

Calc III Midterm Essay Review

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Chapter 1

Definition review

1. definition review
2. proposition review
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1.1, 1.2, 1.3, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6 3.1, 3.2

Definition 1.1 (standard basis in \mathbb{R}^3). The vectors

$$i = (1, 0, 0), j = (0, 1, 0), k = (0, 0, 1)$$

are called the standard basis vectors of \mathbb{R}^3 , and for any vector $a = (a_1, a_2, a_3) \in \mathbb{R}^3$, we can write

$$a = a_1 i + a_2 j + a_3 k$$

Definition 1.2 (Equation of a line). A line l in \mathbb{R}^3 through the tip of $a = (a_1, a_2, a_3)$ pointing in the direction of a vector $v = (v_1, v_2, v_3)$ is given by

$$l(t) = a + tv$$

where $t \in \mathbb{R}$. Alternatively, a line passing through two points $P = (x_1, y_1, z_1), Q = (x_2, y_2, z_2)$ is given by

$$l(t) = (x(t), y(t), z(t))$$

where

$$\begin{cases} x(t) = x_1 + (x_2 - x_1)t \\ y(t) = y_1 + (y_2 - y_1)t \\ z(t) = z_1 + (z_2 - z_1)t \end{cases}$$

Chapter 2

Practice Problems

Problem 2.1. Find the equation of the line passing through $(1, 0, 2)$ in the direction $(2, -1, 3)$.

Proof. By definition 1.2 The line is given by

$$l(t) = (1 + 2t, -t, 2 + 3t)$$

□

Problem 2.2. In which direction does the line

$$l(t) = (3 - 2t, 2 + 5t, 1 + t)$$

point?

Proof. In the direction of the vector $(-2, 5, 1)$.

□

Problem 2.3. Compute the following limits if they exist; if the limits don't exist, please explain why.

1.

$$\lim_{(x,y) \rightarrow (1,1)} \frac{x^2 + y^2 - 2xy}{x - y}$$

Problem 2.4. Find the tangent plane of $f(x, y) = \ln(x + y) - 2x$ at $(1, 2)$.

Problem 2.5.

Chapter 3

Answer Key