

# Calc III Sections

Fall 2025

Hui Sun

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## Calc III-Week 6 (9/29-10/3)

Topics: 1-9 in Section 3.1 and questions 1-11 in Section 3.2.

**Proposition 1.** Let  $f(x, y)$  be twice continuously differentiable, then

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$$

**Definition 1 (First order Taylor expansion).** Let  $f : U \subset \mathbb{R}^n \rightarrow \mathbb{R}$  be differentiable at  $a \in U$ , then

$$f(x) = f(a) + \sum_{i=1}^n \frac{\partial f}{\partial x_i}(a)(x_i - a_i) + R_1(a, x)$$

where

$$\frac{R_1(a, x)}{\|x - a\|} \rightarrow 0 \text{ as } x \rightarrow a$$

**Definition 2 (Alternative Definition (First-order)).** Let  $f : U \subset \mathbb{R}^n \rightarrow \mathbb{R}$  be differentiable at  $a \in U$ . Then

$$f(a + h) = f(a) + \sum_{i=1}^n h_i \frac{\partial f}{\partial x_i}(a) + R_1(a, h)$$

where  $R_1(a, h)/\|h\| \rightarrow 0$  as  $h \rightarrow 0$ .

**Definition 3 (Second order Taylor expansion).** Let  $f : U \subset \mathbb{R}^n \rightarrow \mathbb{R}$  be twice continuously differentiable at  $a \in U$ , then

$$f(x) = f(a) + \sum_{i=1}^n \frac{\partial f}{\partial x_i}(a)(x_i - a_i) + \frac{1}{2} \sum_{i,j=1}^n \frac{\partial^2 f}{\partial x_i \partial x_j}(a)(x_i - a_i)(x_j - a_j) + R_2(a, x)$$

where

$$\frac{R_2(a, x)}{\|x - a\|^2} \rightarrow 0 \text{ as } x \rightarrow a$$

**Problem 1.** Find all the second partial derivatives of  $f(x, y) = xy + \log(x - y)$ .  
(This includes  $\partial^2 f / \partial x^2$ ,  $\partial^2 f / \partial x \partial y$ ,  $\partial^2 f / \partial y \partial x$ ,  $\partial^2 f / \partial y^2$ ).

**Problem 2.** Write the second-order Taylor expansion for the following function,

$$f(x, y) = e^{x+y}$$

centered at  $(x, y) = (0, 0)$ .