Numerical Analysis assignment No. 3

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1 Assignment Content

To investigate Δx dependence of the Trapezoid method and the Simpson $\frac{1}{3}$ method.

The governing equation (exact solution) is

$$I = \int_{3.1}^{3.9} \frac{1}{x} \mathrm{d}x = \ln \left| \frac{3.9}{3.1} \right| = 0.229574441645... \tag{1}$$

2 Resultsit and Discussions

The results of both methods is shown as Table 1. I used Python code which was distributed in the class. And I investigated at imax = 9, 51, 81, 201, 401, 801, 8001, 80001.

Table 1 Results

Δx	$\log(\Delta x)$	imax	$\log(\varepsilon)$ of the Trapeziod	$\log(\varepsilon)$ of the Simpson 1/3
0.1	-1.0	9	-4.50	-7.67
0.016	-1.80	51	-6.09	-10.8
0.01	-2.0	81	-6.50	-11.7
0.004	-2.40	201	-7.29	-13.3
0.002	-2.70	401	-7.89	-14.5
0.001	-3.0	801	-8.50	-16.1
0.0001	-4.0	8001	-10.5	-15.5
0.00001	-5.0	80001	-12.5	-16.0

The last 3 lines from the result of the Simpson method (under the single line), there is no decreasing of error. This is highly possible to be Machine Zero. Therefore, I don't plot the last 3 lines of result. I use just the 5 lines above.

From the 5 results, I made fitting line by using Least–squares method. In the method, the fitting line is written by $y = Ax + b$, and the parameter r is Correlation coefficient. The fitting result is shown as Table 2. Since both $|r| > 0.999$, the fitting is right.

Table 2 Fitting line parameters

	Trapezoid	Simpson 1/3
A	1.994	4.028
B	-2.505	-3.618
r	0.999995	0.99988

And plot result and fitting line are shown in Figure 1. From these result, the Simpson 1/3 method's slope is 2 times steeper, so Simpson 1/3 converges faster than Trapezoid.

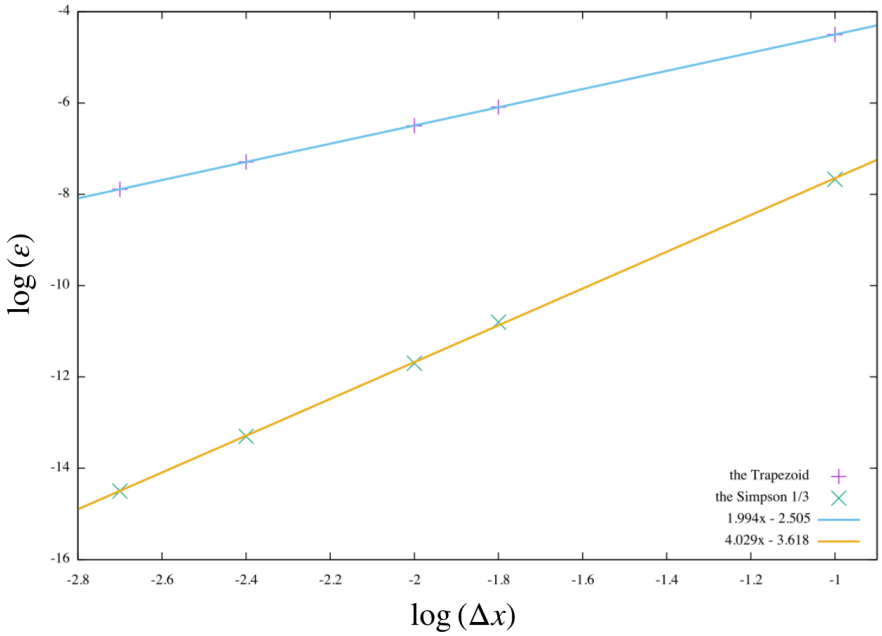

Figure 1 Plots from results and fitting line.