Calc III Sections

Fall 2025

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Calc III-Week 4 (9/15-9/19)

Topics: (1) Partial derivatives. (2) Definition of total derivatives.

Definition 1 (tangent plane). Let $f : \mathbb{R}^2 \to \mathbb{R}$ (and f is smooth enough), then the **tangent plane** to the graph f in $\mathbb{R}^3$ is the given by

$$z = f(x_0, y_0) + \frac{\partial f}{\partial x}(x_0, y_0) + \frac{\partial f}{\partial y}(x_0, y_0)$$

Problem 1. Compute $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ for the following functions:

1. $$x^3y^4 - xy^2$$
2. $$x^2 \sin(2y) + 3$$
3. $$\ln\left(\frac{y}{x}\right) + \ln\left(\frac{1}{x+y}\right) - \ln\left(\frac{x}{2}\right)$$

   You may use the following identities to simply the equation first:

   $$\ln\left(\frac{a}{b}\right) = \ln a - \ln b, \quad \ln\left(\frac{1}{a}\right) = -\ln a$$

Proof. We have

1. $$\partial x : 3x^2y^4 - y^2, \quad \partial y : 4x^3y^3 - 2xy$$
2. $$\partial x : 2x\sin(2y), \quad \partial y : 2x^2\cos(2y)$$
3. $$\partial x : -\frac{2}{x} - \frac{1}{x+y}, \quad \partial y : \frac{1}{y} - \frac{1}{x+y}$$

□

Problem 2. Compute the plane tangent to the graph of $f(x, y) = x^2y + 2xy - y^2$ at $(1, 2)$.

Proof. We have

$$\frac{\partial f}{\partial x}(1,2) = 2xy + 2y|_{(1,2)} = 8, \quad \frac{\partial f}{\partial y}(1,2) = x^2 + 2x - 2y|_{(1,2)} = -1$$

and $f(1, 2) = 2$, thus the plane is given by

$$z = 2 + 8x - y$$

□