# 1 Converging-Diverging Verification (CDV) Nozzle

The Converging-Diverging Verification (CDV) Nozzle is a verification case involving the flow of inviscid, non-heat-conducting air through a converging-diverging nozzle. Details can be found on the NASA validation site [National Aeronautics and Space Administration, 2021]. This is a classic one-dimensional, steady, compressible flow problem discussed in most compressible flow textbooks, such as Ref. [Anderson, 1984].

This case allows the verification of a CFD code in the following manner:

- Verify through comparison with analytic solutions
- Verify mass conservation through a duct
- Verify of constancy of total pressure through a duct (isentropic flow)
- Verify consistency of axisymetric and three-dimensional flow domains

This case involves steady, inviscid, non-heat-conducting flow through a converging-diverging nozzle. The plenum total pressure and total temperature are assumed constant. The values used in this case are presented in Tab. 1.

| Variable | Symbol | Units | Value |
|---|---|---|---|
| Plenum Total Pressure | $p_t$ | psi | 1.0 |
| Plenum Total Temperature | $T_t$ | R | 100.0 |
| Exit Static Pressure | $p_{\text{exit}}$ | psi | 0.89, 0.75, 0.16 |

Table 1: Table of conditions (adapted from NASA [National Aeronautics and Space Administration, 2021]).

The nature of the flow is determined by the exit static pressure. Three values of exit static pressure are examined which result in three types of flows:

- Subsonic, isentropic flow ($p_{\text{exit}}/p_t = 0.89$)
- Supersonic flow with a normal shock in the diffusing section ($p_{\text{exit}}/p_t = 0.0.75$)
- Supersonic, isentropic flow ($p_{\text{exit}}/p_t = 0.16$)

## 1.1 Geometry

The geometry is an axisymmetric converging-diverging duct. Figure 1 shows the general shape of the nozzle. It has an area of 2.5 in$^2$ at the inflow ($x = 0$ in), an area of 1.0 in$^2$ at the throat ($x = 5$ in), and an area of 1.5 in$^2$ at the exit ($x = 10$ in). The nozzle area has the form

$$A(x) = \begin{cases} 1.75 - 0.75\cos(\pi(0.2x - 1.0)), & 0 \le x < 5 \\ 1.25 - 0.25\cos(\pi(0.2x - 1.0)), & 5 \le x \le 10 \end{cases} \tag{1}$$

This nozzle geometry is from Ref. [Liou, 1987], which also discusses some CFD computations of this nozzle.

The ratios of select areas are listed in Tab. 2

| Ratio | Value |
|---|---|
| $A_{\text{throat}}/A_{\text{inlet}}$ | $1.0/2.5 = 0.4$ |
| $A_{\text{exit}}/A_{\text{throat}}$ | $1.5/1.0 = 1.5$ |
| $A_{\text{exit}}/A_{\text{inlet}}$ | $1.5/2.5 = 0.6$ |

Table 2: Ratio of areas.

## 1.2 Nondimensionalizations

In order to simplify the task of simulating the nozzles with `rhoCentralFoam`, we set $p_t = 1$ Pa, $T_t = 1$ K, and $p_{\text{exit}} = \{0.89, 0.75, 0.16\}$ Pa. The fluid is such that $W = 8314.5$ kg/kmol, and $c_p = 3.5$ J/kg-K, so that the gas constant is $R = \mathcal{R}/W = 1$ J/kg-K and $\gamma = 1.4$ the ratio of specific heats. We also let the diameter of the nozzle inlet be equal to $D = 1$ m, but keep the geometry unchanged.

## 1.3 Analytical solutions

### 1.3.1 Case 1 - Isentropic flow and subsonic exit

Let us begin with the case of subsonic, isentropic flow (case 1). In this case, we use the isentropic flow tables with $\gamma = 1.4$ (air) and consider the ratio $p_{\text{exit}}/p_t = 0.89$, which gives $M_{\text{exit}} = 0.41144$ and $T_{\text{exit}}/T_t = 0.96725$, giving $T_{\text{exit}} = 0.96725$ K and a sound speed $a_{\text{exit}} = \sqrt{\gamma T_{\text{exit}}} = 1.1637$ m/s, which implies an exit speed $u_{\text{exit}} = 0.47878$ m/s.

