Calc III Sections

Fall 2025

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Calc III-Week 8 (Fall Break)

Topics: (1) Acceleration and Arc Length, (2) Vector Fields.

Proposition 1 (Newton's second law)**.** Let $c(t)$ be a path of a particle with mass m and $a(t) = c''(t)$ be the acceleration, then

$$F(c(t)) = ma(t)$$

where F is the force applying on the particle.

Definition 1 (arc length)**.** Let $c(t) = (x(t), y(t), z(t))$ be a path, then the length of the path in $\mathbb{R}^3$ from $t_0 \le t \le t_1$ is

$$\begin{aligned} L_{t_0 \to t_1}(c) &= \int_{t_0}^{t_1} \left(x(t)^2 + y(t)^2 + z(t)^2\right)^{\frac{1}{2}} dt \\ &= \int_{t_0}^{t_1} \|c'(t)\| dt \end{aligned}$$

More generally, if $c(t) = (x_1(t), \ldots, x_n(t))$ is a path in $\mathbb{R}^n$, then

$$L_{t_0 \to t_1}(c) = \int_{t_0}^{t_1} \left(\sum_{i=1}^{n} x_i(t)^2\right)^{\frac{1}{2}} dt$$

Definition 2 (vector field)**.** A vector field is a function $F : A \subset \mathbb{R}^n \to \mathbb{R}^n$ that assigns $x \in \mathbb{R}^n$ to another vector $F(x) \in \mathbb{R}^n$.

Problem 1. Find the velocity, speed, and acceleration of the following path at $t = 0$:

$$c(t) = (\cos t, 2t, -\sin t)$$

Proof. The velocity is

$$c'(t) = (-\sin t, 2, -\cos t), \quad c'(0) = (0, 2, -1)$$

And the speed is

$$\|c'(t)\| = \sqrt{5}$$

□

Problem 2. Find the length of the curve above from $t = 0$ to $t = 2$.

Proof.

$$\begin{aligned} L_{0 \to 2} \|c'(t)\| dt &= \int_0^2 \left(\sin^2 t + 4 + \cos^2 t\right)^{\frac{1}{2}} dt \\ &= \int_0^2 \sqrt{5} dt \\ &= 2\sqrt{5} \end{aligned}$$

□