Calc III Sections

Fall 2025

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Calc III-Week 11 (11/3-7)

Topic: theorem of change of variables.

Definition 0.1 (injective, surjective)**.** Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a map, then T is **injective or one-to-one** if

$$Tx = Ty$$

then

$$x = y$$

T is called **surjective or onto** if for all $y \in \mathbb{R}^2$, there exists $x \in \mathbb{R}^2$ such that

$$Tx = y$$

Definition 0.2 (linear map)**.** Suppose $T : \mathbb{R}^2 \to \mathbb{R}^2$, then T is linear if and only if for all $x, y \in \mathbb{R}^2, \lambda \in \mathbb{R}$.

$$\begin{cases} T(x + y) = Tx + Ty \\ T(\lambda x) = \lambda Tx \end{cases}$$

Proposition 0.1. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear map, then there exists a 2×2 matrix $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ such that

$$Tx = Ax$$

Proposition 0.2. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$, if T is injective, then it is also surjective. The converse is also true: if T is surjective, then it is also injective.