Additive Combinatorics Chapter 1 exercises

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Problem 0.1. We have the following expected value formulas.

$$\mathbb{E}(X) = \int_0^\infty P(X \geq \lambda) d\lambda$$

and

$$\mathbb{E}(X^p) = p \int_0^\infty \lambda^{p-1} P(X > \lambda) d\lambda$$

Proof. Change the order of integration.

$$\begin{aligned} \mathbb{E}(X) &= \int_0^\infty \lambda P(X = \lambda) d\lambda \\ &= \end{aligned}$$

□

Problem 0.2. When does equality hold in Markov's inequality?

Proof. In the proof of Markov's inequality, we used that $X \geq \lambda I(X \geq \lambda)$. For equality to hold, we need $X = \lambda I(X \geq \lambda)$, but this is not possible for $X \not\equiv 0$. Suppose $X = a$ for some $a > 0$, then take $\lambda = a/2, a/3$, we reach a contradiction. □

Warning 0.1. This is a warning.