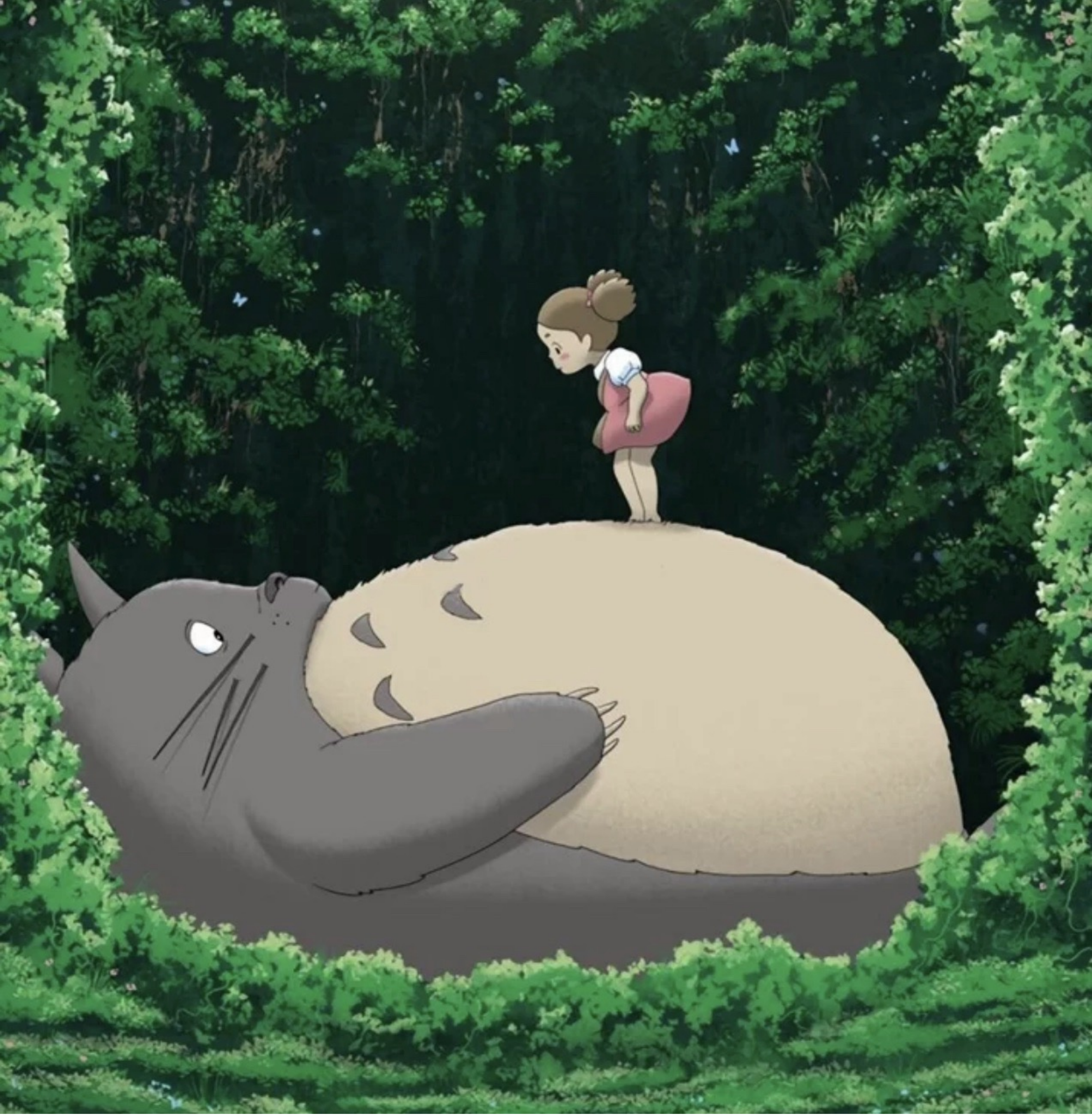

Fourier Analysis

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Date: August 26, 2023

Contents

Chapter 1 Fourier Series and Integrals

We will go through the book's notes in this document. Chapter 1 is organized as follows:

1. Fourier coefficients and Series
2. Criteria for pointwise convergence
3. Convergence in norm
4. summability methods
5. The fourier transform of L^1 functions
6. Schwartz class and tempered distributions
7. Fourier transform on L^p, for $1 < p \leq 2$
8. Convergence and summability of Fourier Integrals
9. Further results

The Lebesgue measure in $\mathbb{R}^n$ will be denoted using dx, and on the unit sphere S^{n-1} will be $d\sigma$.

Let $a = (a_1, ..., a_n) \in \mathbb{N}^n$ be a multiindex, and $f : \mathbb{R}^n \to \mathbb{C}$, then

$$D^a f = \frac{\partial^{|a|} f}{\partial_{x_1}^{a_1} ... \partial_{x_n}^{a_n}}$$

where $|a| = a_1 + ... + a_n$.

Theorem 1.1 (Minkowski's integral inequality.)

Given $(X, \mu), (Y, \nu)$ as σ-finite measure spaces, we have the following inequality

$$\left(\int_X \left| \int_Y f(x,y) d\nu(y) \right|^p d\mu(x) \right)^{1/p} \leq \int_Y \left(\int_X |f(x,y)^p d\mu(x) \right)^{1/p} d\nu(y)$$

♡

Taking ν to be the counting measure over a two point set $S = 1, 2$ gives the usual Minkowski inequality

$$\|f_1 + f_2\|_{L^p} \leq \|f_!\|_{L^p} + \|f_2\|_{L^p}$$

We will use $\mathcal{D}$ to denote the space of test functions, i.e. C_c^∞, and $\mathcal{S}$ to denote the space of Schwartz functions.