Calc III Sections

Fall 2025

Hui Sun

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Calc III-Week 1 (8/25-29)

1 Logistics

- TA: Hui.
- Email: hsun95@jh.edu.
- Office Hour (tentative): Tuesday 4-6 PM, Krieger 211.
- Biweekly Quizzes: 10-15 min, 10%.
- Attendance: 5%. (If you can't make it, email me).

2 Icebreaking Activity

- In a group of three or four:
  1. Learn each other names, year, pronouns.
  2. Find something in common and different among you and share with the entire class.
  3. Play Buzz if you have time, with prime 7: say the number if it doens't contain or is not divisible by 7, say buzz otherwise.

3 Some Math

Problem 1. Draw the following vectors in $\mathbb{R}^2$:

$$u = (1, 2), \quad v = (3, -2)$$

Compute $u + v, u - v$, and draw them in the plane.

Proof.

$$u + v = (4, 0), \quad u - v = (-2, 4)$$

□

Problem 2. Consider the following vectors in $\mathbb{R}^3$:

$$u = (1, 2, 3), \quad , v = (-2, 1, 4)$$

1. Compute their norms.
2. Two vectors $a, b \in \mathbb{R}^3$ are called **orthognal** if $a \cdot b = 0$. Are u, v orthogonal? If not, find a nonzero vector orthogonal to u.

Proof. 1.

$$\|u\| = (u \cdot u)^{\frac{1}{2}} = \sqrt{14}, \quad \|v\| = \sqrt{21}$$

2. We check

$$u \cdot v = -2 + 2 + 12 = 12 \neq 0$$

thus not orthogonal. A vector that is orthognal to u: $(-3, 0, 1)$. Note that this vector is **not** unique! For example, $(-1, -1, 1)$ is another such vector.

□

Problem 3. Let $u, v \in \mathbb{R}^3$, suppose that u, v are orthongal, show that

$$\|u+v\|^2 = \|u\|^2 + \|v\|^2$$

Bonus: is the converse true? (meaning assuming $\|u+v\|^2 = \|u\|^2 + \|v\|^2$, is it true that $u \cdot v = 0$?)

Proof. We have

$$\begin{aligned} \|u+v\|^2 &= (u+v)\cdot(u+v) \\ &= u\cdot u + u\cdot v + v\cdot u + v\cdot v \\ &= \|u\|^2 + \|v\|^2 \end{aligned}$$

because $u \cdot v = v \cdot u = 0$. The converse is also true: we know by definition that

$$\|u+v\|^2 = \|u\|^2 + \|v\|^2 + 2u\cdot v$$

given the assumption, we also have

$$\|u+v\|^2 = \|u\|^2 + \|v\|^2$$

Thus equating them we get

$$\|u\|^2 + \|v\|^2 + 2u\cdot v = \|u\|^2 + \|v\|^2 \Rightarrow u\cdot v = 0$$

□

4 Reminders

1. First HW due this Friday.
2. First Quiz next Tuesday.