Calc III Sections

Fall 2025

Hui Sun

September 2, 2025

Calc III-Week 2

Definition 1 (cross product)**.** Let $a = (a_1, a_2, a_3), b = (b_1, b_2, b_3)$ be vectors in $\mathbb{R}^3$, the cross product of a, b is the vector $a \times b$,

$$a \times b = \begin{bmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix}$$

where i, j, k are the standard vectors in $\mathbb{R}^3$.

Proposition 1. Here are some properties of the cross product:

1. $a \times b$ is perpendicular to vectors a, b.
2. The length of the cross product is the area of the parallelogram:

   $$\|a \times b\| = \|a\|\|b\| \sin\theta$$

   where θ is the angle between them. (Compare this with the dot product).
3. $a \times b = -b \times a$, and $a \times (b + c) = a \times b + a \times c$. Moreover, $a \times b = 0$ iff a, b are parallel or either a or b are 0.
4. (HW) The cross product is **not** associative! For example, compute

   $$(i \times i) \times j, \quad i \times (i \times j)$$

Definition 2 (Plane in three dimensions)**.** A perpendicular vector and a normal vector uniquely define a plane in $\mathbb{R}^3$: given the plane $\mathcal{P}$ passing containing the point (x_0, y_0, z_0) that has a normal vector (A, B, C) is given by the equation:

$$\mathcal{P} = A(x - x_0) + B(y - y_0) + C(z - z_0)$$

Problem 1. Let $\vec{u} = (1, 2, 3), \vec{v} = (0, 1, 1)$ be vectors in $\mathbb{R}^3$, compute the area of the parallelogram spanned by these two vectors.

Problem 2. Compute the plane containg all three points:

$$(1, 0, 2), \quad (2, -1, 0), \quad (-1, 2, 3)$$