

## 1 Basic

## 1.1 .vimrc

```

1  linenumber, relative-linenumber, mouse, cindent, expandtab,
1  shiftwidth, softtabstop, nowrap, ignorecase(when search), noVi-
1  compatible, backspace
2  normu when enter insert mode
2
2  | se nu rnu mouse=a cin et sw=2 sts=2 nowrap ic nocp bs=2
2  syn on

```

## 1.2 Default Code

```

2 所有模板的 define 都在這
3
3  #include<bits/stdc++.h>
3  using namespace std;
3
4  #ifdef LOCAL // ===== Local ===== g++ -DLOCAL ...
4  void dbg() { cerr << '\n'; }
4  template<class T, class ...U> void dbg(T a, U ...b) {
4      cerr << a << ' ', dbg(b...); }
4  template<class T> void org(T l, T r) {
5      while (l != r) cerr << *l++ << ' '; cerr << '\n'; }
5  #define DEBUG(args...) \
5      (dbg("#> (" + string(#args) + ") = (" , args, ")"))
5  #define ORANGE(args...) \
5      (cerr << "#> [" + string(#args) + "] = " , org(args))
6  #else // ===== OnlineJudge =====
6  #define DEBUG(...) ((void)0)
6  #define ORANGE(...) ((void)0)
6  #endif

```

```
#define ll long long
#define ld long double
#define INF 0x3f3f3f3f
#define LLINF 0x3f3f3f3f3f3f3f3f
#define NINF 0xc1c1c1c1
#define NLLINF 0xc1c1c1c1c1c1c1c1
#define X first
#define Y second
#define PB emplace_back
#define pll pair<ll, ll>
#define MEM(a,n) memset(a, n, sizeof(a))
#define io ios::sync_with_stdio(0); cin.tie(0); cout.tie(0);
const int MXN = + 5;
```

```

7 #define ll long long
7 #define ld long double
7 #define INF 0x3f3f3f3f
7 #define LLINF 0x3f3f3f3f3f3f3f3f
7 #define NINF 0xc1c1c1c1
7 #define NLLINF 0xc1c1c1c1c1c1c1c1
8 #define X first
8 #define Y second
8 #define PB emplace_back
9 #define pll pair<ll, ll>
9 #define MEM(a,n) memset(a, n, sizeof(a))
10 #define io ios::sync_with_stdio(0); cin.tie(0); cout.
10 tie(0);
10 const int MXN = + 5;

```

```
void sol(){};
```

[illegible]

### 1.3 Common Sense

```

12 陣列過大時本機的指令：
13 windows: g++ -WL,-stack,40000000 a.cpp
13 linux: ulimit -s unlimited
14 1e7 的 int 陣列 = 4e7 byte = 40 mb
14 STL 式模板函式名稱定義：
14 .init(n, ...) ⇒ 初始化並重置全部變數，0-base
14 .addEdge(u, v, ...) ⇒ 加入一條邊，有向圖為  $u \rightarrow v$ ，無向圖為  $u \leftrightarrow v$ 
14 .run() ⇒ 執行並回傳答案
14 .build() ⇒ 查詢前處理
14 .query(...) ⇒ 查詢並回傳答案
14 memset 設 -0x3f 的值是 -0x3e3e3e3f / 0xc1c1c1c1

```

## 1.4 Useful STL

```

15 // unique
15 sort(a.begin(), a.end());
15 a.resize(unique(a.begin(), a.end()) - a.begin());
15 // O(n) a[k] = kth small, a[i] < a[k] if i < k
15 nth_element(a.begin(), a.begin()+k, a.end());
16 // stable_sort(a.begin(), a.end())
16 // lower_bound: first element >= val
16 // upper_bound: first element > val
16 // set_union, set_intersection, set_difference,
16 // set_symmetric_difference
16 set_union(a.begin(), a.end(), b.begin(), b.end(),
17 inserter(c, c.begin()));
17 //next_permutation prev_permutation(sort/reverse first)
17 do{ for(auto i : a) cout << i << ' ';
17 } while(next_permutation(a.begin(), a.end()));

```

```
// unique
sort(a.begin(), a.end());
a.resize(unique(a.begin(), a.end()) - a.begin());
// O(n) a[k] = kth small, a[i] < a[k] if i < k
nth_element(a.begin(), a.begin()+k, a.end());
// stable_sort(a.begin(), a.end())
// lower_bound: first element >= val
// upper_bound: first element > val
// set_union, set_intersection, set_difference,
// set_symmetric_difference
set_union(a.begin(), a.end(), b.begin(), b.end(),
    inserter(c, c.begin()));
//next_permutation prev_permutation(sort/reverse first)
do{ for(auto i : a) cout << i << ' ';
} while(next_permutation(a.begin(), a.end()));
```

```

15 // unique
15 sort(a.begin(), a.end());
15 a.resize(unique(a.begin(), a.end()) - a.begin());
15 // 0(n) a[k] = kth small, a[i] < a[k] if i < k
15 nth_element(a.begin(), a.begin()+k, a.end());
16 // stable_sort(a.begin(), a.end())
16 // lower_bound: first element >= val
16 // upper_bound: first element > val
16 // set_union, set_intersection, set_difference,
16 // set_symmetric_difference
16 set_union(a.begin(), a.end(), b.begin(), b.end(),
17 inserter(c, c.begin()));
17 //next_permutation prev_permutation(sort/reverse first)
17 do{ for(auto i : a) cout << i << ' ';
17 } while(next_permutation(a.begin(), a.end()));

