

シミュレーション —常微分方程式—

学籍番号：16426

4 年 電子情報工学科 23 番

福澤 大地

提出日：2020 年 1 月 23 日

1 目的

常微分方程式の解法であるオイラー法、ホイン法を応用して物理現象などを解析する。連立微分方程式や高階微分方程式の数値解をプログラムを使用して解くことで目標の達成とする。

2 開発環境

プログラムの開発、実行を行った環境を表 1 に示す。

表 1 開発環境

CPU	Intel Core i5-7400 @ 3.0GHz
メモリ	8GB
OS	Microsoft Windows 10 Home
システム	64bit
コンパイラ	GCC 7.4.0

3 課題 6：生存競争モデル

3.1 課題内容

式 (1) に示す、生物の生存競争モデルの連立微分方程式をオイラー法で解くプログラムを作成する。

$$\begin{cases} \frac{dy_1}{dx} = ay_1 - cy_1y_2 \\ \frac{dy_2}{dx} = by_2 + dy_1y_2 \end{cases} \quad (1)$$

3.2 プログラムリスト

課題 6 のプログラムリストをリスト 1 に示す。

リスト 1 課題 6 のプログラム

```
1 #include <stdio.h>
2 #include <math.h>
3
4 #define a 1
5 #define b 1
6 #define c 1
7 #define d 1
8 #define x_0 0.0
9 #define y1_0 10
10 #define y2_0 10
11
12 #define H 0.1
13 #define STEP 10
14
15 double y1p(double, double);
16 double y2p(double, double);
```

```

17 void euler(double *, double *, double *, double, int);
18
19 int main(void) {
20     int i;
21     double x[STEP + 1];
22     double y1[STEP + 1];
23     double y2[STEP + 1];
24
25     euler(x, y1, y2, H, STEP);
26
27     for (i = 0; i <= STEP; i++)
28         printf("i = %2d, x = %.16f, y1 = %.16f, y2 = %.16f\n", i, x[i], y1[i], y2[i]);
29
30     return 0;
31 }
32
33 // y1'(y1, y2)
34 double y1p(double y1, double y2) {
35     return a * y1 - c * y1 * y2;
36 }
37
38 // y2'(y1, y2)
39 double y2p(double y1, double y2) {
40     return -b * y2 + d * y1 * y2;
41 }
42
43 // オイラー法
44 void euler(double *x, double *y1, double *y2, double h, int step) {
45     int i;
46
47     x[0] = x_0;
48     y1[0] = y1_0;
49     y2[0] = y2_0;
50
51     for (i = 0; i <= step - 1; i++) {
52         x[i + 1] = x[i] + h;
53         y1[i + 1] = y1[i] + h * y1p(y1[i], y2[i]);
54         y2[i + 1] = y2[i] + h * y2p(y1[i], y2[i]);
55     }
56 }

```

3.3 実行結果

課題 6 の実行結果をリスト 2 に示す。

リスト 2 課題 6 の実行結果

```

i = 0, x = 0.0000000000000000, y1 = 10.0000000000000000, y2 = 10.0000000000000000
i = 1, x = 0.1000000000000000, y1 = 0.9999999999999996, y2 = 19.0000000000000000
i = 2, x = 0.2000000000000000, y1 = -0.7999999999999997, y2 = 19.0000000000000000
i = 3, x = 0.3000000000000000, y1 = 0.6399999999999999, y2 = 15.5800000000000001
i = 4, x = 0.4000000000000000, y1 = -0.2931200000000000, y2 = 15.0191199999999991
i = 5, x = 0.5000000000000000, y1 = 0.1178084454400000, y2 = 13.0769675545599995
i = 6, x = 0.6000000000000000, y1 = -0.0244684318832032, y2 = 11.9233285209712019

```

i = 7,	x = 0.7000000000000000,	y1 = 0.0022592401021203,	y2 = 10.7018211537004380
i = 8,	x = 0.7999999999999999,	y1 = 0.0000673657607164,	y2 = 9.6340568366820101
i = 9,	x = 0.8999999999999999,	y1 = 0.0000092017800292,	y2 = 8.6707160535705672
i = 10,	x = 0.9999999999999999,	y1 = 0.0000021433558501,	y2 = 7.8036524268156926

3.4 考察

まずはパラメータ a, b, c, d の値が全て 1 の場合について手計算で求めたオイラー法の答えを確認する。例として、初期条件が $y_1(x_0) = 10, y_2(x_0) = 10, h = 0.1$ であるときの $y_1(x_1), y_2(x_1)$ を求める。手計算で求めた結果を次式に示す。

$$\begin{aligned}
 y_1(x_1) &= y_1(x_0) + h \{ a y_1(x_0) - c y_1(x_0) y_2(x_0) \} \\
 &= 10 + 0.1(1 \times 10 - 1 \times 10 \times 10) \\
 &= 10 - 9 = \underline{1} \\
 y_2(x_1) &= y_2(x_0) + h \{ -b y_1(x_0) + d y_1(x_0) y_2(x_0) \} \\
 &= 10 + 0.1(-1 \times 10 + 1 \times 10 \times 10) \\
 &= 10 + 9 = \underline{19}
 \end{aligned} \tag{2}$$

これは、リスト 2 の結果と一致しているので、正しく計算が行えている。

次に変数 y_1, y_2 を縦軸、時間 x を横軸としてプロットしたグラフを図 1 に示す。

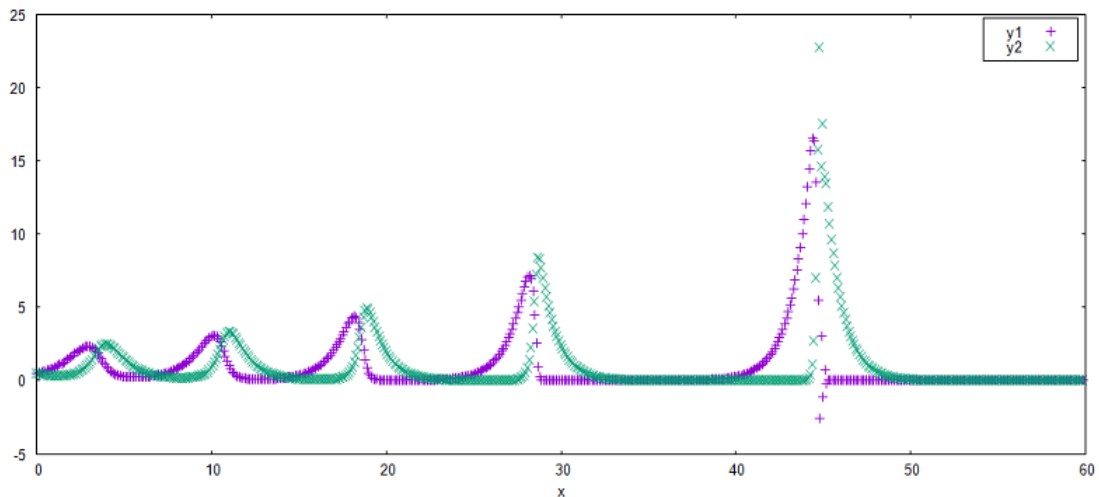


図 1 生存競争モデル

生存競争モデルであるため y_1 を被食者、 y_2 を捕食者として考えると、次のような循環があることがわかる。

1. 被食者が増える。
2. 捕食者が増える。
3. 捕食者に食べられて被食者が減る。
4. 被食者が少なくなったため、捕食者が減る。
5. 捕食者が減ったため、被食者がまた増えて (1) に戻る。

次に、初期条件を $y_1(0) = 10, y_2(0) = 0$ とし、パラメータ a, b, c, d を変更した場合について考える。パラメータの値を $a = 1, b = 1, c = 0.01, d = 0.01$ とした場合のグラフを図 2 に、 $a = 0.01, b = 1, c = 1, d = 0.01$ とし

た場合のグラフを図 3 に、 $a = 1, b = 0.01, c = 0.01, d = 1$ とした場合のグラフを図 4 に、 $a = b = c = d = 1$ とした場合のグラフを図 5 に示す。

