False Discovery Rates, A New Deal

Matthew Stephens

2014/7/8

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- Organized researchers get more done (and better!).
- Many of them are more organized than I am!
- Thought: I should get organized; I should help others get organized.

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- While doing research, record what you did and what the outcome was.
- Use version control (git) and internet repositories (bitbucket, github) to organize notes, code, etc.
- Use *knitr* to help make your research reproducible.
- Talk about the tools you find useful!

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- An amateur example: http://github.com/stephens999/ash

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- This talk was written with knitr (with RStudio)!

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- "publishing figures or results without the complete software environment could be compared to a mathematician publishing an announcement of a mathematical theorem without giving the proof" (Buckheit and Donohoe)
- "an article about a computational result is advertising, not scholarship. The actual scholarship is the full software environment, code and data, that produced the result." [Claerbout]

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- Reproducing work is also the first step to extending it.
- ullet Helps communications among researchers (eg student + advisor).
- If you do not publish code implementing your methods, your methods will likely go unused.

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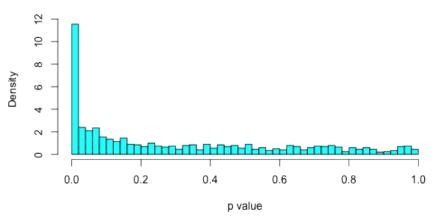
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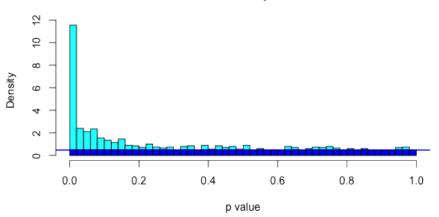
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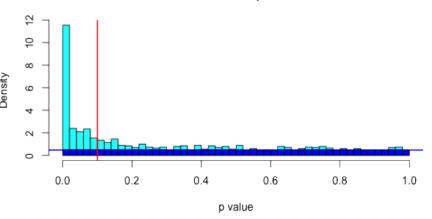
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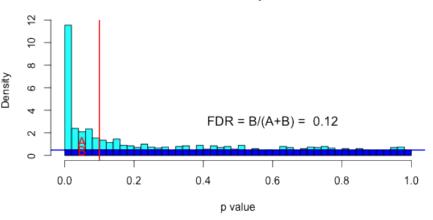
$$fdr(P) = Pr(\beta_j = 0 | p_j = P).$$

• The fdr is more relevant, but slightly harder to estimate than FDR because it involves density estimation rather than tail-area estimation.









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- The original paper introducing FDR (Benjamini and Hochberg, 1995) has been cited 21,787 times (May 2014) according to Google Scholar.
- That is three times a day for the last 19 years!

Problem 1: The Zero Assumption (ZA)

• The standard qvalue approach assumes that all the *p* values near 1 are null.

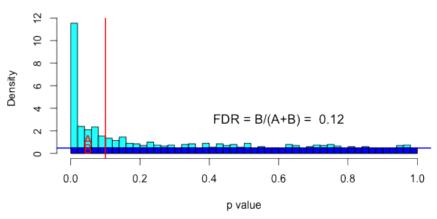
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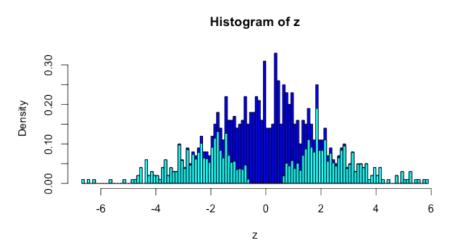
Problem 1: The Zero Assumption (ZA)

- The standard qvalue approach assumes that all the p values near 1 are null.
- Analogously, one can assume that all Z scores near 0 are null. Efron refers to this as the "Zero Assumption".
- Seems initially natural.

