Demonstration of Straight-line fit

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Project Github: https://github.com/joyfuldahye/MFCVML_Assignments/tree/master/Assignment06 (https://github.com/joyfuldahye/MFCVML_Assignments/tree/master/Assignment06)

Requirements

- Find a line that fits the given data
- The approximating line is obtained by the least square approximate solution

Essential Visualisation: Line Fitting

- Plot the noisy data
- Plot the clean data
- Plot the line that fits the noisy data by the least squre error

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Import libraries

In [1]:

```
import numpy as np
import matplotlib.pyplot as plt
```

Assign noisy and clean outcome data

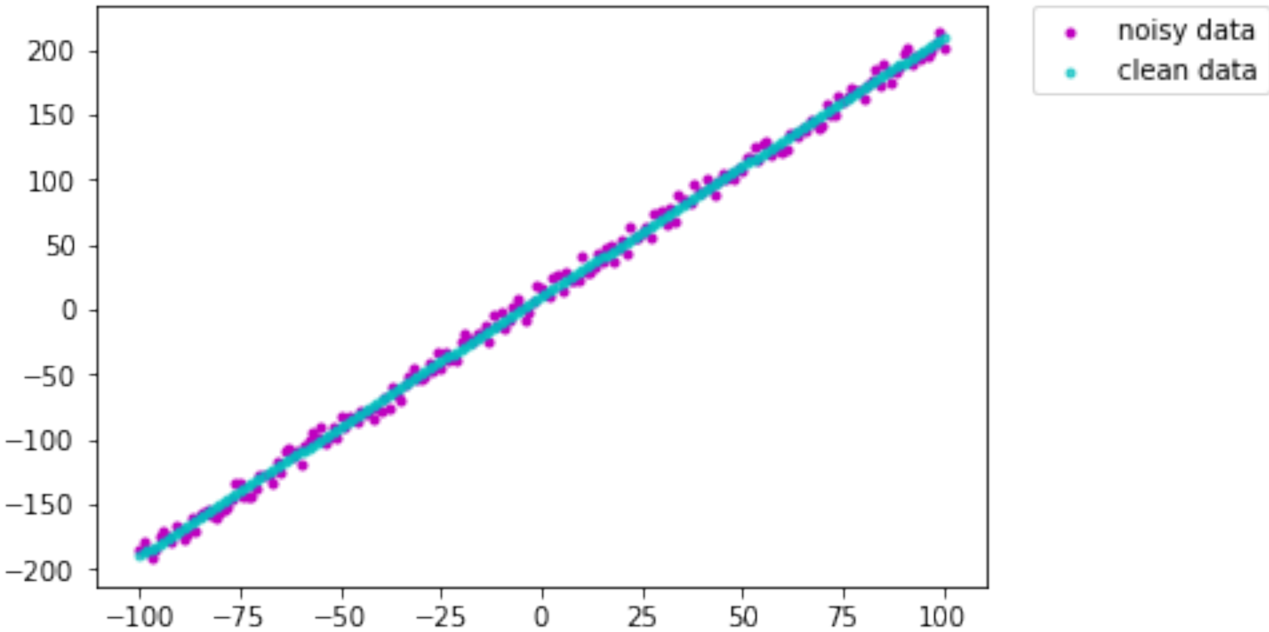
In [2]:

```
num     = 201
std     = 20
a       = 2
b       = 10

n       = np.random.rand(num)
nn      = n - np.mean(n)
x       = np.linspace(-100,100,num)
y1      = a * x + nn * std + b
y2      = a * x + b
```

Plot the outcome data

In [3]:

```
plt.figure(1)
plt.scatter(x, y1, c = 'm', alpha=1, marker = '.', label = "noisy data")
plt.scatter(x, y2, c = 'c', alpha=.7, marker = '.', label = "clean data")
plt.legend(bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.)
plt.show()

# x  : x-coordinate data
# y1 : (noisy) y-coordinate data
# y2 : (clean) y-coordinate data
# y = f(x) = a * x + b
```