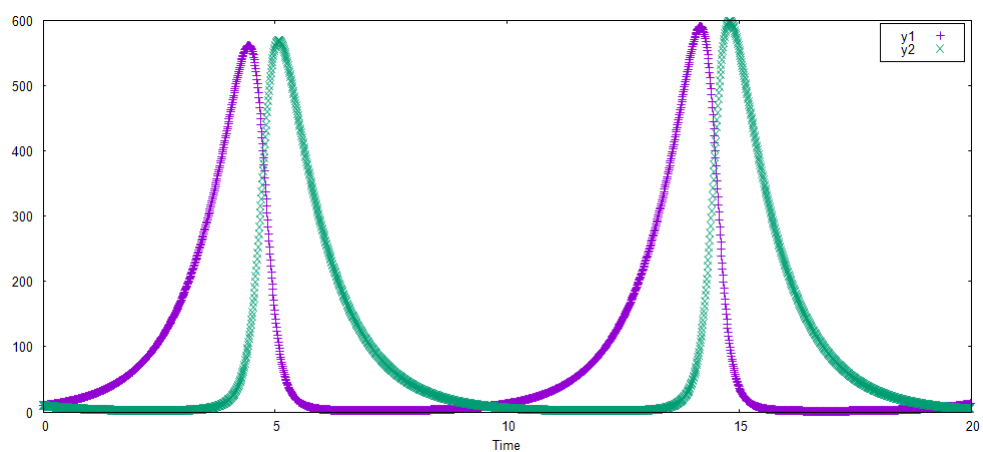


図 2 $y_2 < a/c, y_1 < b/d$

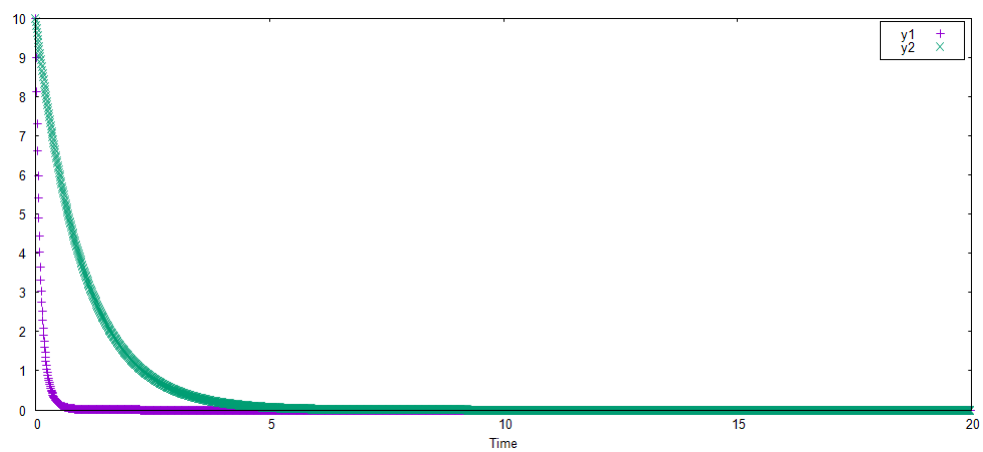


図 3 $y_2 > a/c, y_1 < b/d$

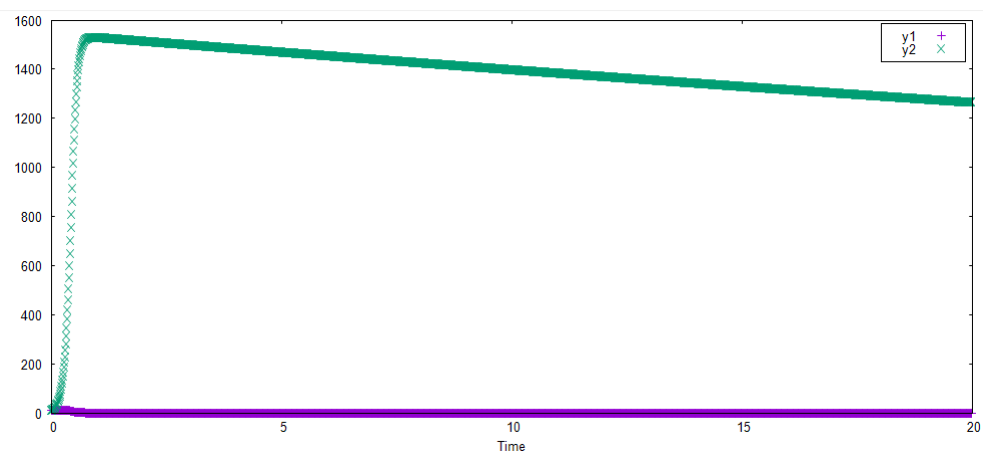


図 4 $y_2 < a/c, y_1 > b/d$

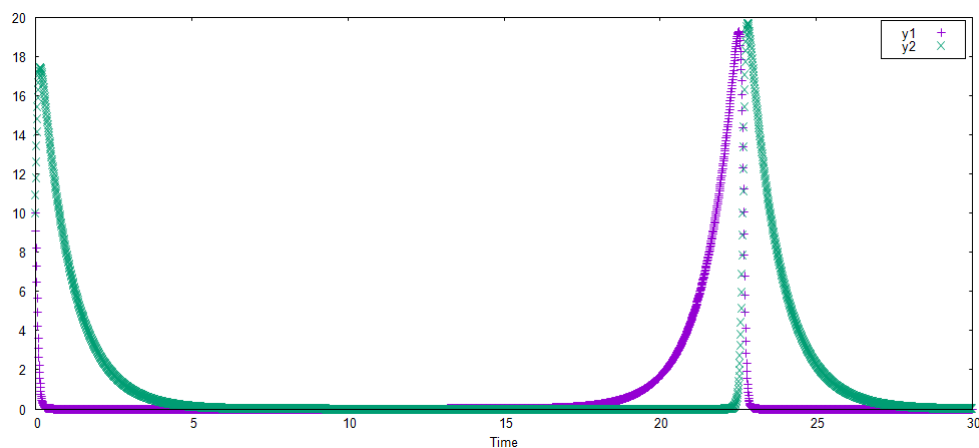


図 5 $y_2 > a/c, y_1 > b/d$

図 2-5 と式 (1) より、パラメータ a, b, c, d には次のような意味があると考えられる。

- a … 被食者の増えやすさ
- b … 捕食者の減りやすさ
- c … 捕食者の数と比例した被食者の減りやすさ
- d … 被食者の数と比例した捕食者の増えやすさ

図 2, 5 では各パラメータのバランスが取れているため、前述した簡条書きのような循環が起こっている。その一方で図 3, 4 ではパラメータ同士のバランスが取れていないため、被食者の数が多くなり、最終的にはどちらも絶滅してしまう。図 3 では a が小さく、 c が大きいため、被食者が増えにくく減りやすくなる。捕食者はパラメータ上では減りにくく増えにくい、被食者が減りやすい状態においては捕食者も減りやすくなってしまふ。図 4 では b が小さく d が大きいため、捕食者が増えやすく減りにくくなる。被食者の数もパラメータ上では増えやすく減りにくいので捕食者が一気に増加するが、被食者が減りすぎて捕食者は少しずつ少なくなっていく。図 3 とは異なり、捕食者が少しづつ減っているのはパラメータ b により減りにくくなっているためだと考えられる。

4 課題 7：ニュートンの運動方程式

式 (3) に示す、ニュートンの運動方程式の高階微分方程式をオイラー法で解くプログラムを作成する。

$$\begin{cases} mv' = -ky - lv \\ y' = v \end{cases} \quad (3)$$

4.1 プログラムリスト

課題 7 のプログラムリストをリスト 3 に示す。

リスト 3 課題 7 のプログラム

```
1 #include <stdio.h>
2 #include <math.h>
3
4 #define l 0
5 #define k 1
```

```

6  #define m 1
7  #define t_0 0
8  #define y_0 10
9  #define v_0 0
10
11 #define H 0.01
12 #define STEP 100
13
14 double yp(double);
15 double vp(double, double);
16 void euler(double *, double *, double *, double, int);
17
18 int main(void) {
19     int i;
20     double t[STEP + 1];
21     double y[STEP + 1];
22     double v[STEP + 1];
23
24     euler(t, y, v, H, STEP);
25
26     for (i = 0; i <= STEP; i++)
27         printf("i = %2d, t = %.16f, y = %.16f, v = %.16f\n", i, t[i], y[i], v[i]);
28
29     return 0;
30 }
31
32 // y'
33 double yp(double v) {
34     return v;
35 }
36
37 // v'
38 double vp(double y, double v) {
39     return (-k * y - l * v) / m;
40 }
41
42 // オイラー法
43 void euler(double *t, double *y, double *v, double h, int step) {
44     int i;
45
46     t[0] = t_0;
47     y[0] = y_0;
48     v[0] = v_0;
49
50     for (i = 0; i <= step - 1; i++) {
51         t[i + 1] = t[i] + h;
52         y[i + 1] = y[i] + h * yp(v[i]);
53         v[i + 1] = v[i] + h * vp(y[i], v[i]);
54     }
55 }

```

4.2 実行結果

課題 7 の実行結果をリスト 4 に示す。

リスト 4 課題 7 の実行結果

```

i = 0, t = 0.0000000000000000, y = 10.0000000000000000, v = 0.0000000000000000
i = 1, t = 0.0100000000000000, y = 10.0000000000000000, v = -0.1000000000000000
i = 2, t = 0.0200000000000000, y = 9.9990000000000006, v = -0.2000000000000000
i = 3, t = 0.0300000000000000, y = 9.9969999999999999, v = -0.2999900000000000
i = 4, t = 0.0400000000000000, y = 9.9940000999999992, v = -0.3999600000000000
i = 5, t = 0.0500000000000000, y = 9.9900004999999990, v = -0.4999000010000000
i = 6, t = 0.0600000000000000, y = 9.9850014999899983, v = -0.5998000060000001
i = 7, t = 0.0700000000000000, y = 9.9790034999299984, v = -0.6996500209999000
i = 8, t = 0.0800000000000000, y = 9.9720069997199996, v = -0.7994400559992000
i = 9, t = 0.0900000000000000, y = 9.9640125991600073, v = -0.8991601259963999
i = 10, t = 0.1000000000000000, y = 9.9550209979000428, v = -0.9988002519880000
i = 11, t = 0.1100000000000000, y = 9.9450329953801635, v = -1.0983504619670004
i = 12, t = 0.1200000000000000, y = 9.9340494907604935, v = -1.1978007919208020
i = 13, t = 0.1300000000000000, y = 9.9220714828412859, v = -1.2971412868284069
i = 14, t = 0.1400000000000000, y = 9.9091000699730021, v = -1.3963620016568197
i = 15, t = 0.1500000000000000, y = 9.8951364499564338, v = -1.4954530023565498
i = 16, t = 0.1600000000000000, y = 9.8801819199328680, v = -1.5944043668561141
i = 17, t = 0.1700000000000000, y = 9.8642378762643066, v = -1.6932061860554428
i = 18, t = 0.1800000000000000, y = 9.8473058144037520, v = -1.7918485648180860
i = 19, t = 0.1900000000000000, y = 9.8293873287555709, v = -1.8903216229621236
i = 20, t = 0.2000000000000000, y = 9.8104841125259501, v = -1.9886154962496794
i = 21, t = 0.2100000000000000, y = 9.7905979575634525, v = -2.0867203373749388
i = 22, t = 0.2200000000000001, y = 9.7697307541897036, v = -2.1846263169505735
i = 23, t = 0.2300000000000001, y = 9.7478844910201978, v = -2.2823236244924705
i = 24, t = 0.2400000000000001, y = 9.7250612547752731, v = -2.3798024694026725
i = 25, t = 0.2500000000000001, y = 9.7012632300812456, v = -2.4770530819504253
i = 26, t = 0.2600000000000001, y = 9.6764926992617415, v = -2.5740657142512378
i = 27, t = 0.2700000000000001, y = 9.6507520421192297, v = -2.6708306412438554
i = 28, t = 0.2800000000000001, y = 9.6240437357067918, v = -2.7673381616650476
i = 29, t = 0.2900000000000001, y = 9.5963703540901406, v = -2.8635785990221154
i = 30, t = 0.3000000000000001, y = 9.5677345680999188, v = -2.9595423025630168
i = 31, t = 0.3100000000000001, y = 9.5381391450742878, v = -3.0552196482440159
i = 32, t = 0.3200000000000001, y = 9.5075869485918485, v = -3.1506010396947590
i = 33, t = 0.3300000000000001, y = 9.4760809381949009, v = -3.2456769091806774
i = 34, t = 0.3400000000000001, y = 9.4436241691030940, v = -3.3404377185626264
i = 35, t = 0.3500000000000001, y = 9.4102197919174682, v = -3.4348739602536575
i = 36, t = 0.3600000000000002, y = 9.3758710523149311, v = -3.5289761581728323
i = 37, t = 0.3700000000000002, y = 9.3405812907332031, v = -3.6227348686959817
i = 38, t = 0.3800000000000002, y = 9.3043539420462427, v = -3.7161406816033136
i = 39, t = 0.3900000000000002, y = 9.2671925352302100, v = -3.8091842210237759
i = 40, t = 0.4000000000000002, y = 9.2291006930199728, v = -3.9018561463760779
i = 41, t = 0.4100000000000002, y = 9.1900821315562116, v = -3.9941471533062778
i = 42, t = 0.4200000000000002, y = 9.1501406600231494, v = -4.0860479746218399
i = 43, t = 0.4300000000000002, y = 9.1092801802769312, v = -4.1775493812220716