Implied distribution of p values under H_1



Implied distribution of Z scores under alternative



FDR problem 2: different measurement precision

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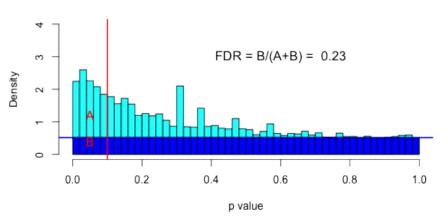
- In some cases the measurement precisions differ among units
- Eg Expression levels of low-expressed genes have less precision than high-expressed genes
- If some effects are measured less precisely than others, those tests "lack power" and dilute signal, increasing FDR

Example: Mouse Heart Data

```
##
        gene
              lv1 lv2 rv1
                                rv2 genelength
## 1
       Itm2a 2236 2174
                        9484 10883
                                           1626
##
      Sergef
               97
                     90
                          341
                                408
                                           1449
    Fam109a 383
                   314
                         1864
                               2384
                                           2331
        Dhx9 2688 2631 18501
                                           4585
## 4
                              20879
              762
                    674
## 5
       Ssu72
                         2806
                               3435
                                           1446
## 8
      Eif2b2
              736
                    762
                         3081
                               3601
                                           1565
```

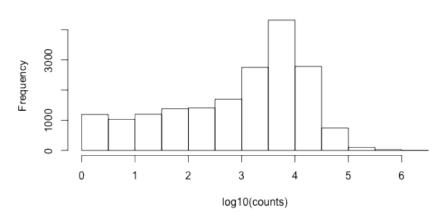
 Data on 150 mouse hearts, dissected into left and right ventricle (courtesy Scott Schmemo, Marcelo Nobrega)

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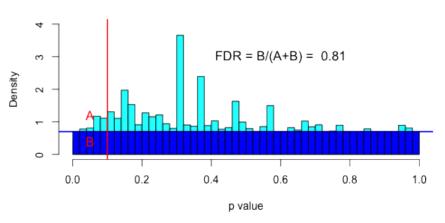


Mouse Data: Counts vary considerably across genes

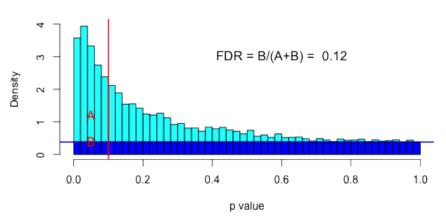
Distribution of total counts



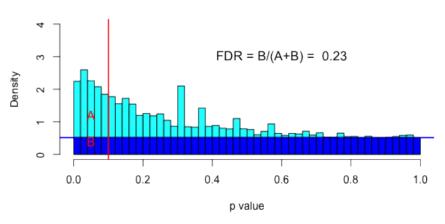
Lower count genes, less power



Higher count genes, more power

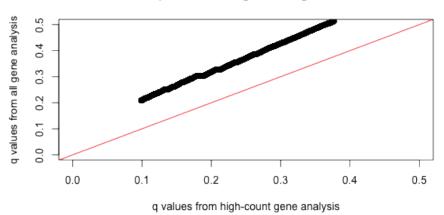


Low-count genes dilute signal at high-count genes



FDR problem 2: low count genes add noise, increase q values

q values for high count genes



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- The ZA, which implies actual effects have a (probably unrealistic) bimodal distribution; causes overestimate of π_0 , losing power.
- By focussing on p values, low-precision measurements can dilute high-precision measurements.

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where f_1 , π_0 are to be estimated from the data.

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- Various semi-parametric approaches taken to estimating f_1 . For example, Efron uses Poisson regression; Muralidharan uses mixture of normal distributions.
- ullet Once f_1 and π_0 estimated, FDR calculations are straightforward.



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• Instead of modelling Z scores, model the effects β ,

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• Instead of modelling Z scores, model the effects β ,

$$\beta_j \sim \pi_0 \delta_0(.) + (1 - \pi_0)g(.)$$

- Constrain g to be unimodal about 0; estimate g from data.
- Incorporate precision of each observation $\hat{\beta}$ into the likelihood. Specifically, approximate likelihood for β_j by a normal:

$$L(\beta_j) \propto \exp(-0.5(\beta_j - \hat{\beta}_j)^2/s_j^2).$$

[From $\hat{\beta}_j \sim N(\beta_j, s_j)$] Or, better, use a t likelihood if s_j estimated using few observations.

