

# Additive Combinatorics Chapter 1 exercises

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**Problem 0.1.** We have the following expected value formulas.

$$\mathbb{E}(X) = \int_0^\infty P(X \geq \lambda) d\lambda$$

and

$$\mathbb{E}(X^p) = p \int_0^\infty \lambda^{p-1} P(X > \lambda) d\lambda$$

*Proof.* Change the order of integration.

$$\begin{aligned} \mathbb{E}(X) &= \int_0^\infty \lambda P(X = \lambda) d\lambda \\ &= \end{aligned}$$

□

**Problem 0.2.** When does equality hold in Markov's inequality?

*Proof.* In the proof of Markov's inequality, we used that  $X \geq \lambda I(X \geq \lambda)$ . For equality to hold, we need  $X = \lambda I(X \geq \lambda)$ , but this is not possible for  $X \neq 0$ . Suppose  $X = a$  for some  $a > 0$ , then take  $\lambda = a/2, a/3$ , we reach a contradiction. □



**Warning 0.1.** This is a warning.