```


i = 44, t = 0.4400000000000002, y = 9.0675046864647104, v = -4.2686421830248413
 i = 45, t = 0.4500000000000002, y = 9.0248182646344617, v = -4.3593172298894887
 i = 46, t = 0.4600000000000002, y = 8.9812250923355670, v = -4.4495654125358337
 i = 47, t = 0.4700000000000003, y = 8.9367294382102092, v = -4.5393776634591898
 i = 48, t = 0.4800000000000003, y = 8.8913356615756172, v = -4.6287449578412918
 i = 49, t = 0.4900000000000003, y = 8.8450482119972040, v = -4.7176583144570481
 i = 50, t = 0.5000000000000002, y = 8.7978716288526329, v = -4.8061087965770204
 i = 51, t = 0.5100000000000002, y = 8.7498105408868625, v = -4.8940875128655463
 i = 52, t = 0.5200000000000002, y = 8.7008696657582068, v = -4.9815856182744147
 i = 53, t = 0.5300000000000002, y = 8.6510538095754619, v = -5.0685943149319970
 i = 54, t = 0.5400000000000003, y = 8.6003678664261418, v = -5.1551048530277512
 i = 55, t = 0.5500000000000003, y = 8.5488168178958635, v = -5.2411085316920127
 i = 56, t = 0.5600000000000003, y = 8.4964057325789426, v = -5.3265966998709713
 i = 57, t = 0.5700000000000003, y = 8.4431397655802325, v = -5.4115607571967610
 i = 58, t = 0.5800000000000003, y = 8.3890241580082652, v = -5.4959921548525630
 i = 59, t = 0.5900000000000003, y = 8.3340642364597404, v = -5.5798823964326454
 i = 60, t = 0.6000000000000003, y = 8.2782654124954131, v = -5.6632230387972431
 i = 61, t = 0.6100000000000003, y = 8.2216331821074409, v = -5.7460056929221972
 i = 62, t = 0.6200000000000003, y = 8.1641731251782197, v = -5.8282220247432717
 i = 63, t = 0.6300000000000003, y = 8.1058909049307868, v = -5.9098637559950538
 i = 64, t = 0.6400000000000003, y = 8.0467922673708365, v = -5.9909226650443612
 i = 65, t = 0.6500000000000004, y = 7.9868830407203930, v = -6.0713905877180698
 i = 66, t = 0.6600000000000004, y = 7.9261691348432119, v = -6.1512594181252735
 i = 67, t = 0.6700000000000004, y = 7.8646565406619589, v = -6.2305211094737061
 i = 68, t = 0.6800000000000004, y = 7.8023513295672222, v = -6.3091676748803254
 i = 69, t = 0.6900000000000004, y = 7.7392596528184185, v = -6.3871911881759980
 i = 70, t = 0.7000000000000004, y = 7.6753877409366584, v = -6.4645837847041818
 i = 71, t = 0.7100000000000004, y = 7.6107419030896164, v = -6.5413376621135484
 i = 72, t = 0.7200000000000004, y = 7.5453285264684808, v = -6.6174450811444450
 i = 73, t = 0.7300000000000004, y = 7.4791540756570365, v = -6.6928983664091302
 i = 74, t = 0.7400000000000004, y = 7.4122250919929451, v = -6.7676899071657006
 i = 75, t = 0.7500000000000004, y = 7.3445481929212884, v = -6.8418121580856299
 i = 76, t = 0.7600000000000005, y = 7.2761300713404324, v = -6.9152576400148424
 i = 77, t = 0.7700000000000005, y = 7.2069774949402836, v = -6.9880189407282467
 i = 78, t = 0.7800000000000005, y = 7.1370973055330014, v = -7.0600887156776500
 i = 79, t = 0.7900000000000005, y = 7.0664964183762251, v = -7.1314596887329795
 i = 80, t = 0.8000000000000005, y = 6.9951818214888952, v = -7.2021246529167415
 i = 81, t = 0.8100000000000005, y = 6.9231605749597280, v = -7.2720764711316308
 i = 82, t = 0.8200000000000005, y = 6.8504398102484121, v = -7.3413080768812282
 i = 83, t = 0.8300000000000005, y = 6.7770267294795996, v = -7.4098124749837124
 i = 84, t = 0.8400000000000005, y = 6.7029286047297623, v = -7.4775827422785088
 i = 85, t = 0.8500000000000005, y = 6.6281527773069771, v = -7.5446120283258065
 i = 86, t = 0.8600000000000005, y = 6.5527066570237187, v = -7.6108935560988762
 i = 87, t = 0.8700000000000006, y = 6.4765977214627295, v = -7.6764206226691130
 i = 88, t = 0.8800000000000006, y = 6.3998335152360379, v = -7.7411865998837399

```

i = 89, t = 0.8900000000000006, y = 6.3224216492372003, v = -7.8051849350361007
i = 90, t = 0.9000000000000006, y = 6.2443697998868393, v = -7.8684091515284731
i = 91, t = 0.9100000000000006, y = 6.1656857083715542, v = -7.9308528495273416
i = 92, t = 0.9200000000000006, y = 6.0863771798762807, v = -7.9925097066110569
i = 93, t = 0.9300000000000006, y = 6.0064520828101697, v = -8.0533734784098190
i = 94, t = 0.9400000000000006, y = 5.9259183480260713, v = -8.1134379992379202
i = 95, t = 0.9500000000000006, y = 5.8447839680336919, v = -8.1726971827181814
i = 96, t = 0.9600000000000006, y = 5.7630569962065099, v = -8.2311450223985183
i = 97, t = 0.9700000000000006, y = 5.6807455459825249, v = -8.2887755923605830
i = 98, t = 0.9800000000000006, y = 5.5978577900589190, v = -8.3455830478204085
i = 99, t = 0.9900000000000007, y = 5.5144019595807148, v = -8.4015616257209977
i = 100, t = 1.0000000000000007, y = 5.4303863433235051, v = -8.4567056453168057

```

4.3 考察

実行結果をグラフにプロットしたものを図6に示す。図6は $l=0$ であるため、単振動になっている。次に、初期条件 k, l を変更した場合のグラフを図7, 8に示す。3つのグラフから、ばね定数を示す k が y, v の振動のしやすさに、摩擦係数の l が振動のしにくさを表していることが分かる。そのため図7は l が大きく、振動が止まっている。図8は k が大きいため図7よりは振動するが、 $l=1$ であるため振動が止まる。

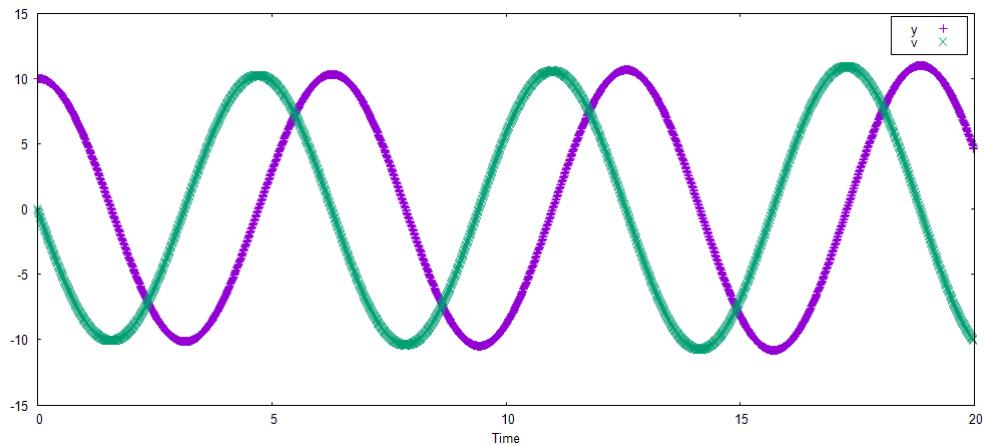


図6 初期条件 $m=1, k=1, l=0$

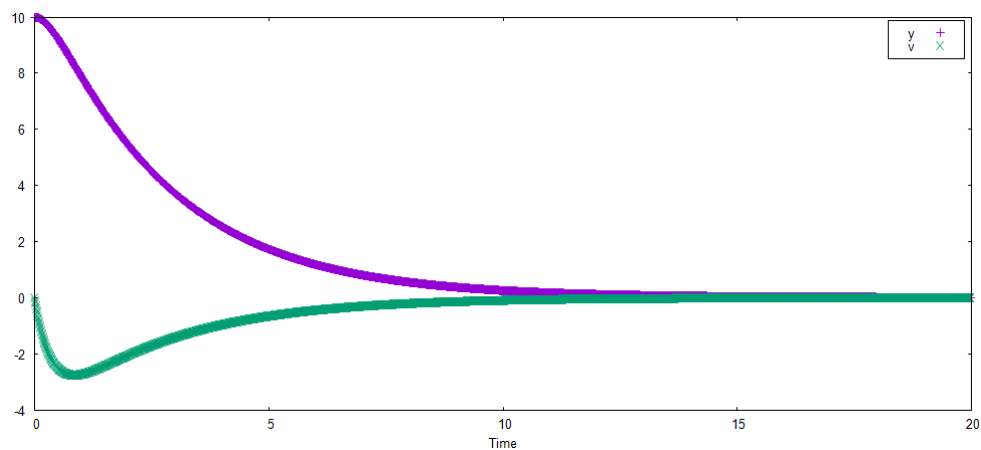


図 7 初期条件 $k = 1, l = 3$

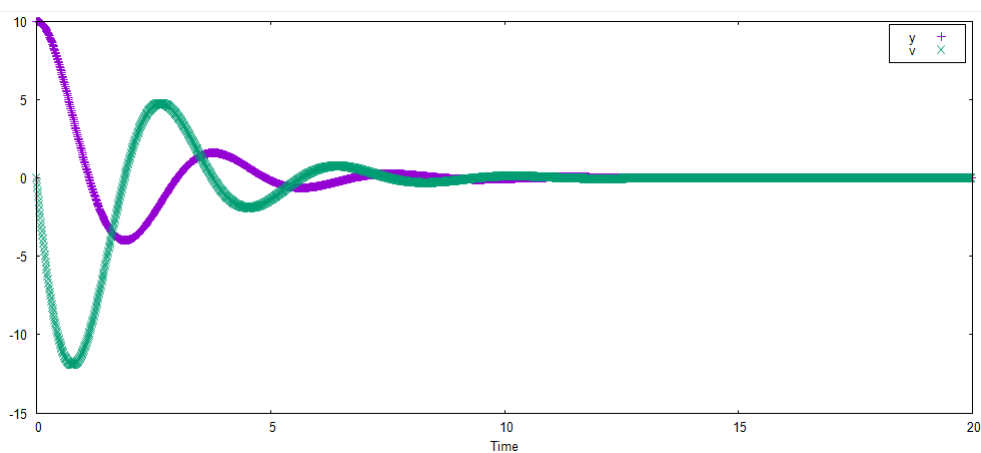


図 8 初期条件 $k = 3, l = 1$

5 課題 8 : RLC 共振回路

式 (4) に示す、RLC 共振回路の微分方程式をホイン法で解くプログラムを作成する。

$$\frac{dV}{dt^2} + \frac{R}{L} \frac{dC}{dt} + \frac{V}{LC} = 0 \quad (4)$$

5.1 プログラムリスト

課題 8 のプログラムリストをリスト 5 に示す。

リスト 5 課題 8 のプログラム

```

1 #include <stdio.h>
2 #include <math.h>
3
4 #define R 1
5 #define C 0.3
6 #define L 10
7 #define t_0 0

```

```

8  #define Q_0 10
9  #define I_0 0
10
11 #define H 0.01
12 #define STEP 100
13
14 double Qp(double);
15 double Ip(double, double);
16 void heun(double *, double *, double *, double, int);
17
18 int main(void) {
19     int i;
20     double t[STEP + 1];
21     double y[STEP + 1];
22     double v[STEP + 1];
23
24     heun(t, y, v, H, STEP);
25
26     for (i = 0; i <= STEP; i++)
27         printf("i = %2d, t = %.16f, y = %.16f, v = %.16f\n", i, t[i], y[i], v[i]);
28
29     return 0;
30 }
31
32 // Q'
33 double Qp(double I) {
34     return I;
35 }
36
37 // I'
38 double Ip(double Q, double I) {
39     return (-R * I - Q / C) / L;
40 }
41
42 // ホイン法
43 void heun(double *t, double *Q, double *I, double h, int step) {
44     int i;
45
46     t[0] = t_0;
47     Q[0] = Q_0;
48     I[0] = I_0;
49
50     for (i = 0; i <= step - 1; i++) {
51         double k1;
52         double k2;
53
54         t[i + 1] = t[i] + h;
55
56         k1 = h * Qp(I[i]);
57         k2 = h * Qp(I[i] + k1);
58         Q[i + 1] = Q[i] + (k1 + k2) / 2;
59
60         k1 = h * Ip(Q[i], I[i]);
61         k2 = h * Ip(Q[i] + h, I[i] + k1);
62         I[i + 1] = I[i] + (k1 + k2) / 2;
63     }