FDR - A New Deal

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- By allowing K large, and σ_k to span a dense grid of values, we get a flexible unimodal symmetric distribution.
- Can approximate, arbitrarily closely, any scale mixture of normals. Includes almost all priors used for sparse regression problems (spike-and-slab, double exponential/Laplace/Bayesian Lasso, horseshoe).

 Alternatively, a mixture of uniforms, with 0 as one end-point of the range, provides still more flexibility, and in particular allows for asymmetry.

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- If allow a very large number of uniforms this provides the non-parametric mle for g; cf Grenander 1953; Cordy + Thomas 1997.

Illustration: g a mixture of 0-centered normals

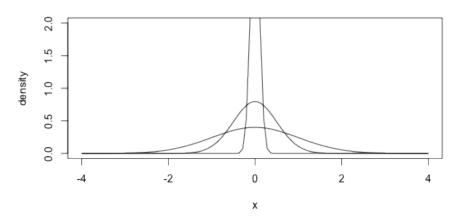


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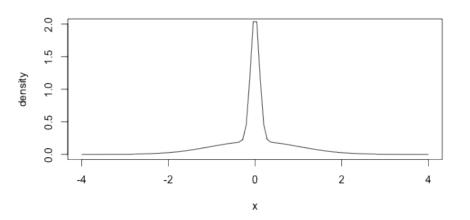


Illustration: g a mixture of 0-anchored uniforms

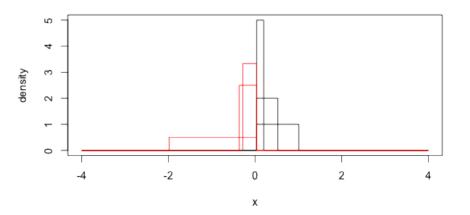
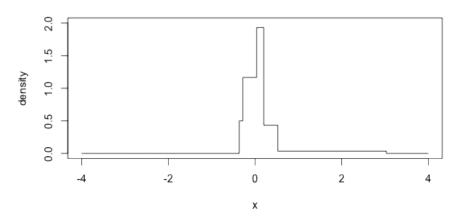


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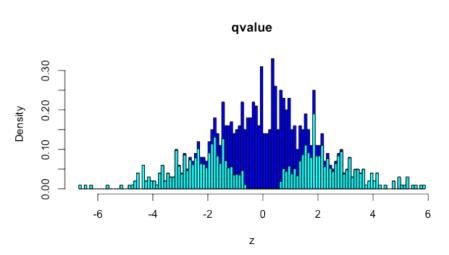
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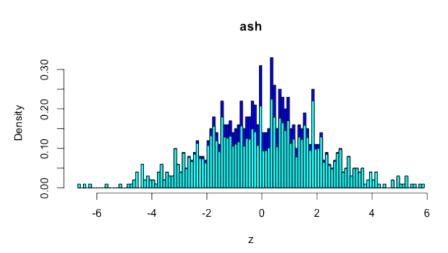
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- So we call the approach "Adaptive Shrinkage" (ASH).

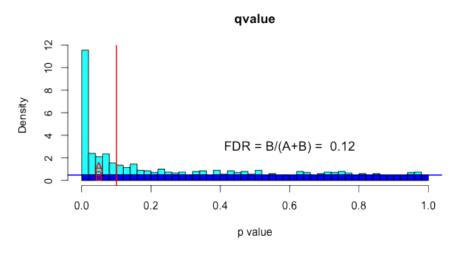
Recall Problem 1: distribution of alternative Z values multimodal



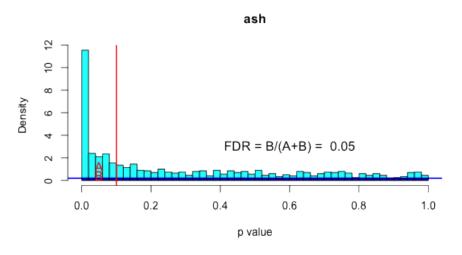
Problem Fixed: distribution of alternative Z values unimodal



