Calc III Practice Exam

1. Let $\vec{F}(x,y,z) = x\,\mathbf{i} + y\,\mathbf{j} + xe^z\,\mathbf{k}$ and let S be the part of the cylinder $x^2+y^2=1$ underneath $z=2$ in the first octant. Suppose S is positively oriented. Calculate
$$\iint_S \vec{F}\cdot d\vec{S}.$$
Hi.

2. Let D be the closed disk of radius 4 centered at $(0,0)$. Find the maximum and minimum of $f(x,y) = x^2 + \frac{1}{4}(y+1)^2 + 1$ on D.
Hi.

3. Let $E \subset \mathbb{R}^3$ be the solid region bounded by $x^2+y^2=9$, $z=0$ and $y+z=9$.
   (a) Sketch the region E and express it as an elementary region as a set in terms of inequalities.
   (b) Find the volume of E using a triple integral.

   Hi.

4. Let $\vec{F} = \langle ye^{xy} - zy, xe^{xy} - xz, -xy\rangle$ be a vector field in $\mathbb{R}^3$. Let C be the intersection of the paraboloid $x = y^2+z^2$ and the cylinder $z^2+y^2=9$. Calculate
$$\int_C \vec{F}\cdot d\vec{r}.$$
Hi.

5. Find the line integral over C, the lines connecting $(1,0,0)$, $(1,1,0)$, $(1,1,1)$ and $(1,0,1)$, oriented clockwise, for the vector field
$$\vec{F} = (x\cos(x), xy - z, e^z + y).$$
Hi.

6. Find
$$\lim_{(x,y)\to(1,2)} \frac{y^3-4x}{x^3+4y^3}.$$
Hi.

7. Let $\vec{F} = (y^2, 2xy + x)$.
   (a) Is $\vec{F}$ conservative? If so, find a potential function for $\vec{F}$. If not, justify your answer.
   (b) Let C be the positively oriented triangle connecting $(0,0)$, $(0,1)$ and $(-1,0)$. Evaluate $\int_C \vec{F}\cdot d\vec{r}$.

   Hi.

8. Let $\vec{F}(x, y, z) = (xz - y^3 \cos(z))\,\mathbf{i} + x^3 e^{-z}\,\mathbf{j} + z e^{x^2+y^2+z^2}\,\mathbf{k}$. Find the flux of the curl of $\vec{F}$ across the upper hemisphere of $x^2 + y^2 + z^2 = 1$ oriented upwards.