### 1.3.2 Case 2 - Normal shock in diffuser and subsonic exit

Next, we consider the case of $p_{\text{exit}}/p_t = 0.75$, resulting in a normal shock in the diffuser past the throat (case 2) and a subsonic exit. In this case, the isentropic flow tables are used together with the normal shock tables to determine the analytical solution. We obtain that the exit Mach number is $M_{\text{exit}} = 0.5019$ and the shock Mach number is $M_{\text{shock}} = 1.6117$. At the exit, $T_{\text{exit}}/T_t = 0.9520$ and the static temperature is $T_{\text{exit}} = 0.9520$ K, giving $a_{\text{exit}} = \sqrt{\gamma T_{\text{exit}}} = 1.1545$ m/s and $u_{\text{exit}} = 0.57946$ m/s.

### 1.3.3 Case 3 - Isentropic flow and supersonic exit

Finally, we consider the case of $p_{\text{exit}}/p_t = 0.16$ (case 3), resulting in the nozzle functioning as intended: air expands from the stagnation conditions, reaching $M_{\text{exit}} = 1.854124$ at the exit. In this case, the isentropic flow tables can be used throughout the nozzle together withe the area ratio and the condition of $M_{\text{throat}} = 1$ (sonic flow at the throat).

## 1.4 Comparisons against rhoCentralFoam

Simulations with `rhoCentralFoam` are executed (see class repository) with the appropriate boundary conditions and the pressure ratio $p/p_t$ and the Mach number $M$ are compared along the centerline of the nozzle for the 3 cases.

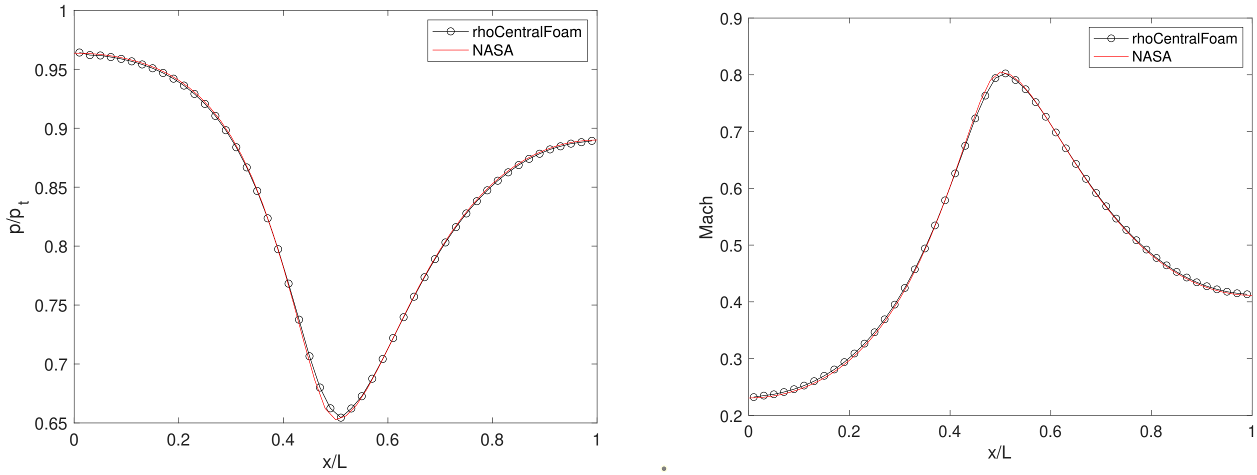


Figure 1: Case 1. Pressure ratio $p/p_t$ (left) and Mach number $M$ (right) along the centerline as a function of $x/L$ where $L$ is the nozzle length. The throat is located at $x/L = 0.5$