Example: FDR estimation

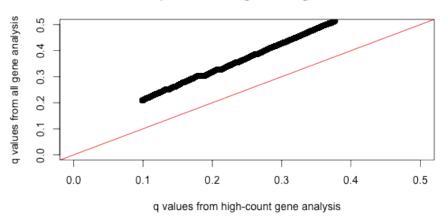


Example: FDR estimation



Recall Problem 2: low count genes add noise, increase q values

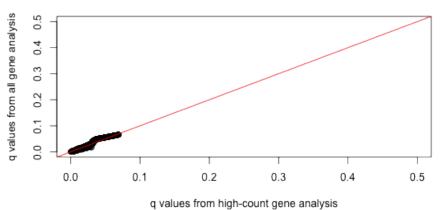
q values for high count genes



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Problem Fixed: incorporating precision reduces influence of low-count genes

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A new problem: an embarrassment of riches

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- In the illustrative example, the maximum q value is 0.18

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- Suggests a change of focus: ask for which β_j are we confident about the sign (cf Gelman et al, 2012).

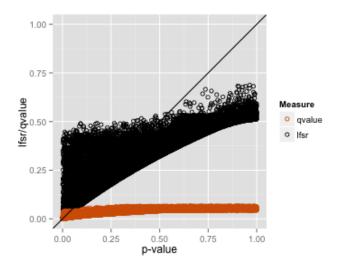
The False Sign Rate

 Suggestion: replace FDR with local false sign rate (Ifsr), the probability that if we say an effect is positive (negative), it is not.

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- Example: suppose we estimate that $\Pr(\beta_j < 0) = 0.95, \Pr(\beta_j = 0) = 0.025$ and $\Pr(\beta_j > 0) = 0.025$. Then we report β_j as a "(negative) discovery", and estimate its Ifsr as 0.05.

Even with many signals, large p values have high lfsr



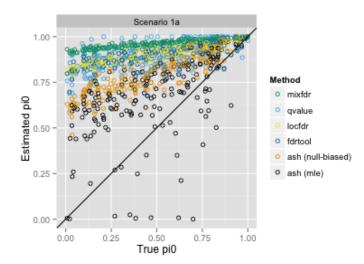
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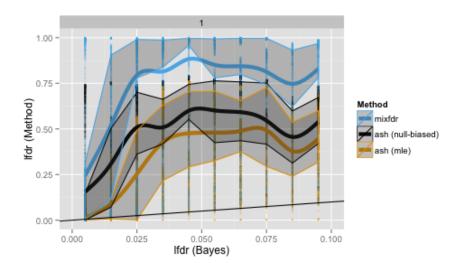
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- So methods for estimating π_0 and FDR, including those presented here, are designed to be "conservative" (i.e. overestimate the FDR).
- The False Sign Rate is much less senstive to π_0 , and hence more identifiable from data!

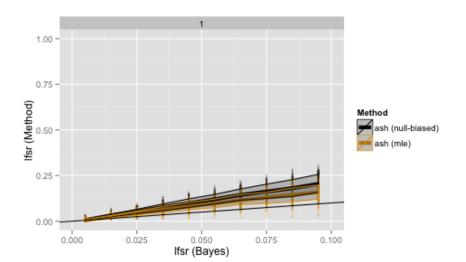
Simulated example: π_0 not identifiable.



Simulated Example: so fdr not identifiable



Simulated Example: fsr much more identifiable



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- ASH provides a generic approach to shrinkage estimation, as well as false discovery (sign) rates.
- But by using two numbers $(\hat{\beta}, s)$ instead of one (p values or z scores) varying precision of measurements is better accounted for.
- Unimodal assumption for effects reduces conservatism
- False Sign Rate preferable to False Discovery Rate: more identifiable, and better representation of information in data for "high-signal" situations.

Other Applications

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- E.g. Currently applying it to wavelet shrinkage applications.

Next steps?

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- Allow for correlations in the measured $\hat{\beta}_j$.

• to the developers of **R**, **knitr**, **Rstudio** and **Pandoc**.