```

```

//next_permutation prev_permutation(sort(reverse first))
do{ for(auto i : a) cout << i << ' ';
} while(next_permutation(a.begin(), a.end()));

```

```
17 //next_permutation prev_permutation(sort/reverse first)
17 do{ for(auto i : a) cout << i << ' ';
17 } while(next_permutation(a.begin(), a.end()));
```

```
do{ for(auto i : a) cout << i << ' ';  
} while(next_permutation(a.begin(), a.end()));
```

```
17 do{ for(auto i : a) cout << i << ' ';  
17 } while(next_permutation(a.begin(), a.end()));
```

## 1.5 Bi/Ternary Search

```
while(l < r){ // first l of check(l) == true
    ll m = (l + r) >> 1;
    if(!check(m)) l = m + 1; else r = m; }
while(l < r){ // last l of check(l) == false
    ll m = (l + r + 1) >> 1;
    if(!check(m)) l = m; else r = m - 1; }
while(l < r){
    ll ml = l + (r - l) / 3, mr = r - (r - l) / 3;
    if(check(ml)>check(mr)) l = ml + 1; else r = mr - 1; }
```

## 1.6 Troubleshoot

提交前：

- 如果樣本不夠，寫幾個簡單的測資。
- 複雜度會不會爛？生成最大的測資試試。
- 記憶體使用是否正常？
- 會 overflow 嗎？
- 確定提交正確的檔案。

WA：

- 記得輸出你的答案！也輸出 debug 看看。
- 測資之間是否重置了所有變數？
- 演算法可以處理整個輸入範圍嗎？
- 再讀一次題目。
- 您是否正確處理所有邊緣測資？
- 您是否正確理解了題目？
- 任何未初始化的變數？
- 有 overflow 嗎？
- 混淆 n, m, i, j 等等？
- 確定演算法有效嗎？
- 哪些特殊情況沒有想到？
- 確定 STL 函數按你的想法執行嗎？
- 寫一些 assert 看看是否有些東西不如預期？
- 寫一些測資來跑你的演算法。
- 產生一些簡單的測資跑演算法看看。
- 再次瀏覽此列表。
- 向隊友解釋你的演算法。
- 請隊友查看您的代碼。
- 去散步，例如去廁所。
- 你的輸出格式正確嗎？(包括空格)
- 重寫，或者讓隊友來做。

RE：

- 您是否在本地測試了所有極端情況？
- 任何未初始化的變數？
- 您是否在任何向量範圍之外閱讀或寫作？
- 任何可能失敗的 assert？
- 任何的除以 0？(例如 mod 0)
- 任何的無限遞迴？
- 無效的 pointer 或 iterator？
- 你是否使用了太多的記憶體？

TLE：

- 有無限迴圈嗎？
- 複雜度是多少？
- 是否正在複製大量不必要的數據？(改用參考)
- 有沒有開 io？
- 避免 vector/map。(使用 array/unordered\_map)
- 你的隊友對你的演算法有什麼看法？

MLE：

- 您的演算法應該需要的最大記憶體是多少？
- 測資之間是否重置了所有變數？

## 2 flow

### 2.1 MinCostFlow \*

```
struct zkwflow{
    static const int MXN = 10000;
    struct Edge{ int v, f, re; ll w; };
    int n, s, t, ptr[MXN]; bool vis[MXN]; ll dis[MXN];
    vector<Edge> E[MXN];
    void init(int _n, int _s, int _t){
        n=_n, s=_s, t=_t;
        for(int i=0; i<n; i++) E[i].clear();
    }
    void addEdge(int u, int v, int f, ll w){
        E[u].emplace_back(v, f, (int)E[v].size(), w);
        E[v].emplace_back(u, 0, (int)E[u].size()-1, -w);
    }
    bool SPFA(){
        fill_n(dis, n, LLMXN); memset(vis, 0, 4 * n);
        queue<int> q; q.push(s); dis[s] = 0;
        while (!q.empty()){
            int u = q.front(); q.pop(); vis[u] = false;
            for(auto &it : E[u]){
                if(it.f > 0 && dis[it.v] > dis[u] + it.w){
                    dis[it.v] = dis[u] + it.w;
                    if(!vis[it.v]){
                        vis[it.v] = 1; q.push(it.v);
                    }
                }
            }
        }
    }
```

```
return dis[t] != LLMXN;
}
int DFS(int u, int nf){
    if(u == t) return nf;
    int res = 0; vis[u] = 1;
    for(int &i = ptr[u]; i < (int)E[u].size(); ++i){
        auto &it = E[u][i];
        if(it.f > 0 && dis[it.v] == dis[u] + it.w && !vis[it.v]){
            int tf = DFS(it.v, min(nf, it.f));
            res += tf, nf -= tf, it.f -= tf;
            E[it.v][it.re].f += tf;
            if(nf == 0){ vis[u] = false; break; }
        }
    }
    return res;
}
pair<int, ll> flow(){
    int flow = 0; ll cost = 0;
    while (SPFA()){
        memset(ptr, 0, 4 * n);
        int f = DFS(s, INF);
        flow += f; cost += dis[t] * f;
    }
    return { flow, cost };
}
} flow;
```

