Columbia HW Problems

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August 4, 2025

Contents

Chapter 1

HW2

Problem 1.1 (1). Determine whether the following statements are true or false. Justify your answers.

(a) Any subring of a field is an integral domain.

(b) The ring $\mathbb{Z}/49\mathbb{Z}$ is an integral domain.

(c) The direct product $F_1 \times F_2$ of two fields is a field.

(d) An element ab of a ring R is invertible if and only if both a and b are invertible.

(e) The ring $\mathbb{Z} \times \mathbb{Z}$ has exactly four idempotents.

Hint: First find all idempotents in the ring $\mathbb{Z}$. An idempotent is an element e such that $e^2 = e$.

Proof. (a) True (b) False (c) False, consider $(1,0) \cdot (0,1)$ (d) False Consider $5 + 5 = 1$ in $\mathbb{Z}/6\mathbb{Z}$ (e) True. □