Calc III Section Notes with Answers

Spring 2026

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Contents

Chapter 1

The Geometry of Euclidean Spaces

Week 1 (1/19-23)

Logistics

- TA: Hui.
- Email: hsun95@jh.edu.
- Office Hours: Tuesday 4-6 PM, Krieger 211; Friday 1-2 PM Zoom.
- Biweekly Quizzes: 15 min, 10%.
- Attendance: 5%. (If you can't make it, email me).

Definition 1 (dot product).

Definition 2 (linear combination).

Definition 3 (standard basis).

Problem 1. Draw the following vectors in $\mathbb{R}^2$:

$$u = (1, 2), \quad v = (3, -2)$$

Compute $u + v, u - v$, and draw them in the plane.

Proof.

$$u + v = (4, 0), \quad u - v = (-2, 4)$$

□

Problem 2. Consider the following vectors in $\mathbb{R}^3$:

$$u = (1, 2, 3), \quad , v = (-2, 1, 4)$$

1. Compute their norms.
2. Two vectors $a, b \in \mathbb{R}^3$ are called **orthognal** if $a \cdot b = 0$. Are u, v orthogonal? If not, find a nonzero vector orthogonal to u.

Proof. 1.

$$\|u\| = (u \cdot u)^{\frac{1}{2}} = \sqrt{14}, \quad \|v\| = \sqrt{21}$$

2. We check

$$u \cdot v = -2 + 2 + 12 = 12 \neq 0$$

thus not orthogonal. A vector that is orthognal to u: $(-3, 0, 1)$. Note that this vector is **not** unique! For example, $(-1, -1, 1)$ is another such vector.

□

Problem 3. Can you express $w = (1, 0)$ as a linear combination of v_1, v_2 for difference choices of v_1, v_2?

1. $v_1 = (1, 1), v_2 = (-2, -2)$.
2. $v_1 = (2, 1), v_2 = (-1, 0)$.

Week 2 (1/26-30)