### 2.2 Dinic

求最大流  $O(N^2 E)$ ，求二分最大匹配  $O(E\sqrt{N})$   
 dinic.init(n, st, en)  $\Rightarrow$  0-base  
 dinic.addEdge(u, v, f)  $\Rightarrow u \rightarrow v$ , flow f units  
 dinic.run()  $\Rightarrow$  return max flow from st to en  
 反向邊為該邊的流量  
 Dinic 玄學：若 TLE，可以先加“正向邊”且每次都 run()，再全加一次每次都 run()。  
 範例 code 待補

```
const int MXN = 10005;
struct Dinic{
    struct Edge{ ll v, f, re; };
    int n, s, t, lvl[MXN];
    vector<Edge> e[MXN];
    void init(int _n, int _s, int _t){
        n = _n; s = _s; t = _t;
        for(int i = 0; i < n; ++i) e[i].clear();
    }
    void addEdge(int u, int v, ll f = 1){
        e[u].push_back({v, f, e[v].size()});
        e[v].push_back({u, 0, e[u].size() - 1});
    }
    bool bfs(){
        memset(lvl, -1, n * 4);
        queue<int> q;
        q.push(s);
        lvl[s] = 0;
        while(!q.empty()){
            int u = q.front(); q.pop();
            for(auto &i : e[u]){
                if(i.f > 0 && lvl[i.v] == -1)
                    lvl[i.v] = lvl[u] + 1, q.push(i.v);
            }
        }
        return lvl[t] != -1;
    }
    ll dfs(int u, ll nf){
        if(u == t) return nf;
        ll res = 0;
        for(auto &i : e[u]){
            if(i.f > 0 && lvl[i.v] == lvl[u] + 1){
                ll tmp = dfs(i.v, min(nf, i.f));
                res += tmp, nf -= tmp, i.f -= tmp;
                e[i.v][i.re].f += tmp;
                if(nf == 0) return res;
            }
        }
        if(!res) lvl[u] = -1;
        return res;
    }
    ll run(ll res){
        while(bfs()) res += dfs(s, LLINF);
        return res;
    }
};
```

### 2.3 Kuhn Munkres 最大完美二分匹配

二分完全圖最大權完美匹配  $O(n^3)$ (不太會跑滿)  
 轉換：  
 最大權匹配 (沒邊就補 0)  
 最小權完美匹配 (權重取負)  
 最大權重積 (ll 改 ld, memset 改 fill, w 取自然對數 log(w)，答案為 exp(ans))  
 二分圖判斷：DFS 建樹記深度  $\rightarrow$  有邊的兩點深度奇偶性相同  $\rightarrow$  奇環  $\rightarrow$  非二分圖  
 二分圖最小頂點覆蓋 = 最大匹配  
 | 最大匹配 | + | 最小邊覆蓋 | = |V|  
 | 最小點覆蓋 | + | 最大獨立集 | = |V|

| 最大匹配 | = | 最小點覆蓋 |  
最大團 = 補圖的最大獨立集

```
const int MXN = 1005;
struct KM{ // 1-base
    int n, mx[MXN], my[MXN], pa[MXN];
    ll g[MXN][MXN], lx[MXN], ly[MXN], sy[MXN];
    bool vx[MXN], vy[MXN];
    void init(int _n){
        n = _n;
        MEM(g, 0);
    }
    void addEdge(int x, int y, ll w){ g[x][y] = w; }
    void augment(int y){
        for(int x, z; y; y = z){
            x = pa[y], z = mx[x], my[y] = x, mx[x] = y;
        }
    }
    void bfs(int st){
        for(int i = 1; i <= n; ++i)
            sy[i] = LLINF, vx[i] = vy[i] = 0;
        queue<int> q; q.push(st);
        for(;;){
            while(!q.empty()){
                int x = q.front(); q.pop();
                vx[x] = 1;
                for(int y = 1; y <= n; ++y)
                    if(!vy[y]){
                        ll t = lx[x] + ly[y] - g[x][y];
                        if(t == 0){
                            pa[y] = x;
                            if(!my[y]){ augment(y); return; }
                            vy[y] = 1, q.push(my[y]);
                        }
                        else if(sy[y] > t) pa[y] = x, sy[y] = t;
                    }
            }
            ll cut = LLINF;
            for(int y = 1; y <= n; ++y)
                if(!vy[y] && cut > sy[y]) cut = sy[y];
            for(int j = 1; j <= n; ++j){
                if(vx[j]) lx[j] -= cut;
                if(vy[j]) ly[j] += cut;
                else sy[j] -= cut;
            }
            for(int y = 1; y <= n; ++y)
                if(!vy[y] && sy[y] == 0){
                    if(!my[y]){ augment(y); return; }
                    vy[y] = 1, q.push(my[y]);
                }
        }
    }
    ll run(){
        MEM(mx, 0), MEM(my, 0), MEM(lx, 0), MEM(ly, 0), MEM(sy, 0);
        for(int x = 1; x <= n; ++x) for(int y = 1; y <= n; ++y)
            lx[x] = max(lx[x], g[x][y]);
        for(int x = 1; x <= n; ++x) bfs(x);
        ll ret = 0;
        for(int y = 1; y <= n; ++y) ret += g[my[y]][y];
        return ret;
    }
};
```