```

5.2 実行結果

課題 8 の実行結果をリスト 6 に示す。

リスト 6 課題 8 の実行結果

```
i = 0, t = 0.0000000000000000, y = 10.000000000000000, v = 0.0000000000000000
i = 1, t = 0.0100000000000000, y = 10.000000000000000, v = -0.0333333333333333
i = 2, t = 0.0200000000000000, y = 9.9996650000000002, v = -0.0666333500000000
i = 3, t = 0.0300000000000000, y = 9.9989953348325002, v = -0.0998989671916750
i = 4, t = 0.0400000000000000, y = 9.9979913502122244, v = -0.1331291042978505
i = 5, t = 0.0500000000000000, y = 9.9966534027140312, v = -0.1663226829398952
i = 6, t = 0.0600000000000000, y = 9.9949818597504851, v = -0.1994786270050057
i = 7, t = 0.0700000000000000, y = 9.9929770995490852, v = -0.2325958626800496
i = 8, t = 0.0800000000000000, y = 9.9906395111291513, v = -0.2656733184852986
i = 9, t = 0.0900000000000000, y = 9.9879694942783743, v = -0.2987099253080512
i = 10, t = 0.1000000000000000, y = 9.9849674595290292, v = -0.3317046164361432
i = 11, t = 0.1100000000000000, y = 9.9816338281338464, v = -0.3646563275913462
i = 12, t = 0.1200000000000000, y = 9.9779690320415533, v = -0.3975639969626513
i = 13, t = 0.1300000000000000, y = 9.9739735138720782, v = -0.4304265652394389
i = 14, t = 0.1400000000000000, y = 9.9696477268914219, v = -0.4632429756445325
i = 15, t = 0.1500000000000000, y = 9.9649921349861952, v = -0.4960121739671358
i = 16, t = 0.1600000000000000, y = 9.9600072126378247, v = -0.5287331085956513
i = 17, t = 0.1700000000000000, y = 9.9546934448964386, v = -0.5614047305503815
i = 18, t = 0.1800000000000000, y = 9.9490513273544074, v = -0.5940259935161097
i = 19, t = 0.1900000000000000, y = 9.9430813661195714, v = -0.6265958538745595
i = 20, t = 0.2000000000000000, y = 9.9367840777881327, v = -0.6591132707367336
i = 21, t = 0.2100000000000000, y = 9.9301599894172288, v = -0.6915772059751297
i = 22, t = 0.2200000000000001, y = 9.9232096384971786, v = -0.7239866242558326
i = 23, t = 0.2300000000000001, y = 9.9159335729234073, v = -0.7563404930704820
i = 24, t = 0.2400000000000001, y = 9.9083323509680490, v = -0.7886377827681146
i = 25, t = 0.2500000000000001, y = 9.9004065412512290, v = -0.8208774665868798
i = 26, t = 0.2600000000000001, y = 9.8921567227120306, v = -0.8530585206856282
i = 27, t = 0.2700000000000001, y = 9.8835834845791393, v = -0.8851799241753718
i = 28, t = 0.2800000000000001, y = 9.8746874263411772, v = -0.9172406591506147
i = 29, t = 0.2900000000000001, y = 9.8654691577167135, v = -0.9492397107205537
i = 30, t = 0.3000000000000001, y = 9.8559292986239715, v = -0.9811760670401480
i = 31, t = 0.3100000000000001, y = 9.8460684791502189, v = -1.0130487193410569
i = 32, t = 0.3200000000000001, y = 9.8358873395208413, v = -1.0448566619624444
i = 33, t = 0.3300000000000001, y = 9.8253865300681191, v = -1.0765988923816499
i = 34, t = 0.3400000000000001, y = 9.8145667111996833, v = -1.1082744112447247
i = 35, t = 0.3500000000000001, y = 9.8034285533666736, v = -1.1398822223968326
i = 36, t = 0.3600000000000002, y = 9.7919727370315854, v = -1.1714213329125136
i = 37, t = 0.3700000000000002, y = 9.7801999526358152, v = -1.2028907531258111
```

i = 38, t = 0.3800000000000002, y = 9.7681109005669011, v = -1.2342894966602602
 i = 39, t = 0.3900000000000002, y = 9.7557062911254651, v = -1.2656165804587369
 i = 40, t = 0.4000000000000002, y = 9.7429868444918544, v = -1.2968710248131681
 i = 41, t = 0.4100000000000002, y = 9.7299532906924817, v = -1.3280518533940993
 i = 42, t = 0.4200000000000002, y = 9.7166063695658718, v = -1.3591580932801224
 i = 43, t = 0.4300000000000002, y = 9.7029468307284059, v = -1.3901887749871591
 i = 44, t = 0.4400000000000002, y = 9.6889754335397846, v = -1.4211429324976028
 i = 45, t = 0.4500000000000002, y = 9.6746929470681842, v = -1.4520196032893149
 i = 46, t = 0.4600000000000002, y = 9.6601001500551273, v = -1.4828178283644760
 i = 47, t = 0.4700000000000003, y = 9.6451978308800648, v = -1.5135366522782927
 i = 48, t = 0.4800000000000003, y = 9.6299867875246683, v = -1.5441751231675560
 i = 49, t = 0.4900000000000003, y = 9.6144678275368349, v = -1.5747322927790530
 i = 50, t = 0.5000000000000002, y = 9.5986417679944047, v = -1.6052072164978306
 i = 51, t = 0.5100000000000002, y = 9.5825094354686016, v = -1.6355989533753090
 i = 52, t = 0.5200000000000002, y = 9.5660716659871792, v = -1.6659065661572465
 i = 53, t = 0.5300000000000002, y = 9.5493293049972987, v = -1.6961291213115530
 i = 54, t = 0.5400000000000003, y = 9.5322832073281170, v = -1.7262656890559516
 i = 55, t = 0.5500000000000003, y = 9.5149342371531045, v = -1.7563153433854883
 i = 56, t = 0.5600000000000003, y = 9.4972832679520796, v = -1.7862771620998896
 i = 57, t = 0.5700000000000003, y = 9.4793311824729756, v = -1.8161502268307645
 i = 58, t = 0.5800000000000003, y = 9.4610788726933261, v = -1.8459336230686529
 i = 59, t = 0.5900000000000003, y = 9.4425272397814854, v = -1.8756264401899192
 i = 60, t = 0.6000000000000003, y = 9.4236771940575768, v = -1.9052277714834880
 i = 61, t = 0.6100000000000003, y = 9.4045296549541675, v = -1.9347367141774254
 i = 62, t = 0.6200000000000003, y = 9.3850855509766848, v = -1.9641523694653606
 i = 63, t = 0.6300000000000003, y = 9.3653458196635579, v = -1.9934738425327507
 i = 64, t = 0.6400000000000003, y = 9.3453114075461041, v = -2.0227002425829852
 i = 65, t = 0.6500000000000004, y = 9.3249832701081452, v = -2.0518306828633315
 i = 66, t = 0.6600000000000004, y = 9.3043623717453681, v = -2.0808642806907200
 i = 67, t = 0.6700000000000004, y = 9.2834496857244257, v = -2.1098001574773679
 i = 68, t = 0.6800000000000004, y = 9.2622461941417775, v = -2.1386374387562412
 i = 69, t = 0.6900000000000004, y = 9.2407528878822767, v = -2.1673752542063536
 i = 70, t = 0.7000000000000004, y = 9.2189707665775025, v = -2.1960127376779019
 i = 71, t = 0.7100000000000004, y = 9.1969008385638400, v = -2.2245490272172401
 i = 72, t = 0.7200000000000004, y = 9.1745441208403076, v = -2.2529832650916850
 i = 73, t = 0.7300000000000004, y = 9.1519016390261356, v = -2.2813145978141587
 i = 74, t = 0.7400000000000004, y = 9.1289744273181039, v = -2.3095421761676653
 i = 75, t = 0.7500000000000004, y = 9.1057635284476195, v = -2.3376651552296006
 i = 76, t = 0.7600000000000005, y = 9.0822699936375617, v = -2.3656826943958933
 i = 77, t = 0.7700000000000005, y = 9.0584948825588825, v = -2.3935939574049803
 i = 78, t = 0.7800000000000005, y = 9.0344392632869628, v = -2.4213981123616128
 i = 79, t = 0.7900000000000005, y = 9.0101042122577280, v = -2.4490943317604916
 i = 80, t = 0.8000000000000005, y = 8.9854908142235352, v = -2.4766817925097357
 i = 81, t = 0.8100000000000005, y = 8.9606001622088129, v = -2.5041596759541771
 i = 82, t = 0.8200000000000005, y = 8.9354333574654738, v = -2.5315271678984868

```

i = 83, t = 0.8300000000000005, y = 8.9099915094280941, v = -2.5587834586301281
i = 84, t = 0.8400000000000005, y = 8.8842757356688615, v = -2.5859277429421388
i = 85, t = 0.8500000000000005, y = 8.8582871618522923, v = -2.6129592201557381
i = 86, t = 0.8600000000000005, y = 8.8320269216897263, v = -2.6398770941427636
i = 87, t = 0.8700000000000006, y = 8.8054961568935912, v = -2.6666805733479308
i = 88, t = 0.8800000000000006, y = 8.7786960171314448, v = -2.6933688708109202
i = 89, t = 0.8900000000000006, y = 8.7516276599797944, v = -2.7199412041882876
i = 90, t = 0.9000000000000006, y = 8.7242922508777028, v = -2.7463967957752007
i = 91, t = 0.9100000000000006, y = 8.6966909630801617, v = -2.7727348725269976
i = 92, t = 0.9200000000000006, y = 8.6688249776112656, v = -2.7989546660805691
i = 93, t = 0.9300000000000006, y = 8.6406954832171561, v = -2.8250554127755629
i = 94, t = 0.9400000000000006, y = 8.6123036763187617, v = -2.8510363536754122
i = 95, t = 0.9500000000000006, y = 8.5836507609643231, v = -2.8768967345881822
i = 96, t = 0.9600000000000006, y = 8.5547379487817121, v = -2.9026358060872406
i = 97, t = 0.9700000000000006, y = 8.5255664589305358, v = -2.9282528235317473
i = 98, t = 0.9800000000000006, y = 8.4961375180540415, v = -2.9537470470869645
i = 99, t = 0.9900000000000007, y = 8.4664523602308179, v = -2.9791177417443846
i = 100, t = 1.0000000000000007, y = 8.4365122269262862, v = -3.0043641773416803

```

5.3 考察

$R = 0, 3, 6, 9, 2\sqrt{L/C}$ の場合それぞれについてプロットしたグラフを図 9-13 に示す。図 9 より刻み幅は 0.01 が減衰しない刻み幅であることがわかった。

