

Calc III Sections

Fall 2025

Hui Sun

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Calc III-Week 10 (10/27-31)

vector fields, divergence and curl, double, triple integral Topics: (1) Acceleration and Arc Length, (2) Vector Fields. computing double and triple integrals

Definition 0.1 (flow line). Let F be a vector field, a flow line of F is a path $c(t)$ satisfying

$$c'(t) = F(c(t))$$

(Tangent vector of the path coincides with the given vector field F).

Definition 0.2 (divergence). Let F be a vector field in \mathbb{R}^3 $F = (F_1, F_2, F_3)$, the divergence of F is the **scalar field** (assigns one number to an given point (x, y, z)),

$$\operatorname{div} F = \nabla \cdot F = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$

More generally, if $F = (F_1, \dots, F_n)$ is a vector field on \mathbb{R}^n , its divergence is

$$\operatorname{div} F = \sum_{i=1}^n \frac{\partial F_i}{\partial x_i} = \frac{\partial F_1}{\partial x_1} + \dots + \frac{\partial F_n}{\partial x_n}$$