## 2.4 Directed MST \*

```
struct DMST {
    struct Edge{ int u, v, c;
        Edge(int u, int v, int c):u(u),v(v),c(c){} };
    int v, e, root;
    Edge edges[MXN];
    int newV(){ return ++v; }
    void addEdge(int u, int v, int c)
        { edges[++e] = Edge(u, v, c); }
    bool con[MXN];
    int mnInW[MXN], prv[MXN], cyc[MXN], vis[MXN];
    int run(){
        memset(con, 0, 4*(V+1));
        int r1 = 0, r2 = 0;
        while(1){
            fill(mnInW, mnInW+V+1, INF);
            fill(prv, prv+V+1, -1);
            for(int i = 1; i <= e; ++i){
                int u=edges[i].u, v=edges[i].v, c=edges[i].c;
                if(u != v && v != root && c < mnInW[v])
                    mnInW[v] = c, prv[v] = u;
            }
            fill(vis, vis+V+1, -1);
            fill(cyc, cyc+V+1, -1);
            r1 = 0;
            bool jf = 0;
            for(int i = 1; i <= v; ++i){
                if(con[i]) continue;
                if(prv[i] == -1 && i != root) return -1;
                if(prv[i] > 0) r1 += mnInW[i];
                int s;
                for(s = i; s != -1 && vis[s] == -1; s = prv[s])
                    vis[s] = i;
```

```
vis[s] = i;
if(s > 0 && vis[s] == i){
    jf = 1; int v = s;
    do{ cyc[v] = s, con[v] = 1;
        r2 += mnInW[v]; v = prv[v];
    }while(v != s);
    con[s] = 0;
}
}
if(!jf) break;
for(int i = 1; i <= e; ++i){
    int &u = edges[i].u;
    int &v = edges[i].v;
    if(cyc[v] > 0) edges[i].c -= mnInW[edges[i].v];
    if(cyc[u] > 0) edges[i].u = cyc[edges[i].u];
    if(cyc[v] > 0) edges[i].v = cyc[edges[i].v];
    if(u == v) edges[i--] = edges[E--];
}
}
return r1+r2;}}
```

## 2.5 SW min-cut (不限 S-T 的 min-cut) \*

```
struct SW{ // 0(V^3)
    int n, vst[MXN], del[MXN];
    int edge[MXN][MXN], wei[MXN];
    void init(int _n){
        n = _n; memset(del, 0, sizeof(del));
        memset(edge, 0, sizeof(edge));
    }
    void addEdge(int u, int v, int w){
        edge[u][v] += w; edge[v][u] += w;
    }
    void search(int &s, int &t){
        memset(vst, 0, sizeof(vst)); memset(wei, 0, sizeof(wei));
        s = t = -1;
        while (true){
            int mx=-1, cur=0;
            for (int i=0; i<n; i++){
                if (!del[i] && !vst[i] && mx<wei[i])
                    cur = i, mx = wei[i];
            }
            if (mx == -1) break;
            vst[cur] = 1;
            s = t; t = cur;
            for (int i=0; i<n; i++){
                if (!vst[i] && !del[i]) wei[i] += edge[cur][i];
            }
        }
    }
    int solve(){
        int res = 2147483647;
        for (int i=0,x,y; i<n-1; i++){
            search(x,y);
            res = min(res,wei[y]);
            del[y] = 1;
            for (int j=0; j<n; j++){
                edge[x][j] = (edge[j][x] += edge[y][j]);
            }
        }
        return res;
    }
} graph;
```

## 2.6 Bounded Max Flow

```
// flow use ISAP
// Max flow with lower/upper bound on edges
// source = 1, sink = n
int in[ N ], out[ N ];
int l[ M ], r[ M ], a[ M ], b[ M ]; // 0-base, a下界, b上界
int solve(){
    flow.init( n ); // n為點的數量, m為邊的數量, 點是1-base
    for( int i = 0 ; i < m ; i ++ ){
        in[ r[ i ] ] += a[ i ];
        out[ l[ i ] ] += a[ i ];
        flow.addEdge( l[ i ], r[ i ], b[ i ] - a[ i ] );
        // flow from l[i] to r[i] must in [a[i], b[i]]
    }
    int nd = 0;
    for( int i = 1 ; i <= n ; i ++ ){
        if( in[ i ] < out[ i ] ){
            flow.addEdge( i, flow.t, out[ i ] - in[ i ] );
            nd += out[ i ] - in[ i ];
        }
    }
```

```

    if( out[ i ] < in[ i ] )
        flow.addEdge( flow.s , i , in[ i ] - out[ i ] );
}
// original sink to source
flow.addEdge( n , 1 , INF );
if( flow.maxflow() != nd )
    return -1; // no solution
int ans = flow.G[ 1 ].back().c; // source to sink
flow.G[ 1 ].back().c = flow.G[ n ].back().c = 0;
// take out super source and super sink
for( size_t i = 0 ; i < flow.G[ flow.s ].size() ; i
    ++ ){
    flow.G[ flow.s ][ i ].c = 0;
    Edge &e = flow.G[ flow.s ][ i ];
    flow.G[ e.v ][ e.r ].c = 0;
}
for( size_t i = 0 ; i < flow.G[ flow.t ].size() ; i
    ++ ){
    flow.G[ flow.t ][ i ].c = 0;
    Edge &e = flow.G[ flow.t ][ i ];
    flow.G[ e.v ][ e.r ].c = 0;
}
flow.addEdge( flow.s , 1 , INF );
flow.addEdge( n , flow.t , INF );
flow.reset();
return ans + flow.maxflow();
}

