Improved function for calculating firing solutions

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Abstract

The following technical note describes a method for calculating firing solutions against targets moving at a steady velocity, with missiles that have a fixed turning radius.

1 Overview

The method uses the following procedure:

1. Select the current target position as the initial aim point.
2. For the current aim point, select the turn direction (§2).
3. For the current aim point and turn direction, calculate the required turn angle (§3).
4. For the missile trajectory determined by the turn angle, calculate the nearest approach betweent missile and target (§4).
5. Compare nearest approach distance against error tolerance:
    (a) If the error is out of tolerance, select the target position at nearest approach as the improved aim point (§5), and repeat the process from step 2.
    (b) If the error is within tolerance, the firing solution has converged.

2 Select turn direction

- The missile has a fixed turn radius r_m.
- For each direction that the missile turns, there is a circular "dead zone" where the missile cannot hit. This zone has radius equal to the missile turn radius, and is centered 1 turn radius away from the missile tube.
- Thus, for all aim points to the left of the missile tube and outside the left dead zone, the missile should turn left. For aim points inside the left dead zone, the missile should turn right and loop around into the zone. Vice versa for all aim points to the right of the missile tube and within the right dead zone.

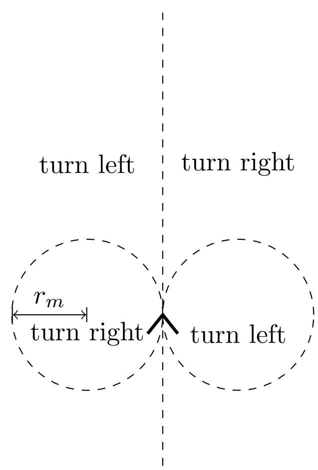

Figure 1: Turn directions for all possible aim points. Direction of fire for the missile tube marked with $\wedge$.

3 Calculate turn angle

Note: This section describes how to calculate the missile turn angle for right-hand turns. Left-hand turns follow an identical process, except reflected across the missile tube centerline.

As shown in Figure 2, the turn angle θ for a given aim point (x_a, y_a) is given by (1).

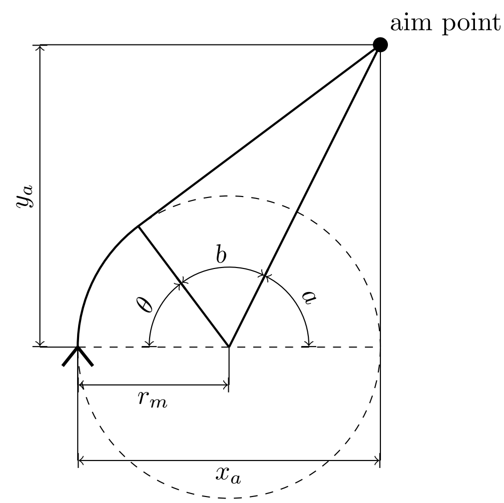

Figure 2: Turn angle θ required to hit a specified aim point. The missile tube is at the origin and marked by $\wedge$.

$$\begin{gathered}\theta = \begin{cases} \pi - (a+b) & \text{if } a+b \leq \pi \\ 3\pi - (a+b) & \text{otherwise} \end{cases} \\ a = \arctan\left(\frac{y_a}{x_a - r_m}\right) \qquad b = \arccos\left(\frac{r_m}{\sqrt{(x_a - r_m)^2 + {y_a}^2}}\right)\end{gathered} \tag{1}$$

Once we have the turn angle, we can calculate:

- the time taken to complete the turn: $t_e = \frac{r_m \theta}{V_m}$
- the target position at the end of the turn: $\mathbf{x_{te}} = \mathbf{x_{t0}} + \mathbf{u_t} t_e$
- the missile position at the end of the turn: $\mathbf{x_{me}} = r_m \begin{pmatrix} 1 - \cos\theta \\ \sin\theta \end{pmatrix}$

4 Calculate nearest approach

After the missile completes its turn, the distance d between the target and missile is given by (2), where:

- $\mathbf{x_{me}}$: Missile position when the missile completes its turn
- $\mathbf{x_{te}}$: Target position when the missile completes its turn
- $\mathbf{u_m}$: Missile velocity
- $\mathbf{u_t}$: Target velocity
- t: Time after the missile completes its turn

$$\begin{aligned} d &= |(\mathbf{x_{te}} + \mathbf{u_t} t) - (\mathbf{x_{me}} + \mathbf{u_m} t)| \\ &= |(\mathbf{x_{te}} - \mathbf{x_{me}}) + (\mathbf{u_t} - \mathbf{u_m}) t| \\ &= |\mathbf{\Delta x_e} + \mathbf{\Delta u} t| \\ &= (\mathbf{\Delta x_e} + \mathbf{\Delta u} t) \cdot (\mathbf{\Delta x_e} + \mathbf{\Delta u} t) \\ &= \sqrt{\mathbf{\Delta x_e} \cdot \mathbf{\Delta x_e} + 2\mathbf{\Delta x_e} \cdot \mathbf{\Delta u} t + \mathbf{\Delta u} \cdot \mathbf{\Delta u} t^2} \end{aligned} \tag{2}$$

Assuming that the nearest approach between the missile and target will occur in this phase of flight, we can minimize their distance apart to calculate the time t_n of the nearest approach. For convenience, we do with with d^2 in (3), but the result is identical.

$$\begin{aligned} \frac{\partial d^2}{\partial t} &= 2\mathbf{\Delta x_e} \cdot \mathbf{\Delta u} + 2\mathbf{\Delta u} \cdot \mathbf{\Delta u} t \\ \frac{\partial d^2}{\partial t}(t_n) &= 0 \Rightarrow \text{nearest approach} \\ t_n &= -\frac{\mathbf{\Delta x_e} \cdot \mathbf{\Delta u}}{\mathbf{\Delta u} \cdot \mathbf{\Delta u}} \end{aligned} \tag{3}$$

By applying the result of (3) to (2), we can find the distance between the target and missile at the nearest approach (4). This is also the distance by which the current firing angle misses the target.

$$d_n = \sqrt{\mathbf{\Delta x_e} \cdot \mathbf{\Delta x_e} + 2\mathbf{\Delta x_e} \cdot \mathbf{\Delta u} t_n + \mathbf{\Delta u} \cdot \mathbf{\Delta u} {t_n}^2} \tag{4}$$

5 Improve aim point

If the current estimate is not within the error tolerance of the firing solution yet, we can improve the estimate by adjusting the aim point to the target position at nearest approach (5).

$$\mathbf{x_a} = \mathbf{x_{te}} + \mathbf{u_t} t_n \tag{5}$$