- to the developers of R, knitr, Rstudio and Pandoc.
- to the several postdoctoral researchers and students who have worked with me on related topics.

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- ashr package: http://www.github.com/stephens999/ash

Pandoc Command used

```
pandoc -s -S -i --template=my.beamer -t beamer -V
theme: CambridgeUS -V colortheme: beaver slides.md -o
slides.pdf
(alternative to produce html slides; but figures would need reworking)
pandoc -s -S -i -t dzslides --mathjax slides.md -o
slides.html
Here is my session info:
print(sessionInfo(), locale=FALSE)
## R version 3.0.2 (2013-09-25)
## Platform: x86_64-apple-darwin10.8.0 (64-bit)
##
## attached base packages:
## [1] splines stats
                            graphics grDevices utils
                                                             datas
   [8] base
                                         ◆□ ト ◆□ ト ◆ □ ト ◆ □ ト ◆ □ ◆ ○ へ ○ ○
```

Some odd things in the data

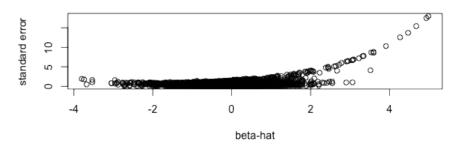


Figure: plot of chunk unnamed-chunk-35

Error: object 'dd' not found

A technicality

• Suppose you estimate $\Pr(\beta_j < 0) = 0.98$, $\Pr(\beta_j > 0) = 0.01$, $\Pr(\beta_j = 0) = 0.01$.

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- Should you declare an fdr of 0.01 or 0.02?

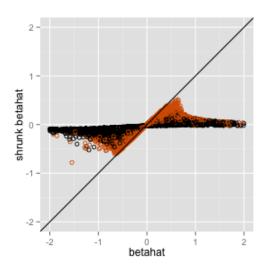
A technicality

- Suppose you estimate $\Pr(\beta_j < 0) = 0.98$, $\Pr(\beta_j > 0) = 0.01$, $\Pr(\beta_j = 0) = 0.01$.
- Should you declare an fdr of 0.01 or 0.02?
- Maybe fsr makes more sense anyway?

Adaptive Shrinkage of point estimates

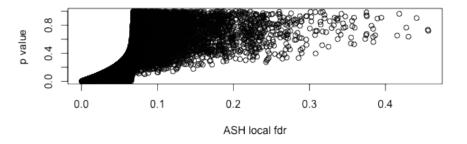
• Recall idea: amount of shrinkage depends on measurement precision, s_j .

Adaptive Shrinkage of point estimates

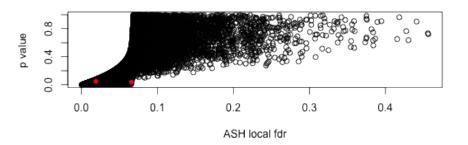


Shrinkage is adaptive to information

Need to fix counts.associate to use fdr method in ash



Shrinkage is adaptive to information

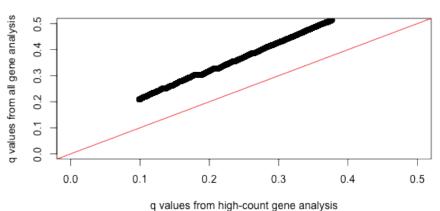


Shrinkage is adaptive to information

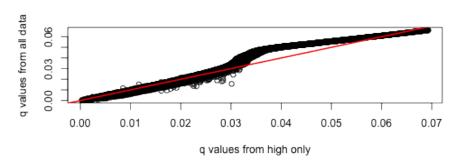
```
## gene lv1 lv2 rv1 rv2 pval zdat.ash$lfdr
## 19422 Mgat5b 7 10 320 452 0.03795 0.06575
## 20432 Sec63 1042 1034 5496 6649 0.04908 0.01895
```

Recall FDR problem 1: q values increased by low count genes

q values for high count genes



ASH q values more robust to inclusion of low count genes



Compare fitted $f(\beta)$, both estimating π_0 and fixing $\pi_0 = 0$.

Error: object 'hh.ash.fdr' not found