```

## 2.7 Flow Method \*

Maximize  $c^T x$  subject to  $Ax \leq b, x \geq 0$ ;  
 with the corresponding symmetric dual problem,  
 Minimize  $b^T y$  subject to  $A^T y \geq c, y \geq 0$ .  
 Maximize  $c^T x$  subject to  $Ax \leq b$ ;  
 with the corresponding asymmetric dual problem,  
 Minimize  $b^T y$  subject to  $A^T y = c, y \geq 0$ .  
 Minimum vertex cover on bipartite graph =  
 Maximum matching on bipartite graph  
 Minimum edge cover on bipartite graph =  
 vertex number - Minimum vertex cover(Maximum matching)  
 Independent set on bipartite graph =  
 vertex number - Minimum vertex cover(Maximum matching)  
 找出最小點覆蓋，做完 dinic 之後，從源點 dfs 只走還有流量的邊，紀錄每個點有沒有被走到，左邊沒被走到的點跟右邊被走到的點就是答案  
 Maximum density subgraph  $(\sum W_e + \sum W_v)/|V|$   
 Binary search on answer:  
 For a fixed D, construct a Max flow model as follow:  
 Let S be Sum of all weight( or inf)  
 1. from source to each node with cap = S  
 2. For each  $(u,v,w)$  in E,  $(u \rightarrow v, \text{cap}=w)$ ,  $(v \rightarrow u, \text{cap}=w)$   
 3. For each node v, from v to sink with cap =  $S + 2 * D - \text{deg}[v] - 2 * (W \text{ of } v)$   
 where  $\text{deg}[v] = \sum \text{weight of edge associated with } v$   
 If  $\text{maxflow} < S * |V|$ , D is an answer.  
 Requiring subgraph: all vertex can be reached from source with edge whose cap > 0.

### • Maximum/Minimum flow with lower bound / Circulation problem

- Construct super source  $S$  and sink  $T$ .
- For each edge  $(x,y,l,u)$ , connect  $x \rightarrow y$  with capacity  $u-l$ .
- For each vertex  $v$ , denote by  $in(v)$  the difference between the sum of incoming lower bounds and the sum of outgoing lower bounds.
- If  $in(v) > 0$ , connect  $S \rightarrow v$  with capacity  $in(v)$ , otherwise, connect  $v \rightarrow T$  with capacity  $-in(v)$ .
  - To maximize, connect  $t \rightarrow s$  with capacity  $\infty$  (skip this in circulation problem), and let  $f$  be the maximum flow from  $S$  to  $T$ . If  $f \neq \sum_{v \in V, in(v) > 0} in(v)$ , there's no solution. Otherwise, the maximum flow from  $s$  to  $t$  is the answer.
  - To minimize, let  $f$  be the maximum flow from  $S$  to  $T$ . Connect  $t \rightarrow s$  with capacity  $\infty$  and let the flow from  $S$  to  $T$  be  $f'$ . If  $f + f' \neq \sum_{v \in V, in(v) > 0} in(v)$ , there's no solution. Otherwise,  $f'$  is the answer.
- The solution of each edge  $e$  is  $l_e + f_e$ , where  $f_e$  corresponds to the flow of edge  $e$  on the graph.

### • Construct minimum vertex cover from maximum matching $M$ on bipartite graph $(X,Y)$

- Redirect every edge:  $y \rightarrow x$  if  $(x,y) \in M$ ,  $x \rightarrow y$  otherwise.
- DFS from unmatched vertices in  $X$ .
- $x \in X$  is chosen iff  $x$  is unvisited.
- $y \in Y$  is chosen iff  $y$  is visited.

### • Maximum density induced subgraph

- Binary search on answer, suppose we're checking answer  $T$
- Construct a max flow model, let  $K$  be the sum of all weights
- Connect source  $s \rightarrow v$ ,  $v \in G$  with capacity  $K$

- For each edge  $(u,v,w)$  in  $G$ , connect  $u \rightarrow v$  and  $v \rightarrow u$  with capacity  $w$
- For  $v \in G$ , connect it with sink  $v \rightarrow t$  with capacity  $K + 2T - (\sum_{e \in E(v)} w(e)) - 2w(v)$
- $T$  is a valid answer if the maximum flow  $f < K|V|$

### • Minimum weight edge cover

- For each  $v \in V$  create a copy  $v'$ , and connect  $u' \rightarrow v'$  with weight  $w(u,v)$ .
- Connect  $v \rightarrow v'$  with weight  $2\mu(v)$ , where  $\mu(v)$  is the cost of the cheapest edge incident to  $v$ .
- Find the minimum weight perfect matching on  $G'$ .

### • Project selection problem

- If  $p_v > 0$ , create edge  $(s,v)$  with capacity  $p_v$ ; otherwise, create edge  $(v,t)$  with capacity  $-p_v$ .
- Create edge  $(u,v)$  with capacity  $w$  with  $w$  being the cost of choosing  $u$  without choosing  $v$ .
- The mincut is equivalent to the maximum profit of a subset of projects.

### • 0/1 quadratic programming

$$\sum_x c_x x + \sum_y c_y \bar{y} + \sum_{xy} c_{xy} x \bar{y} + \sum_{xyx'y'} c_{xyx'y'} (x \bar{y} + x' \bar{y}')$$

can be minimized by the mincut of the following graph:

- Create edge  $(x,t)$  with capacity  $c_x$  and create edge  $(s,y)$  with capacity  $c_y$ .
- Create edge  $(x,y)$  with capacity  $c_{xy}$ .
- Create edge  $(x,y)$  and edge  $(x',y')$  with capacity  $c_{xyx'y'}$ .

