Model One

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1 Optimal Stopping Time Problem

When consumers allocate their asset between consumption, housing and saving, we hope to seek the optimal consumption path and the amount of money spend on housing, as well as the timing for buying the house.

2 Variables

T: Time interval

S: Time point upgrading house

A_t: A_t is the financial asset at t

K_0: K_0 is the initial housing property

K_s: K_s is the amount of property acquired at time s

C_t: consumption at t

λ: down payment rate

Y_t: income at t

3 Objective Function

$$max_{\{C_t,S,K_s\}}E\{\sum_{t=0}^{S-1}\beta^t[\frac{1}{\rho}(C_t)^\alpha+\beta(K_0)^\alpha]^{\frac{\rho}{\alpha}}+\sum_{t=s}^{T}\beta^t[\frac{1}{\rho}(C_t)^\alpha+\beta(K_0+K_S)^\alpha]^{\frac{\rho}{\alpha}}\}$$

4 Recursive Equation

a) Before buying house

$$V(t-1, A_{t-1}, K_0) = max_{A_t}[\frac{1}{\rho}(C_t)^\alpha + \beta(K_0)\alpha]\frac{\rho}{\alpha} + \beta V(t, A_t, K_0)$$

s.t.

$$C_t = (1+r)A_t - A_{t+1} + Y_t$$

b) At the point s of purchasing house

$$V(t-1, A_{t-1}, K_0) = max_{A_t}[\frac{1}{\rho}(C_t)^\alpha + \beta(K_0 + K_S)^\alpha]^{\frac{\rho}{\alpha}} + \beta V(t, A_t, K_0 + K_s)$$

s.t.

$$C_t = (1+r)A_t - A_{t+1} - \lambda K_s + Y_t$$

c) After purchasing house

$$V(t-1, A_{t-1}, K_0 + K_S) = maxA_t[\frac{1}{\rho}(C_t)^\alpha + \beta(K_0 + K_S)^\alpha]^{\frac{\rho}{\alpha}} + (t, A_t, K_0 + K_S)$$

s.t.

$$C_t = (1+r)A_t - A_{t+1} - 1.05 * \frac{1-\lambda}{T-s+1}K_S + Y_t$$

5 Steps of Computing

1. Assume the point of purchasing s=1

2. At the end of decision making period T, since there is no need to keep saving for next period, i.e. $A_{T+1} = 0$. Then for arbitrary A_T, K_S we have $C_T = A_T + Y_T - \lambda K_s$ and compute the value of $V(T, K_T, A_T)$

3. For arbitrary value of A_{T-1}, search the corresponding optimal A_T in bellman equation 4(c), using the updated $V(t, K_T, A_T)$ to compute V(T-1,K$_{T-1}, A_{T-1}).For post-purchasing house period, use equatic$ $purchasing house period, use 4(a). Use the" optsearch" function to find global optimal value.$

4. Calculate the aggregate utility, then find the corresponding optimal K_s which yield the highest aggregate utility.

5. Iterate the above steps for all possible s, search for optimal s and corresponding optimal K_s

6. Plug in the value of s and K_s, calculate poicy function, i.e., the correspondence from A_t to optimal A_{t+1}

7. Compute the optimal consumption and asset path $\{C_t\}_{t=1,...T}$ and $\{A_t\}_{t=1,...T}$