

1. Let $\vec{F}(x, y, z) = x\mathbf{i} + y\mathbf{j} + xe^z\mathbf{k}$ and let S be the part of the cylinder $x^2 + y^2 = 1$ underneath $z = 2$ in the first octant. Suppose S is positively oriented. Calculate

$$\iint_S \vec{F} \cdot d\vec{S}.$$

2. Let D be the closed disk of radius 4 centered at $(0, 0)$. Find the maximum and minimum of $f(x, y) = x^2 + \frac{1}{4}(y + 1)^2 + 1$ on D .
3. Let $E \subset \mathbb{R}^3$ be the solid region bounded by $x^2 + y^2 = 9$, $z = 0$ and $y + z = 9$.

- (a) Sketch the region E and express it as an elementary region as a set in terms of inequalities.
- (b) Find the volume of E using a triple integral.

4. Let $\vec{F} = \langle ye^{xy} - zy, xe^{xy} - xz, -xy \rangle$ be a vector field in \mathbb{R}^3 . Let C be the intersection of the paraboloid $x = y^2 + z^2$ and the cylinder $z^2 + y^2 = 9$. Calculate

$$\int_C \vec{F} \cdot d\vec{r}.$$

5. Find the line integral over C , the lines connecting $(1, 0, 0)$, $(1, 1, 0)$, $(1, 1, 1)$ and $(1, 0, 1)$, oriented clockwise, for the vector field

$$\vec{F} = (x \cos(x), xy - z, e^z + y).$$

6. Find $\lim_{(x,y) \rightarrow (1,2)} \frac{y^3 - 4x}{x^3 + 4y^3}$.

7. Let $\vec{F} = (y^2, 2xy + x)$.

- (a) Is \vec{F} conservative? If so, find a potential function for \vec{F} . If not, justify your answer.
- (b) Let C be the positively oriented triangle connecting $(0, 0)$, $(0, 1)$ and $(-1, 0)$. Evaluate $\int_C \vec{F} \cdot d\vec{r}$.

8. Let $\vec{F}(x, y, z) = (xz - y^3 \cos(z))\mathbf{i} + x^3 e^{-z}\mathbf{j} + ze^{x^2+y^2+z^2}\mathbf{k}$. Find the flux of the curl of \vec{F} across the upper hemisphere of $x^2 + y^2 + z^2 = 1$ oriented upwards.