## 3 Math

### 3.1 Fast Pow & Inverse & Combination

$\text{fpow}(a,b,m) = a^b \pmod{m}$   
 $\text{fa}[i] = i! \pmod{MOD}$   
 $\text{fi}[i] = i!^{-1} \equiv 1 \pmod{MOD}$   
 $\text{c}(a,b) = \binom{a}{b} \pmod{MOD}$

```

ll fpow(ll a, ll b, ll m){
    ll ret = 1;
    a %= m;
    while(b){
        if(b&1) ret = ret * a % m;
        a = a * a % m;
        b >>= 1; }
    return ret; }

```

```

ll fa[MXN], fi[MXN];
void init(){
    fa[0] = 1;
    for(ll i = 1; i < MXN; ++i)
        fa[i] = fa[i-1] * i % MOD;
    fi[MXN-1] = fpow(fa[MXN-1], MOD-2, MOD);
    for(ll i = MXN-1; i > 0; --i)
        fi[i-1] = fi[i] * i % MOD; }

```

```

ll c(ll a, ll b){
    return fa[a] * fi[b] % MOD * fi[a-b] % MOD; }

```

### 3.2 Ext GCD

```

//a * p.first + b * p.second = gcd(a, b)
pair<ll, ll> extgcd(ll a, ll b) {
    pair<ll, ll> res;
    if (a < 0) {
        res = extgcd(-a, b);
        res.first *= -1;
        return res;
    }
    if (b < 0) {
        res = extgcd(a, -b);
        res.second *= -1;
        return res;
    }
    if (b == 0) return {1, 0};
    res = extgcd(b, a % b);
    return {res.second, res.first - res.second * (a / b)};
}

```

### 3.3 Sieve 質數篩

```
const int MXN = 2e9 + 5; // 2^27 約0.7s, 2^30 約6~7s
bool np[MXN]; // np[i] = 1 -> i is'n a prime
vector<int> plist; // prime list
void sieveBuild(int n){
    MEM(np, 0);
    for(int i = 2, sq = sqrt(n); i <= sq; ++i)
        if(!np[i])
            for(int j = i * i; j <= n; j += i) np[j] = 1;
    for(int i = 2; i <= n; ++i) if(!np[i]) plist.PB(i); }
```

### 3.4 FFT \*

```
// const int MAXN = 262144;
// (must be 2^k)
// before any usage, run pre_fft() first
typedef long double ld;
typedef complex<ld> cplx; //real() ,imag()
const ld PI = acos(-1);
const cplx I(0, 1);
cplx omega[MAXN+1];
void pre_fft(){
    for(int i=0; i<=MAXN; i++)
        omega[i] = exp(i * 2 * PI / MAXN * I);
}
// n must be 2^k
void fft(int n, cplx a[], bool inv=false){
    int basic = MAXN / n;
    int theta = basic;
    for (int m = n; m >= 2; m >= 1) {
        int mh = m >> 1;
        for (int i = 0; i < mh; i++) {
            cplx w = omega[inv ? MAXN - (i * theta % MAXN) : i * theta % MAXN];
            for (int j = i; j < n; j += m) {
                int k = j + mh;
                cplx x = a[j] - a[k];
                a[j] += a[k];
                a[k] = w * x;
            }
            theta = (theta * 2) % MAXN;
        }
        int i = 0;
        for (int j = 1; j < n - 1; j++) {
            for (int k = n >> 1; k > (i ^ k); k >= 1);
            if (j < i) swap(a[i], a[j]);
        }
        if(inv) for (i = 0; i < n; i++) a[i] /= n;
    }
    cplx arr[MAXN+1];
    inline void mul(int _n, ll a[], int _m, ll b[], ll ans[])
    {
        int n=1, sum=_n+_m-1;
        while(n<sum)
            n<=1;
        for(int i=0; i<n; i++)
        {
            double x=(i<_n?a[i]:0), y=(i<_m?b[i]:0);
            arr[i]=complex<double>(x+y, x-y);
        }
        fft(n, arr);
        for(int i=0; i<n; i++)
            arr[i]=arr[i]*arr[i];
        fft(n, arr, true);
        for(int i=0; i<sum; i++)
            ans[i]=(long long int)(arr[i].real()/4+0.5);
    }
}
```