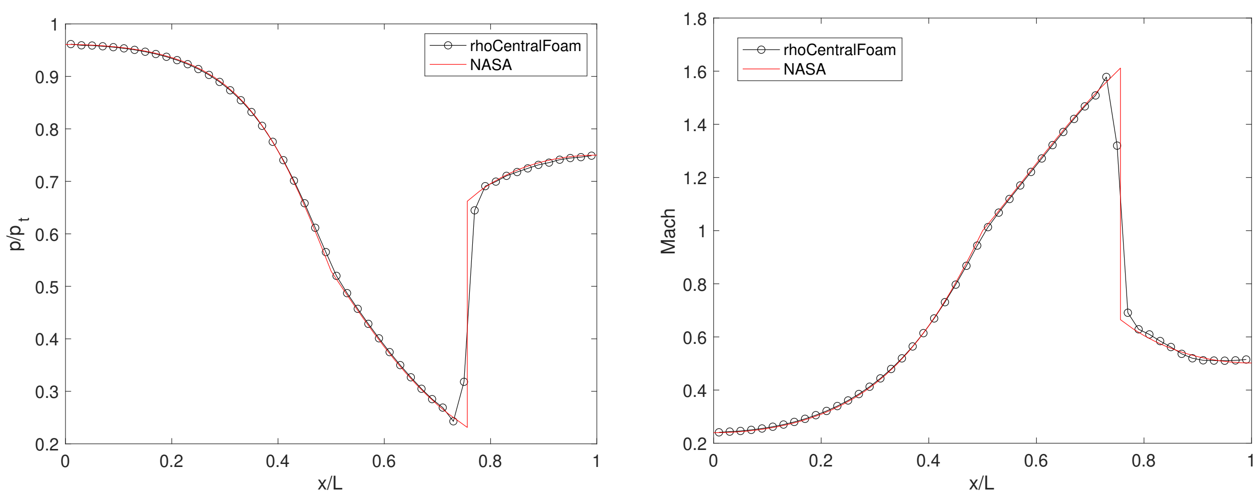


Figure 2: Case 2. Pressure ratio $p/p_t$ (left) and Mach number $M$ (right) along the centerline as a function of $x/L$ where $L$ is the nozzle length. The throat is located at $x/L = 0.5$

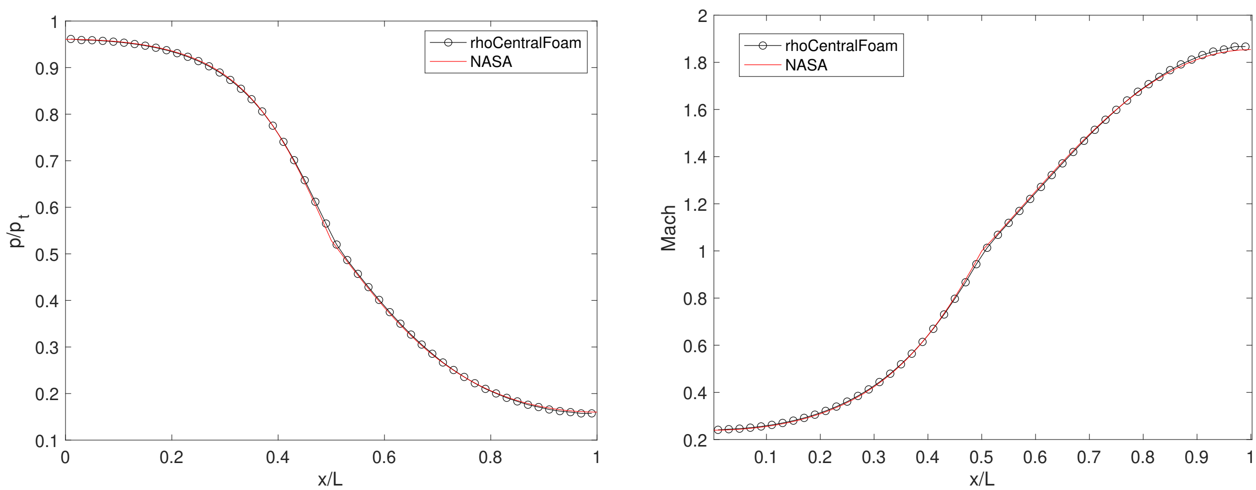


Figure 3: Case 3. Pressure ratio $p/p_t$ (left) and Mach number $M$ (right) along the centerline as a function of $x/L$ where $L$ is the nozzle length. The throat is located at $x/L = 0.5$

# References

National Aeronautics and Space Administration. Converging-Diverging Verification (CDV) Nozzle, 2021. `https://www.grc.nasa.gov/www/wind/valid/cdv/cdv.html`.

J D Anderson. *Modern Compressible Flow*. McGraw Hill, New York, 1984.

M S Liou. A generalized procedure for constructing an upwind-based TVD scheme. 1987. NASA TM 88926.