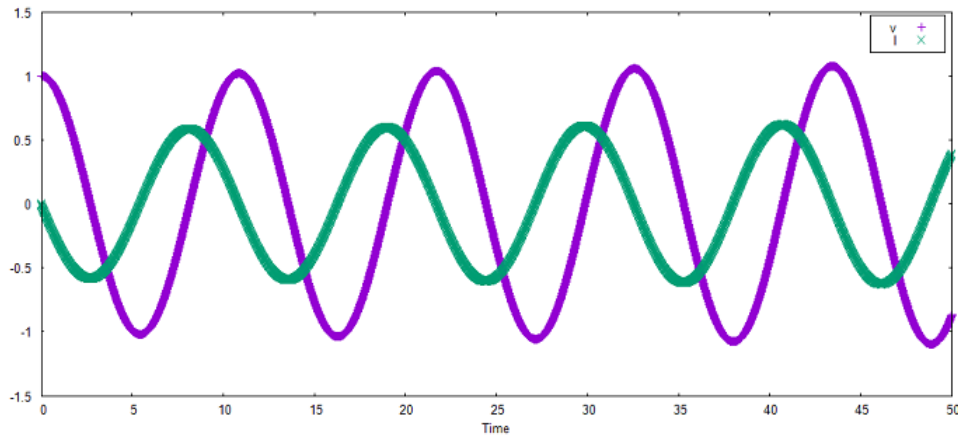


図 9 $R = 0$

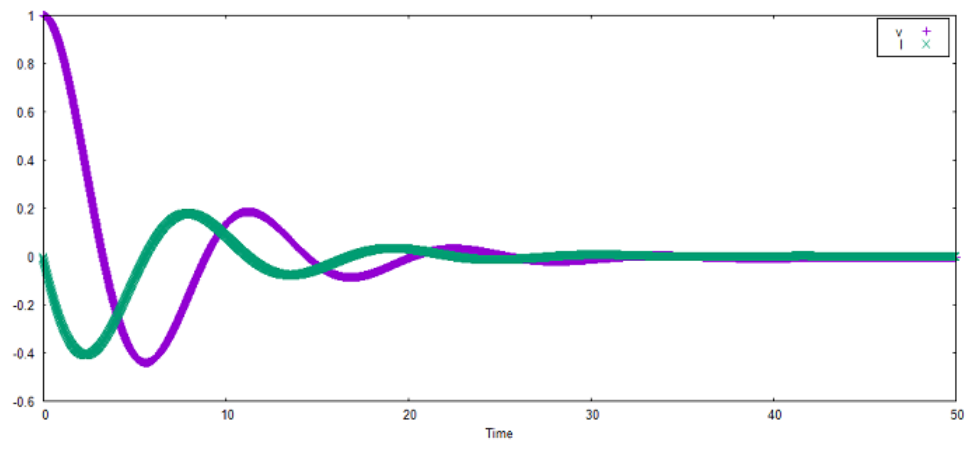


図 10 $R = 3$

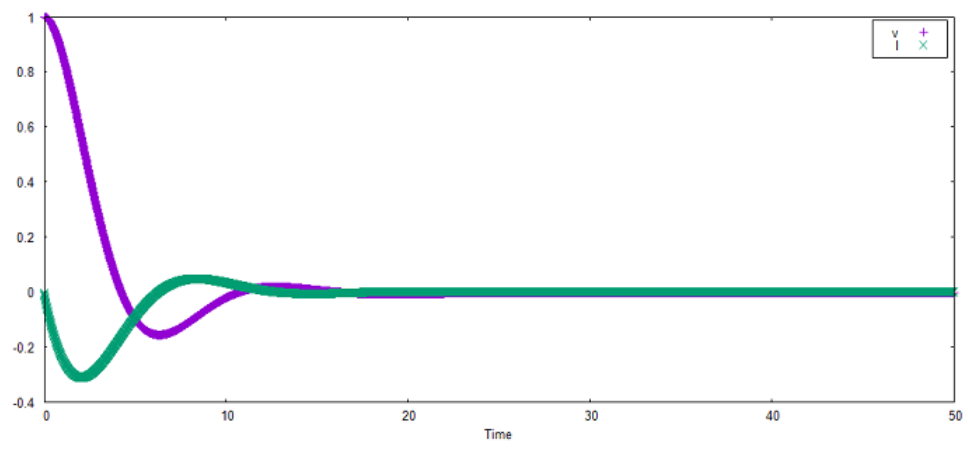


図 11 $R = 6$

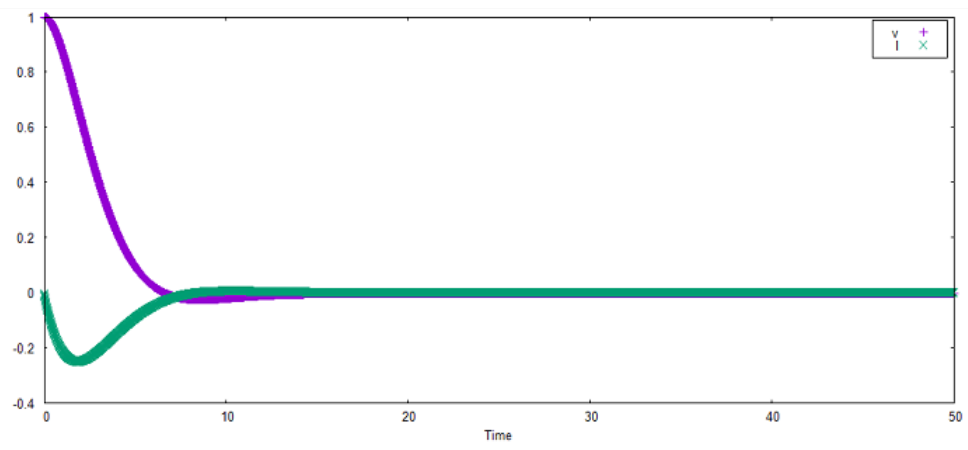


図 12 $R = 9$

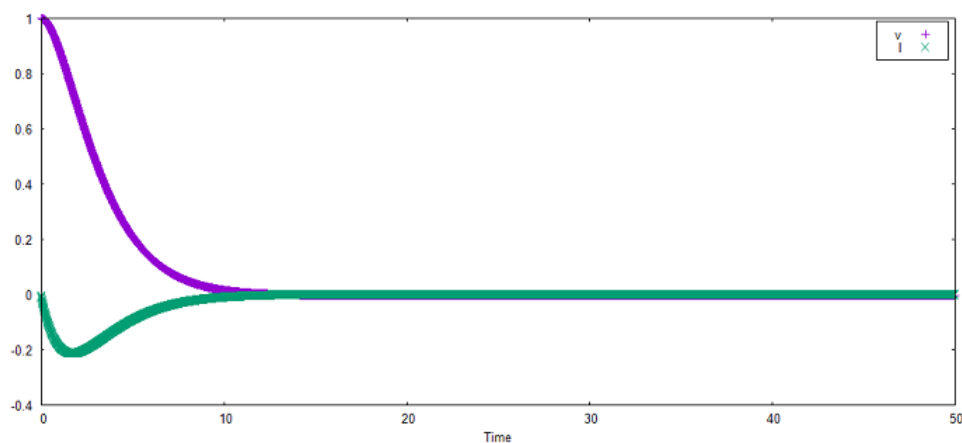


図 13 $R = 2\sqrt{L/C}$

図 9-13 より、 R が大きければ大きいほど振動が収まりやすくなっているため、 R は振動のしやすさに関連していることがわかる。これは R が抵抗の値であり、 R と V , I が反比例の関係にあるためだと考えられる。

6 課題 9：ローレンツ力

式 (5), (6) に示す、ローレンツ力についての連立微分方程式をホイン法で解くプログラムを作成する。

$$\frac{d^2x}{dt} = \frac{q}{m}v_yB_0, \quad \frac{dx}{dt} = v_x \quad (5)$$

$$\frac{d^2y}{dt} = -\frac{q}{m}v_xB_0, \quad \frac{dy}{dt} = v_y \quad (6)$$

6.1 プログラムリスト

課題 9 のプログラムリストをリスト 7 に示す。

リスト 7 課題 9 のプログラム

```

1  #include <stdio.h>
2  #include <math.h>
3
4  #define qB_m 2
5  #define t_0 0
6  #define x_0 0.1
7  #define y_0 0
8  #define vx_0 1
9  #define vy_0 0.1
10
11 #define H 0.01
12 #define STEP 100
13
14 double xp(double);
15 double yp(double);
16 double vxp(double);
17 double vyp(double);
18 void heun(double *, double *, double *, double *, double *, double, int);

```

```

19
20 int main(void) {
21     int i;
22     double t[STEP + 1];
23     double x[STEP + 1];
24     double y[STEP + 1];
25     double vx[STEP + 1];
26     double vy[STEP + 1];
27
28     heun(t, x, y, vx, vy, H, STEP);
29
30     for (i = 0; i <= STEP; i++) {
31         printf("t = %.2f, x = %.6f, y = %.6f, vx = %.6f, vy = %.6f\n", t[i], x[i], y[i], vx
32             [i], vy[i]);
33     }
34     return 0;
35 }
36
37 // x'(vx)
38 double xp(double vx) {
39     return vx;
40 }
41
42 // y'(vy)
43 double yp(double vy) {
44     return vy;
45 }
46
47 // vx'(vy)
48 double vxp(double vy) {
49     return qB_m * vy;
50 }
51
52 // vy'(vx)
53 double vyp(double vx) {
54     return -qB_m * vx;
55 }
56
57 // ホイン法
58 void heun(double *t, double *x, double *y, double *vx, double *vy, double h, int step) {
59     int i;
60
61     t[0] = t_0;
62     x[0] = x_0;
63     y[0] = y_0;
64     vx[0] = vx_0;
65     vy[0] = vy_0;
66
67     for (i = 0; i <= step - 1; i++) {
68         double k1;
69         double k2;
70
71         t[i + 1] = t[i] + h;
72
73         k1 = h * xp(vx[i]);

```

```

74     k2 = h * xp(vx[i] + k1);
75     x[i + 1] = x[i] + (k1 + k2) / 2;
76
77     k1 = h * yp(vy[i]);
78     k2 = h * yp(vy[i] + k1);
79     y[i + 1] = y[i] + (k1 + k2) / 2;
80
81     k1 = h * vxp(vy[i]);
82     k2 = h * vxp(vy[i] + k1);
83     vx[i + 1] = vx[i] + (k1 + k2) / 2;
84
85     k1 = h * vyp(vx[i]);
86     k2 = h * vyp(vx[i] + k1);
87     vy[i + 1] = vy[i] + (k1 + k2) / 2;
88 }
89 }

```

6.2 実行結果

課題 9 の実行結果をリスト 8 に示す。

リスト 8 課題 9 の実行結果

```

t = 0.00, x = 0.100000, y = 0.000000, vx = 1.000000, vy = 0.100000
t = 0.01, x = 0.110050, y = 0.001005, vx = 1.002020, vy = 0.080200
t = 0.02, x = 0.120120, y = 0.001811, vx = 1.003640, vy = 0.060360
t = 0.03, x = 0.130207, y = 0.002418, vx = 1.004859, vy = 0.040488
t = 0.04, x = 0.140306, y = 0.002825, vx = 1.005677, vy = 0.020592
t = 0.05, x = 0.150413, y = 0.003031, vx = 1.006093, vy = 0.000679
t = 0.06, x = 0.160524, y = 0.003038, vx = 1.006107, vy = -0.019241
t = 0.07, x = 0.170635, y = 0.002845, vx = 1.005718, vy = -0.039162
t = 0.08, x = 0.180743, y = 0.002451, vx = 1.004927, vy = -0.059075
t = 0.09, x = 0.190842, y = 0.001858, vx = 1.003734, vy = -0.078973
t = 0.10, x = 0.200930, y = 0.001064, vx = 1.002139, vy = -0.098847
t = 0.11, x = 0.211001, y = 0.000071, vx = 1.000142, vy = -0.118689
t = 0.12, x = 0.221053, y = -0.001122, vx = 0.997744, vy = -0.138492
t = 0.13, x = 0.231080, y = -0.002514, vx = 0.994947, vy = -0.158247
t = 0.14, x = 0.241079, y = -0.004105, vx = 0.991750, vy = -0.177947
t = 0.15, x = 0.251046, y = -0.005893, vx = 0.988156, vy = -0.197584
t = 0.16, x = 0.260977, y = -0.007879, vx = 0.984164, vy = -0.217150
t = 0.17, x = 0.270868, y = -0.010061, vx = 0.979778, vy = -0.236636
t = 0.18, x = 0.280715, y = -0.012439, vx = 0.974998, vy = -0.256036
t = 0.19, x = 0.290514, y = -0.015012, vx = 0.969826, vy = -0.275341
t = 0.20, x = 0.300261, y = -0.017779, vx = 0.964264, vy = -0.294543
t = 0.21, x = 0.309951, y = -0.020740, vx = 0.958314, vy = -0.313636
t = 0.22, x = 0.319582, y = -0.023892, vx = 0.951979, vy = -0.332610
t = 0.23, x = 0.329150, y = -0.027234, vx = 0.945260, vy = -0.351459
t = 0.24, x = 0.338650, y = -0.030767, vx = 0.938161, vy = -0.370175
t = 0.25, x = 0.348078, y = -0.034487, vx = 0.930683, vy = -0.388751