### 3.5 NTT \*

```
// Remember coefficient are mod P
/* p=a*2^n+1
   n    2^n    p    a    root
   16   65536   65537   1    3
   20   1048576 7340033   7    3 */
// (must be 2^k)
template<LL P, LL root, int MAXN>
struct NTT{
    static LL bigmod(LL a, LL b) {
        LL res = 1;
        for (LL bs = a; b; b >>= 1, bs = (bs * bs) % P)
```

```
        if(b&1) res=(res*bs)%P;
        return res;
    }
    static LL inv(LL a, LL b) {
        if(a==1) return 1;
        return (((LL)(a-inv(b*a,a))*b+1)/a)%b;
    }
    LL omega[MAXN+1];
    NTT() {
        omega[0] = 1;
        LL r = bigmod(root, (P-1)/MAXN);
        for (int i=1; i<=MAXN; i++)
            omega[i] = (omega[i-1]*r)%P;
    }
    // n must be 2^k
    void tran(int n, LL a[], bool inv_ntt=false){
        int basic = MAXN / n, theta = basic;
        for (int m = n; m >= 2; m >= 1) {
            int mh = m >> 1;
            for (int i = 0; i < mh; i++) {
                LL w = omega[i*theta%MAXN];
                for (int j = i; j < n; j += m) {
                    int k = j + mh;
                    LL x = a[j] - a[k];
                    if (x < 0) x += P;
                    a[j] += a[k];
                    if (a[j] > P) a[j] -= P;
                    a[k] = (w * x) % P;
                }
            }
            theta = (theta * 2) % MAXN;
        }
        int i = 0;
        for (int j = 1; j < n - 1; j++) {
            for (int k = n >> 1; k > (i ^ k); k >= 1);
            if (j < i) swap(a[i], a[j]);
        }
        if (inv_ntt) {
            LL ni = inv(n, P);
            reverse(a+1, a+n);
            for (i = 0; i < n; i++)
                a[i] = (a[i] * ni) % P;
        }
    }
};
const LL P=2013265921, root=31;
const int MAXN=4194304;
NTT<P, root, MAXN> ntt;
```

### 3.6 Linear Recurrence \*

```
// Usage: linearRec({0, 1}, {1, 1}, k) //k'th fib
typedef vector<ll> Poly;
//S: 前i項的值, tr: 遞迴系數, k: 求第k項
ll linearRec(Poly& S, Poly& tr, ll k) {
    int n = tr.size();
    auto combine = [&](Poly& a, Poly& b) {
        Poly res(n * 2 + 1);
        rep(i, 0, n+1) rep(j, 0, n+1)
            res[i+j] = (res[i+j] + a[i]*b[j])%mod;
        for(int i = 2*n; i > n; --i) rep(j, 0, n)
            res[i-1-j] = (res[i-1-j] + res[i]*tr[j])%mod;
        res.resize(n + 1);
        return res;
    };
    Poly pol(n + 1), e(pol);
    pol[0] = e[1] = 1;
    for (++k; k; k /= 2) {
        if (k % 2) pol = combine(pol, e);
        e = combine(e, e);
    }
    ll res = 0;
    rep(i, 0, n) res = (res + pol[i+1]*S[i])%mod;
    return res;
}
```

### 3.7 Miller Rabin

isprime(n) ⇒ 判斷 n 是否為質數  
記得填 magic number

```
// magic numbers when n <
// 4,759,123,141 : 2, 7, 61
```



```
// 1,122,004,669,633 : 2, 13, 23, 1662803
// 3,474,749,660,383 : 2, 3, 5, 7, 11, 13
// 2^64 : 2, 325, 9375, 28178, 450775,
// 9780504, 1795265022
// Make sure testing integer is in range [2, n^2] if
// you want to use magic.
vector<ll> magic = {};
bool witness(ll a, ll n, ll u, ll t){
    if(!a) return 0;
    ll x = fpow(a, u, n);
    while(t--){
        ll nx = x * x % n;
        if(nx == 1 && x != 1 && x != n - 1) return 1;
        x = nx;
    }
    return x != 1;
}
bool isprime(ll n){
    if(n < 2) return 0;
    if(~n & 1) return n == 2;
    ll u = n - 1, t = 0;
    while(~u & 1) u >>= 1, t++;
    for(auto i : magic){
        ll a = i % n;
        if(witness(a, n, u, t)) return 0;
    }
    return 1;
}
```

### 3.8 Faulhaber ( $\sum_{i=1}^n i^p$ ) \*

```
/* faulhaber' s formula -
 * cal power sum formula of all p=1~k in O(k^2) */
#define MAXK 2500
const int mod = 1000000007;
int b[MAXK]; // bernoulli number
int inv[MAXK+1]; // inverse
int cm[MAXK+1][MAXK+1]; // combinactories
int co[MAXK][MAXK+2]; // coeeficient of x^j when p=i
inline int getinv(int x){
    int a=x, b=mod, a0=1, a1=0, b0=0, b1=1;
    while(b){
        int q, t;
        q=a/b; t=b; b=a-b*q; a=t;
        t=b0; b0=a0-b0*q; a0=t;
        t=b1; b1=a1-b1*q; a1=t;
    }
    return a0<0?a0+mod:a0;
}
inline void pre(){
    /* combinational */
    for(int i=0; i<=MAXK; i++){
        cm[i][0]=cm[i][i]=1;
        for(int j=1; j<i; j++){
            cm[i][j]=add(cm[i-1][j-1], cm[i-1][j]);
        }
    }
    /* inverse */
    for(int i=1; i<=MAXK; i++) inv[i]=getinv(i);
    /* bernoulli */
    b[0]=1; b[1]=getinv(2); // with b[1] = 1/2
    for(int i=2; i<=MAXK; i++){
        if(i&1){ b[i]=0; continue; }
        b[i]=1;
        for(int j=0; j<i; j++){
            b[i]=sub(b[i],
                mul(cm[i][j], mul(b[j], inv[i-j+1])));
        }
    }
    /* faulhaber */
    // sigma_x=1~n {x^p} =
    // 1/(p+1) * sigma_j=0~p {C(p+1, j)*Bj*n^(p-j+1)}
    for(int i=1; i<=MAXK; i++){
        co[i][0]=0;
        for(int j=0; j<=i; j++){
            co[i][i-j+1]=mul(inv[i+1], mul(cm[i+1][j], b[j]));
        }
    }
}
/* sample usage: return f(n,p) = sigma_x=1~n (x^p) */
inline int solve(int n, int p){
    int sol=0, m=n;
    for(int i=1; i<=p+1; i++){
        sol=add(sol, mul(co[p][i], m));
        m = mul(m, n);
    }
    return sol;
}
```

```
}
```

