

Metric Geometry

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A metric is a function $d : X \times X \rightarrow \mathbb{R} \cup \{\infty\}$ such that it is positive-definite, symmetric, triangle inequality. A semi-metric is a function d such that it does **not** satisfy

$$d(x, y) = 0 \Rightarrow x = y$$

Definition 0.1 (isometry). An isometry is a bijective $f : X \rightarrow Y$ is a function such that for any $x_1, x_2 \in X$,

$$d(f(x_1), f(x_2)) = d(x_1, x_2)$$

Example 0.1. Another example of a metric on \mathbb{R} (that is different from $|x - y|$) is $\log |x - y|$.