Define the least squares fit:

- $\hat{f(x)} = avg(y^d) + \rho \frac{std(y^d)}{std(x^d)}(x - avg(x^d))$
  - $std(x^d) = \frac{\|x^d - avg(x^d)\mathbf{1}\|}{\sqrt{N}}$
  - $\rho = \frac{(x^d - avg(x^d)\mathbf{1})^T (y^d - avg(y^d)\mathbf{1})}{N std(x^d) std(y^d)}$

compute the standard deviation of x:

- $std(x^d) = \frac{\|x^d - avg(x^d)\mathbf{1}\|}{\sqrt{N}}$

In [4]:

```
mean_x = np.mean(x)
demean_x = x - mean_x
norm_x = np.linalg.norm(demean_x)
std_x = norm_x / np.sqrt(num)
transposed_demean_x = np.transpose(demean_x)
```

compute the standard deviation of noisy data y_1 :

- $std(y^d) = \frac{\|y^d - avg(y^d)\mathbf{1}\|}{\sqrt{N}}$

In [5]:

```
mean_y1 = np.mean(y1)
demean_y1 = y1 - mean_y1
norm_y1 = np.linalg.norm(demean_y1)
std_y1 = norm_y1 / np.sqrt(num)
```

compute the correlation coefficient (ρ) of x and y_1 :

- $\rho = \frac{(x^d - avg(x^d)\mathbf{1})^T (y^d - avg(y^d)\mathbf{1})}{N std(x^d) std(y^d)}$

In [6]:

```
rho_xy = transposed_demean_x.dot(demean_y1) / (num*std_x*std_y1)
# print(std_x, std_y1)
# print(transposed_demean_x)
# print(transposed_demean_x.dot(demean_y1))
# print(rho)
```

compute the least squares fit

- $\hat{f(x)} = avg(y^d) + \rho \frac{std(y^d)}{std(x^d)}(x - avg(x^d))$

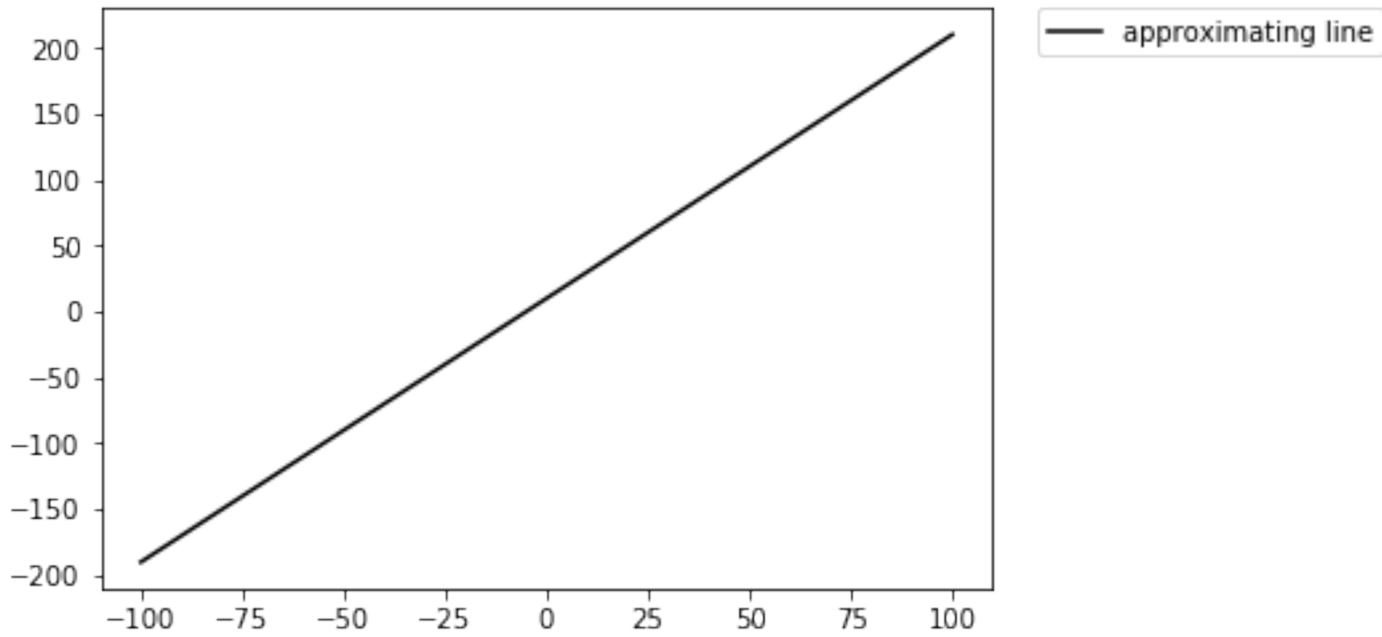
In [7]:

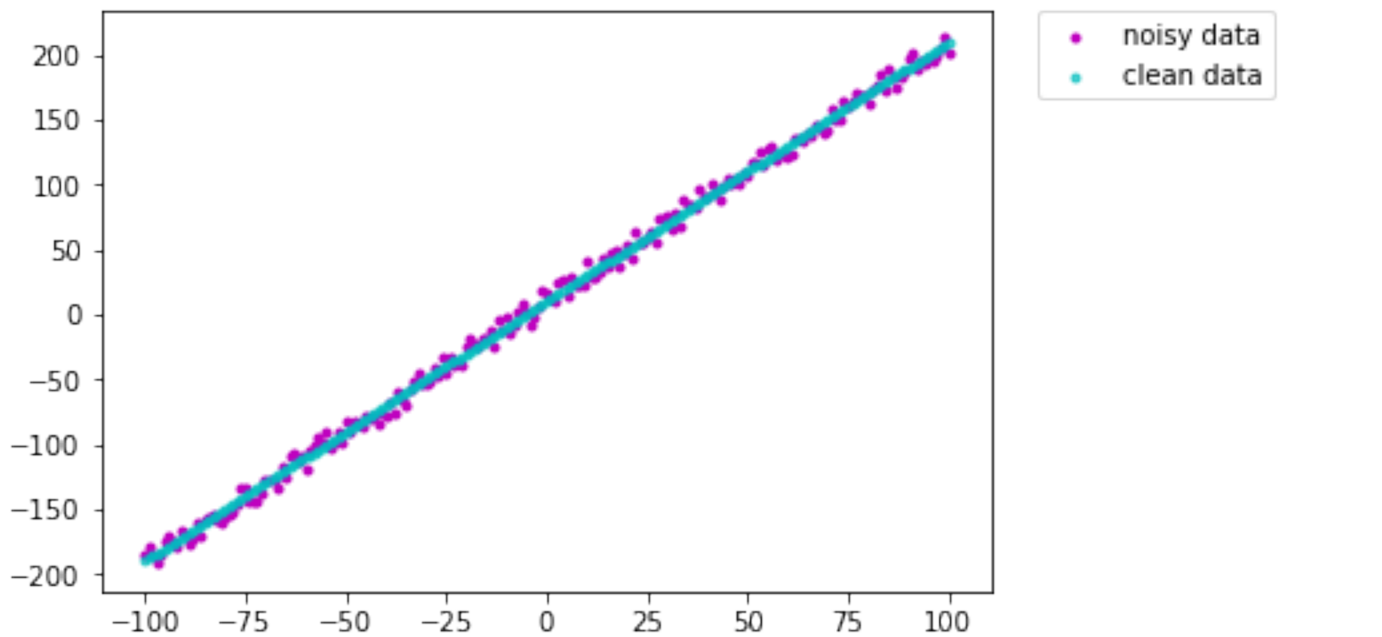
```
y = mean_y1 + rho_xy*(std_y1 / std_x)*demean_x
```