```

t = 0.26, x = 0.357432, y = -0.038394, vx = 0.922830, vy = -0.407179
t = 0.27, x = 0.366706, y = -0.042486, vx = 0.914605, vy = -0.425451
t = 0.28, x = 0.375898, y = -0.046762, vx = 0.906011, vy = -0.443560
t = 0.29, x = 0.385003, y = -0.051219, vx = 0.897051, vy = -0.461499
t = 0.30, x = 0.394019, y = -0.055858, vx = 0.887729, vy = -0.479260
t = 0.31, x = 0.402940, y = -0.060674, vx = 0.878048, vy = -0.496838
t = 0.32, x = 0.411765, y = -0.065667, vx = 0.868012, vy = -0.514223
t = 0.33, x = 0.420488, y = -0.070835, vx = 0.857625, vy = -0.531409
t = 0.34, x = 0.429107, y = -0.076176, vx = 0.846890, vy = -0.548390
t = 0.35, x = 0.437619, y = -0.081687, vx = 0.835813, vy = -0.565159
t = 0.36, x = 0.446018, y = -0.087367, vx = 0.824396, vy = -0.581708
t = 0.37, x = 0.454304, y = -0.093213, vx = 0.812646, vy = -0.598031
t = 0.38, x = 0.462471, y = -0.099223, vx = 0.800566, vy = -0.614121
t = 0.39, x = 0.470516, y = -0.105395, vx = 0.788160, vy = -0.629973
t = 0.40, x = 0.478437, y = -0.111727, vx = 0.775435, vy = -0.645578
t = 0.41, x = 0.486231, y = -0.118215, vx = 0.762394, vy = -0.660932
t = 0.42, x = 0.493893, y = -0.124857, vx = 0.749044, vy = -0.676027
t = 0.43, x = 0.501420, y = -0.131651, vx = 0.735388, vy = -0.690858
t = 0.44, x = 0.508811, y = -0.138594, vx = 0.721432, vy = -0.705419
t = 0.45, x = 0.516062, y = -0.145684, vx = 0.707183, vy = -0.719703
t = 0.46, x = 0.523169, y = -0.152917, vx = 0.692645, vy = -0.733706
t = 0.47, x = 0.530130, y = -0.160290, vx = 0.677824, vy = -0.747420
t = 0.48, x = 0.536942, y = -0.167802, vx = 0.662726, vy = -0.760841
t = 0.49, x = 0.543602, y = -0.175448, vx = 0.647357, vy = -0.773963
t = 0.50, x = 0.550108, y = -0.183227, vx = 0.631723, vy = -0.786780
t = 0.51, x = 0.556457, y = -0.191134, vx = 0.615830, vy = -0.799289
t = 0.52, x = 0.562646, y = -0.199167, vx = 0.599685, vy = -0.811482
t = 0.53, x = 0.568673, y = -0.207322, vx = 0.583293, vy = -0.823356
t = 0.54, x = 0.574535, y = -0.215597, vx = 0.566661, vy = -0.834905
t = 0.55, x = 0.580230, y = -0.223988, vx = 0.549796, vy = -0.846125
t = 0.56, x = 0.585755, y = -0.232491, vx = 0.532704, vy = -0.857011
t = 0.57, x = 0.591109, y = -0.241104, vx = 0.515392, vy = -0.867558
t = 0.58, x = 0.596289, y = -0.249823, vx = 0.497868, vy = -0.877763
t = 0.59, x = 0.601292, y = -0.258645, vx = 0.480137, vy = -0.887621
t = 0.60, x = 0.606118, y = -0.267565, vx = 0.462207, vy = -0.897128
t = 0.61, x = 0.610763, y = -0.276581, vx = 0.444085, vy = -0.906279
t = 0.62, x = 0.615226, y = -0.285690, vx = 0.425778, vy = -0.915072
t = 0.63, x = 0.619505, y = -0.294886, vx = 0.407294, vy = -0.923503
t = 0.64, x = 0.623598, y = -0.304167, vx = 0.388639, vy = -0.931567
t = 0.65, x = 0.627504, y = -0.313529, vx = 0.369821, vy = -0.939262
t = 0.66, x = 0.631221, y = -0.322969, vx = 0.350848, vy = -0.946585
t = 0.67, x = 0.634747, y = -0.332482, vx = 0.331727, vy = -0.953531
t = 0.68, x = 0.638081, y = -0.342065, vx = 0.312466, vy = -0.960100
t = 0.69, x = 0.641221, y = -0.351714, vx = 0.293072, vy = -0.966286
t = 0.70, x = 0.644166, y = -0.361425, vx = 0.273553, vy = -0.972089

```

t = 0.71, x = 0.646916, y = -0.371195, vx = 0.253917, vy = -0.977506
t = 0.72, x = 0.649468, y = -0.381019, vx = 0.234171, vy = -0.982533
t = 0.73, x = 0.651821, y = -0.390893, vx = 0.214324, vy = -0.987170
t = 0.74, x = 0.653975, y = -0.400814, vx = 0.194383, vy = -0.991413
t = 0.75, x = 0.655928, y = -0.410778, vx = 0.174357, vy = -0.995262
t = 0.76, x = 0.657681, y = -0.420780, vx = 0.154252, vy = -0.998714
t = 0.77, x = 0.659231, y = -0.430818, vx = 0.134078, vy = -1.001769
t = 0.78, x = 0.660578, y = -0.440885, vx = 0.113843, vy = -1.004423
t = 0.79, x = 0.661723, y = -0.450980, vx = 0.093553, vy = -1.006677
t = 0.80, x = 0.662663, y = -0.461097, vx = 0.073218, vy = -1.008530
t = 0.81, x = 0.663399, y = -0.471233, vx = 0.052846, vy = -1.009979
t = 0.82, x = 0.663930, y = -0.481383, vx = 0.032444, vy = -1.011026
t = 0.83, x = 0.664256, y = -0.491544, vx = 0.012022, vy = -1.011668
t = 0.84, x = 0.664377, y = -0.501711, vx = -0.008414, vy = -1.011906
t = 0.85, x = 0.664292, y = -0.511881, vx = -0.028855, vy = -1.011740
t = 0.86, x = 0.664002, y = -0.522049, vx = -0.049292, vy = -1.011168
t = 0.87, x = 0.663507, y = -0.532211, vx = -0.069717, vy = -1.010192
t = 0.88, x = 0.662806, y = -0.542363, vx = -0.090123, vy = -1.008812
t = 0.89, x = 0.661900, y = -0.552502, vx = -0.110501, vy = -1.007027
t = 0.90, x = 0.660790, y = -0.562622, vx = -0.130843, vy = -1.004840
t = 0.91, x = 0.659475, y = -0.572721, vx = -0.151141, vy = -1.002249
t = 0.92, x = 0.657956, y = -0.582794, vx = -0.171386, vy = -0.999256
t = 0.93, x = 0.656233, y = -0.592836, vx = -0.191571, vy = -0.995863
t = 0.94, x = 0.654308, y = -0.602845, vx = -0.211688, vy = -0.992070
t = 0.95, x = 0.652181, y = -0.612815, vx = -0.231728, vy = -0.987878
t = 0.96, x = 0.649852, y = -0.622743, vx = -0.251683, vy = -0.983290
t = 0.97, x = 0.647322, y = -0.632625, vx = -0.271545, vy = -0.978307
t = 0.98, x = 0.644593, y = -0.642457, vx = -0.291307, vy = -0.972930
t = 0.99, x = 0.641666, y = -0.652235, vx = -0.310960, vy = -0.967162
t = 1.00, x = 0.638541, y = -0.661955, vx = -0.330497, vy = -0.961005

```

6.3 考察

$q = m = 1$, $B = 1, 2, 10$ のときの計算結果をプロットしたグラフを図 14-16 に示す。

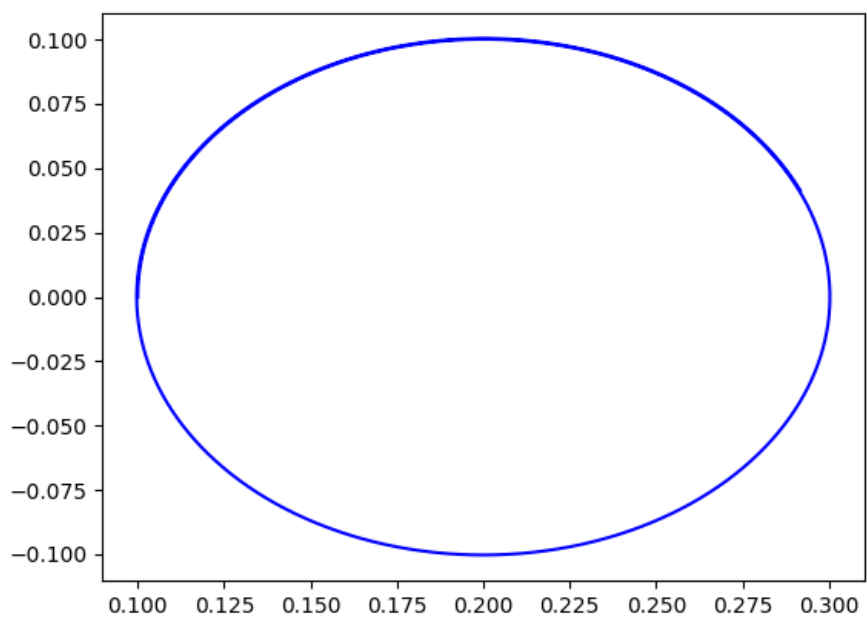


图 14 $B = 1$

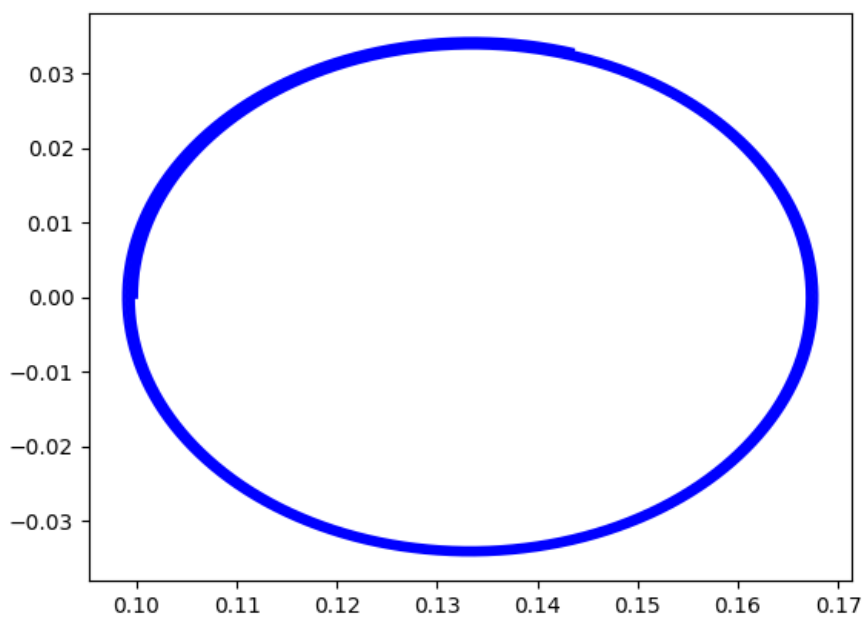


图 15 $B = 3$

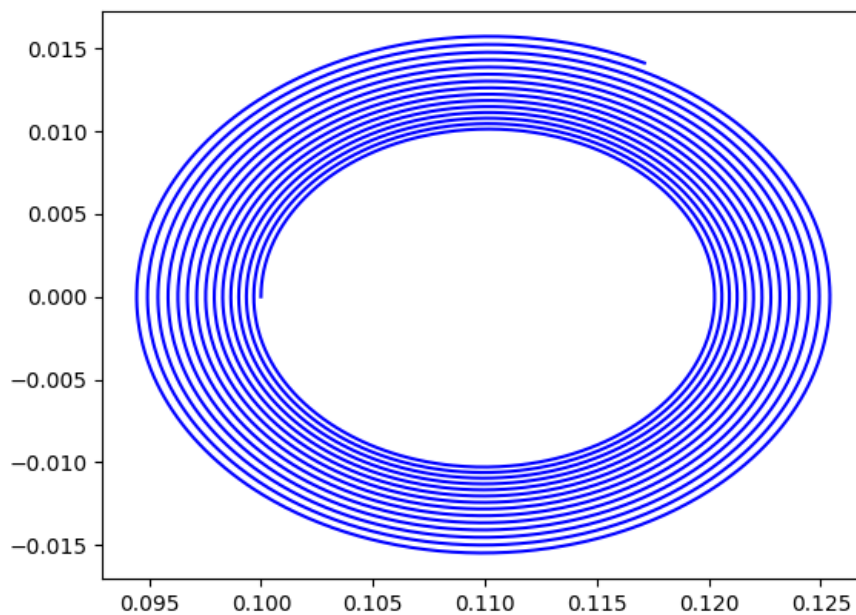


図 16 $B = 10$

誤差はそれぞれ次のようになった。

$B = 1, 3, 10$ のときの誤差

B: 1 , 誤差: 0.0092120809404796
 B: 3 , 誤差: 0.08735045022264068
 B: 10 , 誤差: 1.4792738717251592