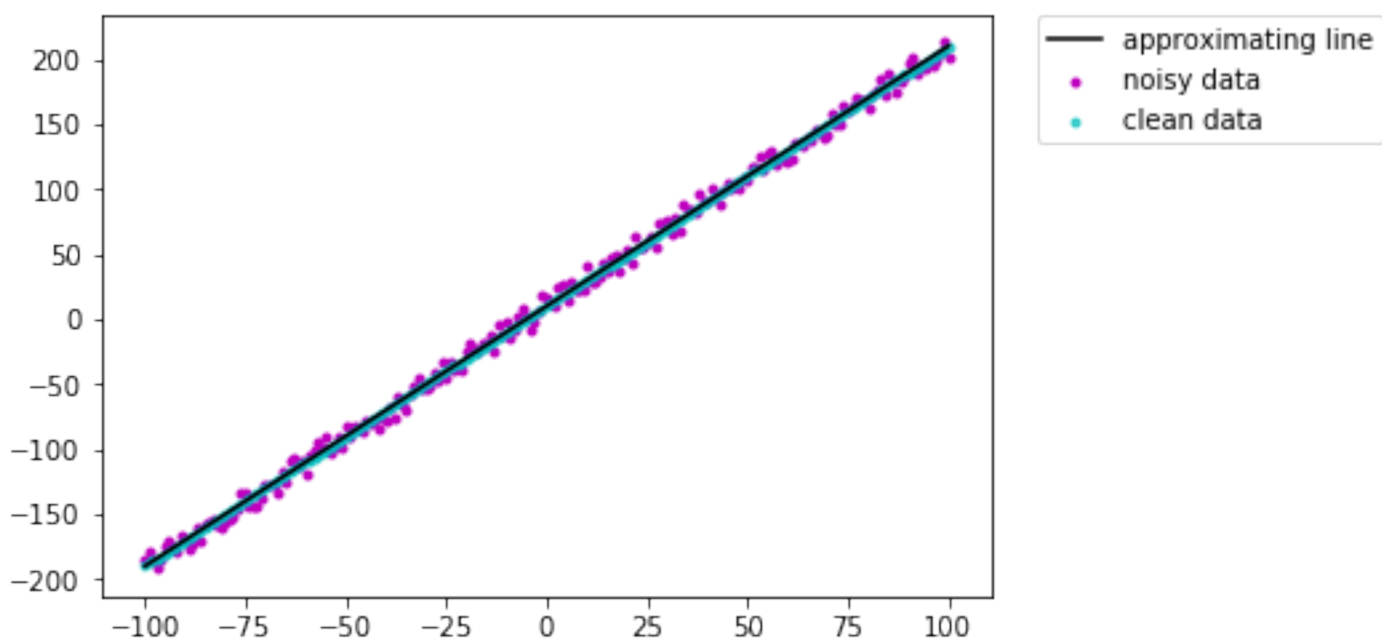
Plot the approximating line and data

In [8]:

```
# plot the approximating line alone
plt.figure(1)
plt.plot(x, y, 'k-', label = "approximating line")
plt.legend(bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.)

# plot the outcome data
plt.figure(2)
plt.scatter(x, y1, c = 'm', alpha=1, marker = '.', label = "noisy data")
plt.scatter(x, y2, c = 'c', alpha=.7, marker = '.', label = "clean data")
plt.legend(bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.)

# plot the approximating line and outcome data together
plt.figure(3)
plt.scatter(x, y1, c = 'm', alpha=1, marker = '.', label = "noisy data")
plt.scatter(x, y2, c = 'c', alpha=.7, marker = '.', label = "clean data")
plt.plot(x, y, 'k-', label = "approximating line")
plt.legend(bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.)

plt.show()
```







The shape of the approximating line which fits the least square error of noisy data is almost exactly overlapped the tendency of clean data.