$t = 0$ の $v_x^2 + v_y^2$ の値を V_0 , n ステップ後の $v_x^2 + v_y^2$ の値を V_n とすると、誤差は $(V_n - V_0)/V_0$ で求められる。 B が大きくなればなるほど誤差が大きくなっており、エネルギー保存則が成り立たなくなっていることがわかる。図 14–16 から B が大きくなればなるほどエネルギー保存則が成り立たなくなり、より外側に軌道を描くようになっていることがわかる。さらに、 B が大きいほど円軌道が小さくなっており、 B と円軌道の大きさは反比例の関係にあると考えられる。

7 課題 10：ローレンツ力と電場

課題 9 に電場から受ける力を追加し、ホイン法で解くプログラムを作成する。

7.1 プログラムリスト

課題 10 のプログラムリストをリスト 9 に示す。

リスト 9 課題 10 のプログラム

```
1 #include <stdio.h>
2 #include <math.h>
3
```

```

4  #define qE 1
5  #define qB_m 1
6  #define t_0 0
7  #define x_0 0.1
8  #define y_0 0
9  #define z_0 0
10 #define vx_0 1
11 #define vy_0 0.1
12 #define vz_0 1
13
14 #define H 0.01
15 #define STEP 100
16
17 double xp(double);
18 double yp(double);
19 double vxp(double);
20 double vvp(double);
21 void heun(double *, double *, double *, double *, double *, double *, double *, double, int
    );
22
23 int main(void) {
24     int i;
25     double t[STEP + 1];
26     double x[STEP + 1];
27     double y[STEP + 1];
28     double z[STEP + 1];
29     double vx[STEP + 1];
30     double vy[STEP + 1];
31     double vz[STEP + 1];
32
33     heun(t, x, y, z, vx, vy, vz, H, STEP);
34
35     for (i = 0; i <= STEP; i++) {
36         printf("t = %.2f, x = (%.6f, %.6f, %.6f), v = (%.6f, %.6f, %.6f)\n", t[i], x[i], y[
            i], z[i], vx[i], vy[i], vz[i]);
37     }
38
39     return 0;
40 }
41
42 // x'(vx)
43 double xp(double vx) {
44     return vx;
45 }
46
47 // y'(vy)
48 double yp(double vy) {
49     return vy;
50 }
51
52 // z'(vz)
53 double zp(double vz) {
54     return qE * vz;
55 }
56
57 // vx'(vy)

```



```

58 double vxp(double vy) {
59     return vy;
60 }
61
62 // vy'(vx)
63 double vyp(double vx) {
64     return -qB_m * vx;
65 }
66
67 // vz'(vz)
68 double vzp(double vz) {
69     return qE;
70 }
71
72 // ホイン法
73 void heun(double *t, double *x, double *y, double *z, double *vx, double *vy, double *vz,
74           double h, int step) {
75     int i;
76     t[0] = t_0;
77     x[0] = x_0;
78     y[0] = y_0;
79     z[0] = z_0;
80     vx[0] = vx_0;
81     vy[0] = vy_0;
82     vz[0] = vz_0;
83
84     for (i = 0; i <= step - 1; i++) {
85         double k1;
86         double k2;
87
88         t[i + 1] = t[i] + h;
89
90         k1 = h * xp(vx[i]);
91         k2 = h * xp(vx[i] + k1);
92         x[i + 1] = x[i] + (k1 + k2) / 2;
93
94         k1 = h * yp(vy[i]);
95         k2 = h * yp(vy[i] + k1);
96         y[i + 1] = y[i] + (k1 + k2) / 2;
97
98         k1 = h * zp(vz[i]);
99         k2 = h * zp(vz[i] + k1);
100        z[i + 1] = z[i] + (k1 + k2) / 2;
101
102        k1 = h * vxp(vy[i]);
103        k2 = h * vxp(vy[i] + k1);
104        vx[i + 1] = vx[i] + (k1 + k2) / 2;
105
106        k1 = h * vyp(vx[i]);
107        k2 = h * vyp(vx[i] + k1);
108        vy[i + 1] = vy[i] + (k1 + k2) / 2;
109
110        k1 = h * zp(vz[i]);
111        k2 = h * zp(vz[i] + k1);
112        vz[i + 1] = vz[i] + (k1 + k2) / 2;

```

```

113     }
114 }

```

7.2 実行結果

課題 10 の実行結果をリスト 10 に示す。

リスト 10 課題 10 の実行結果

```

t = 0.00, x = (0.100000, 0.000000, 0.000000), v = (1.000000, 0.100000, 1.000000)
t = 0.01, x = (0.110050, 0.001005, 0.010050), v = (1.001005, 0.090050, 1.010050)
t = 0.02, x = (0.120110, 0.001910, 0.020201), v = (1.001910, 0.080090, 1.020201)
t = 0.03, x = (0.130179, 0.002715, 0.030454), v = (1.002715, 0.070121, 1.030454)
t = 0.04, x = (0.140257, 0.003420, 0.040810), v = (1.003420, 0.060144, 1.040810)
t = 0.05, x = (0.150341, 0.004024, 0.051270), v = (1.004024, 0.050160, 1.051270)
t = 0.06, x = (0.160431, 0.004528, 0.061835), v = (1.004528, 0.040170, 1.061835)
t = 0.07, x = (0.170527, 0.004932, 0.072507), v = (1.004932, 0.030175, 1.072507)
t = 0.08, x = (0.180626, 0.005235, 0.083286), v = (1.005235, 0.020176, 1.083286)
t = 0.09, x = (0.190729, 0.005438, 0.094173), v = (1.005438, 0.010174, 1.094173)
t = 0.10, x = (0.200834, 0.005540, 0.105169), v = (1.005540, 0.000170, 1.105169)
t = 0.11, x = (0.210939, 0.005542, 0.116276), v = (1.005542, -0.009836, 1.116276)
t = 0.12, x = (0.221045, 0.005443, 0.127495), v = (1.005443, -0.019841, 1.127495)
t = 0.13, x = (0.231150, 0.005244, 0.138826), v = (1.005244, -0.029845, 1.138826)
t = 0.14, x = (0.241253, 0.004944, 0.150271), v = (1.004944, -0.039847, 1.150271)
t = 0.15, x = (0.251352, 0.004543, 0.161831), v = (1.004543, -0.049846, 1.161831)
t = 0.16, x = (0.261448, 0.004042, 0.173508), v = (1.004042, -0.059841, 1.173508)
t = 0.17, x = (0.271538, 0.003441, 0.185302), v = (1.003441, -0.069832, 1.185302)
t = 0.18, x = (0.281623, 0.002739, 0.197214), v = (1.002739, -0.079816, 1.197214)
t = 0.19, x = (0.291701, 0.001937, 0.209246), v = (1.001937, -0.089793, 1.209246)
t = 0.20, x = (0.301770, 0.001034, 0.221399), v = (1.001034, -0.099762, 1.221399)
t = 0.21, x = (0.311830, 0.000032, 0.233674), v = (1.000032, -0.109723, 1.233674)
t = 0.22, x = (0.321881, -0.001071, 0.246072), v = (0.998929, -0.119673, 1.246072)
t = 0.23, x = (0.331920, -0.002274, 0.258595), v = (0.997726, -0.129612, 1.258595)
t = 0.24, x = (0.341947, -0.003576, 0.271244), v = (0.996424, -0.139540, 1.271244)
t = 0.25, x = (0.351961, -0.004979, 0.284020), v = (0.995021, -0.149454, 1.284020)
t = 0.26, x = (0.361961, -0.006481, 0.296925), v = (0.993519, -0.159355, 1.296925)
t = 0.27, x = (0.371946, -0.008082, 0.309959), v = (0.991918, -0.169240, 1.309959)
t = 0.28, x = (0.381915, -0.009783, 0.323124), v = (0.990217, -0.179110, 1.323124)
t = 0.29, x = (0.391867, -0.011583, 0.336421), v = (0.988417, -0.188962, 1.336421)
t = 0.30, x = (0.401800, -0.013482, 0.349852), v = (0.986518, -0.198797, 1.349852)
t = 0.31, x = (0.411715, -0.015480, 0.363418), v = (0.984520, -0.208613, 1.363418)
t = 0.32, x = (0.421609, -0.017577, 0.377120), v = (0.982423, -0.218409, 1.377120)
t = 0.33, x = (0.431482, -0.019772, 0.390961), v = (0.980228, -0.228184, 1.390961)
t = 0.34, x = (0.441334, -0.022065, 0.404940), v = (0.977935, -0.237937, 1.404940)
t = 0.35, x = (0.451162, -0.024456, 0.419059), v = (0.975544, -0.247668, 1.419059)
t = 0.36, x = (0.460966, -0.026945, 0.433321), v = (0.973055, -0.257374, 1.433321)

```

$t = 0.37, x = (0.470745, -0.029532, 0.447726), v = (0.970468, -0.267056, 1.447726)$
 $t = 0.38, x = (0.480499, -0.032216, 0.462275), v = (0.967784, -0.276712, 1.462275)$
 $t = 0.39, x = (0.490225, -0.034997, 0.476971), v = (0.965003, -0.286342, 1.476971)$
 $t = 0.40, x = (0.499923, -0.037874, 0.491815), v = (0.962126, -0.295944, 1.491815)$
 $t = 0.41, x = (0.509592, -0.040849, 0.506808), v = (0.959151, -0.305517, 1.506808)$
 $t = 0.42, x = (0.519232, -0.043919, 0.521951), v = (0.956081, -0.315060, 1.521951)$
 $t = 0.43, x = (0.528841, -0.047085, 0.537247), v = (0.952915, -0.324573, 1.537247)$
 $t = 0.44, x = (0.538417, -0.050347, 0.552696), v = (0.949653, -0.334055, 1.552696)$
 $t = 0.45, x = (0.547961, -0.053705, 0.568301), v = (0.946295, -0.343504, 1.568301)$
 $t = 0.46, x = (0.557472, -0.057157, 0.584062), v = (0.942843, -0.352920, 1.584062)$
 $t = 0.47, x = (0.566947, -0.060704, 0.599982), v = (0.939296, -0.362301, 1.599982)$
 $t = 0.48, x = (0.576387, -0.064345, 0.616062), v = (0.935655, -0.371647, 1.616062)$
 $t = 0.49, x = (0.585790, -0.068080, 0.632303), v = (0.931920, -0.380957, 1.632303)$
 $t = 0.50, x = (0.595156, -0.071908, 0.648708), v = (0.928092, -0.390229, 1.648708)$
 $t = 0.51, x = (0.604484, -0.075830, 0.665277), v = (0.924170, -0.399464, 1.665277)$
 $t = 0.52, x = (0.613771, -0.079845, 0.682013), v = (0.920155, -0.408659, 1.682013)$
 $t = 0.53, x = (0.623019, -0.083952, 0.698917), v = (0.916048, -0.417815, 1.698917)$
 $t = 0.54, x = (0.632225, -0.088151, 0.715992), v = (0.911849, -0.426930, 1.715992)$
 $t = 0.55, x = (0.641389, -0.092442, 0.733237), v = (0.907558, -0.436002, 1.733237)$
 $t = 0.56, x = (0.650510, -0.096823, 0.750656), v = (0.903177, -0.445033, 1.750656)$
 $t = 0.57, x = (0.659587, -0.101296, 0.768250), v = (0.898704, -0.454019, 1.768250)$
 $t = 0.58, x = (0.668619, -0.105859, 0.786021), v = (0.894141, -0.462961, 1.786021)$
 $t = 0.59, x = (0.677605, -0.110512, 0.803971), v = (0.889488, -0.471858, 1.803971)$
 $t = 0.60, x = (0.686545, -0.115254, 0.822101), v = (0.884746, -0.480708, 1.822101)$
 $t = 0.61, x = (0.695436, -0.120085, 0.840413), v = (0.879915, -0.489512, 1.840413)$
 $t = 0.62, x = (0.704280, -0.125005, 0.858909), v = (0.874995, -0.498267, 1.858909)$
 $t = 0.63, x = (0.713073, -0.130012, 0.877591), v = (0.869988, -0.506973, 1.877591)$
 $t = 0.64, x = (0.721817, -0.135107, 0.896461), v = (0.864893, -0.515629, 1.896461)$
 $t = 0.65, x = (0.730509, -0.140289, 0.915520), v = (0.859711, -0.524235, 1.915520)$
 $t = 0.66, x = (0.739149, -0.145558, 0.934771), v = (0.854442, -0.532789, 1.934771)$
 $t = 0.67, x = (0.747736, -0.150912, 0.954216), v = (0.849088, -0.541291, 1.954216)$
 $t = 0.68, x = (0.756269, -0.156352, 0.973856), v = (0.843648, -0.549739, 1.973856)$
 $t = 0.69, x = (0.764748, -0.161877, 0.993693), v = (0.838123, -0.558134, 1.993693)$
 $t = 0.70, x = (0.773171, -0.167486, 1.013729), v = (0.832514, -0.566473, 2.013729)$
 $t = 0.71, x = (0.781538, -0.173179, 1.033967), v = (0.826821, -0.574756, 2.033967)$
 $t = 0.72, x = (0.789847, -0.178956, 1.054409), v = (0.821044, -0.582983, 2.054409)$
 $t = 0.73, x = (0.798099, -0.184815, 1.075056), v = (0.815185, -0.591153, 2.075056)$
 $t = 0.74, x = (0.806292, -0.190756, 1.095910), v = (0.809244, -0.599264, 2.095910)$
 $t = 0.75, x = (0.814424, -0.196778, 1.116974), v = (0.803222, -0.607316, 2.116974)$
 $t = 0.76, x = (0.822497, -0.202882, 1.138249), v = (0.797118, -0.615308, 2.138249)$
 $t = 0.77, x = (0.830508, -0.209066, 1.159739), v = (0.790934, -0.623239, 2.159739)$
 $t = 0.78, x = (0.838457, -0.215329, 1.181444), v = (0.784671, -0.631109, 2.181444)$
 $t = 0.79, x = (0.846343, -0.221672, 1.203368), v = (0.778328, -0.638916, 2.203368)$
 $t = 0.80, x = (0.854165, -0.228093, 1.225511), v = (0.771907, -0.646661, 2.225511)$
 $t = 0.81, x = (0.861923, -0.234592, 1.247878), v = (0.765408, -0.654341, 2.247878)$

t = 0.82,	x = (0.869615,	-0.241168,	1.270469),	v = (0.758832,	-0.661957,	2.270469)
t = 0.83,	x = (0.877241,	-0.247821,	1.293287),	v = (0.752179,	-0.669507,	2.293287)
t = 0.84,	x = (0.884801,	-0.254549,	1.316335),	v = (0.745451,	-0.676992,	2.316335)
t = 0.85,	x = (0.892292,	-0.261353,	1.339614),	v = (0.738647,	-0.684409,	2.339614)
t = 0.86,	x = (0.899716,	-0.268232,	1.363127),	v = (0.731768,	-0.691758,	2.363127)
t = 0.87,	x = (0.907070,	-0.275184,	1.386877),	v = (0.724816,	-0.699040,	2.386877)
t = 0.88,	x = (0.914354,	-0.282209,	1.410865),	v = (0.717791,	-0.706251,	2.410865)
t = 0.89,	x = (0.921568,	-0.289307,	1.435094),	v = (0.710693,	-0.713393,	2.435094)
t = 0.90,	x = (0.928711,	-0.296476,	1.459566),	v = (0.703524,	-0.720465,	2.459566)
t = 0.91,	x = (0.935781,	-0.303717,	1.484285),	v = (0.696283,	-0.727465,	2.484285)
t = 0.92,	x = (0.942779,	-0.311028,	1.509252),	v = (0.688972,	-0.734393,	2.509252)
t = 0.93,	x = (0.949703,	-0.318409,	1.534470),	v = (0.681591,	-0.741248,	2.534470)
t = 0.94,	x = (0.956553,	-0.325858,	1.559942),	v = (0.674142,	-0.748030,	2.559942)
t = 0.95,	x = (0.963328,	-0.333376,	1.585669),	v = (0.666624,	-0.754738,	2.585669)
t = 0.96,	x = (0.970028,	-0.340961,	1.611655),	v = (0.659039,	-0.761371,	2.611655)
t = 0.97,	x = (0.976651,	-0.348613,	1.637902),	v = (0.651387,	-0.767928,	2.637902)
t = 0.98,	x = (0.983197,	-0.356331,	1.664413),	v = (0.643669,	-0.774409,	2.664413)
t = 0.99,	x = (0.989666,	-0.364113,	1.691190),	v = (0.635887,	-0.780814,	2.691190)
t = 1.00,	x = (0.996057,	-0.371961,	1.718237),	v = (0.628039,	-0.787141,	2.718237)

7.3 考察

$B = E = 1.0$, $B = 1.0, E = 2.0$, $B = 0.5, E = 1.0$ のときの実行結果をグラフにプロットしたものを、図 17, 18, 19 に示す。図 18 を見ると、17 に比べて、 z 軸方向への加速度が大きいことが分かる。図 19 を見ると、17 に比べて、円運動の半径は大きく、 z 軸方向への加速度は小さいことが分かる。

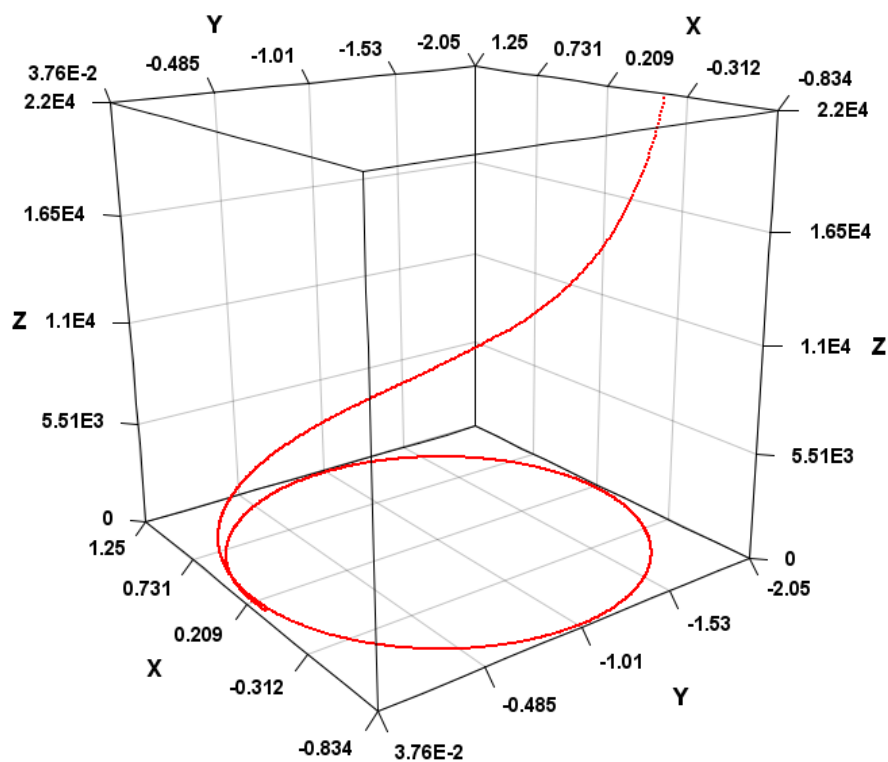


图 17 $B = E = 1.0$

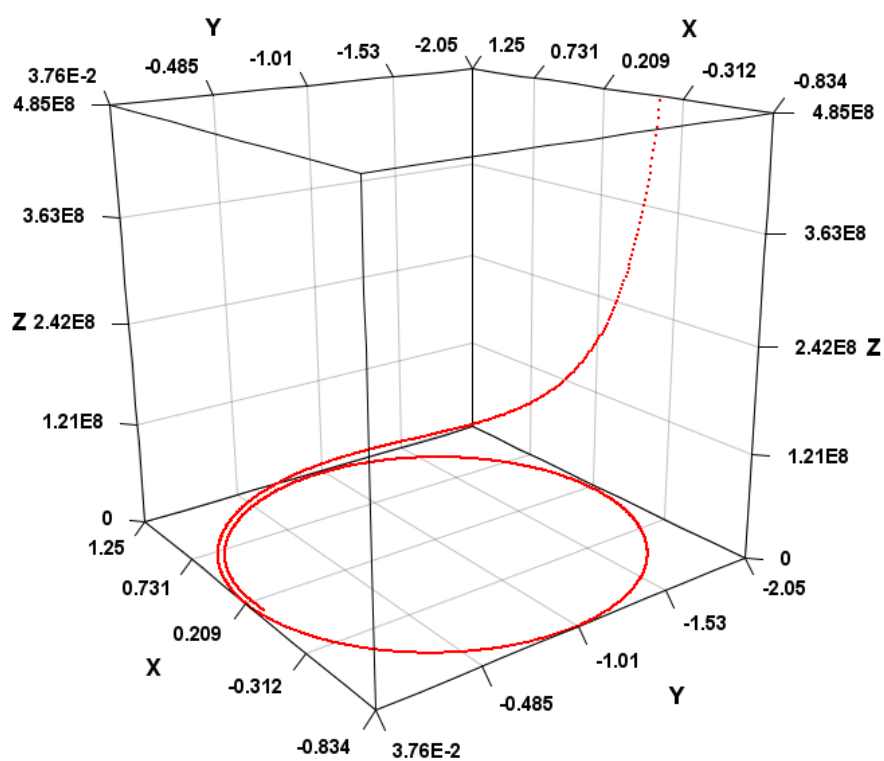


图 18 $B = 1.0, E = 2.0$

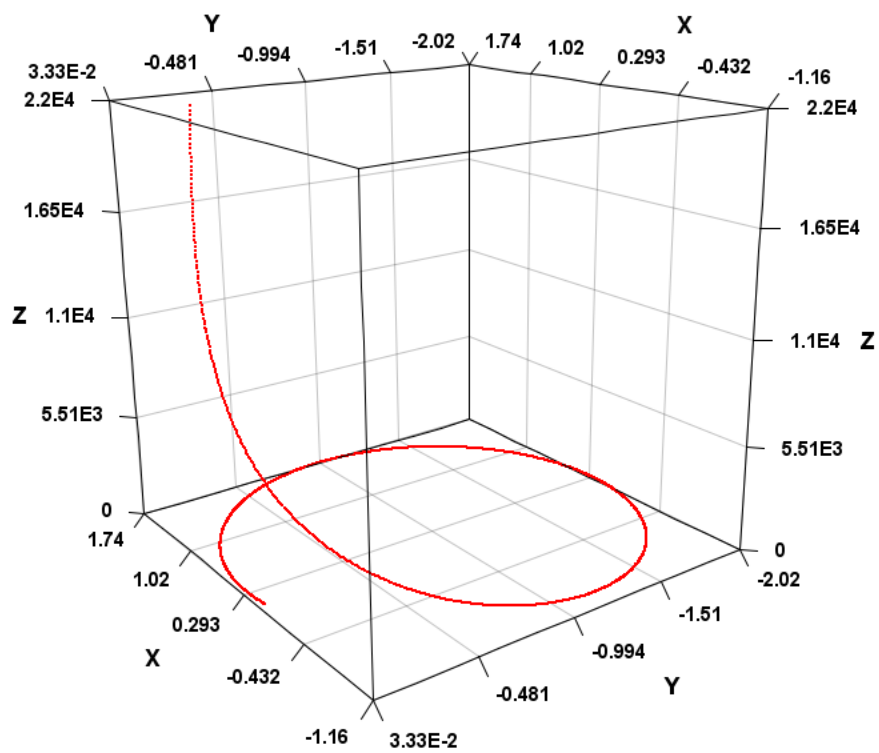


图 19 $B = 0.5, E = 1.0$