### 3.9 Chinese Remainder \*

```
LL x[N], m[N];
LL CRT(LL x1, LL m1, LL x2, LL m2){
    LL g = __gcd(m1, m2);
    if((x2 - x1) % g) return -1; // no sol
    m1 /= g; m2 /= g;
    pair<LL, LL> p = gcd(m1, m2);
    LL lcm = m1 * m2 * g;
    LL res = p.first * (x2 - x1) * m1 + x1;
    return (res % lcm + lcm) % lcm;
}
LL solve(int n){ // n>=2, be careful with no solution
    LL res=CRT(x[0], m[0], x[1], m[1]), p=m[0]/__gcd(m[0], m[1])*m[1];
    for(int i=2; i<n; i++){
        res=CRT(res, p, x[i], m[i]);
        p=p/__gcd(p, m[i])*m[i];
    }
    return res;
}
```

### 3.10 Pollard Rho \*

```
// does not work when n is prime O(n^(1/4))
LL f(LL x, LL mod){ return add(mul(x, x, mod), 1, mod); }
LL pollard_rho(LL n){
    if(!(n&1)) return 2;
    while(true){
        LL y=2, x=rand()%(n-1)+1, res=1;
        for(int sz=2; res==1; sz*=2){
            for(int i=0; i<sz && res<=1; i++){
                x = f(x, n);
                res = __gcd(abs(x-y), n);
            }
            y = x;
        }
        if (res!=0 && res!=n) return res;
    }
}
```

### 3.11 Josephus Problem \*

```
int josephus(int n, int m){ //n人每m次
    int ans = 0;
    for (int i=1; i<=n; ++i)
        ans = (ans + m) % i;
    return ans;
}
```

### 3.12 Gaussian Elimination \*

```
const int GAUSS_MOD = 1000000007LL;
struct GAUSS{
    int n;
    vector<vector<int>>> v;
    int ppow(int a, int k){
        if(k == 0) return 1;
        if(k % 2 == 0) return ppow(a * a % GAUSS_MOD, k >> 1);
        if(k % 2 == 1) return ppow(a * a % GAUSS_MOD, k >> 1) * a % GAUSS_MOD;
    }
    vector<int> solve(){
        vector<int> ans(n);
        REP(now, 0, n){
            REP(i, now, n) if(v[now][now] == 0 && v[i][now] != 0)
                swap(v[now], v[i]); // det = -det;
            if(v[now][now] == 0) return ans;
            int inv = ppow(v[now][now], GAUSS_MOD - 2);
            REP(i, 0, n) if(i != now){
                int tmp = v[i][now] * inv % GAUSS_MOD;
                REP(j, now, n + 1) (v[i][j] += GAUSS_MOD - tmp * v[now][j] % GAUSS_MOD) %= GAUSS_MOD;
            }
            REP(i, 0, n) ans[i] = v[i][n + 1] * ppow(v[i][i], GAUSS_MOD - 2) % GAUSS_MOD;
        }
    }
}
```

```

        return ans;
    }
    // gs.v.clear() , gs.v.resize(n , vector<int>(n + 1 , 0));
} gs;

```

### 3.13 歐拉函數降冪公式

```

ll eulerFunction(ll x) {
    ll ret = x;
    for(ll i = 2; i * i <= x; ++i) {
        if(x % i == 0) {
            ret -= ret / i;
            while(x % i == 0) x /= i;
        }
    }
    if(x > 1) ret -= ret / x;
    return ret;
}

ll eulerPow(ll a, string b, ll mod) {
    ll ret = eulerFunction(mod);
    ll p = 0;
    for(ll i = 0; i < b.size(); ++i) {
        p = (p * 10 + b[i] - '0') % ret;
    }
    p += ret;
    return fastPow(a, p, mod);
}

```

### 3.14 貝爾數 Bell

```

ll bell[MXN][MXN];

void bellf(int n) {
    bell[1][1] = 1;
    for(int i = 2; i <= n; ++i) {
        bell[i][1] = bell[i - 1][i - 1];
        for(int j = 2; j <= i; ++j) {
            bell[i][j] = bell[i - 1][j - 1] + bell[i][j - 1];
        }
    }
}

```

### 3.15 Result \*

- Lucas' Theorem :  
For  $n, m \in \mathbb{Z}^*$  and prime  $P$ ,  $C(m, n) \bmod P = \prod C(m_i, n_i)$  where  $m_i$  is the  $i$ -th digit of  $m$  in base  $P$ .
- Stirling approximation :  
$$n! \approx \sqrt{2\pi n} \left(\frac{n}{e}\right)^n e^{\frac{1}{12n}}$$
- Stirling Numbers(permutation  $|P| = n$  with  $k$  cycles):  
 $S(n, k) = \text{coefficient of } x^k \text{ in } \prod_{i=0}^{n-1} (x + i)$
- Stirling Numbers(Partition  $n$  elements into  $k$  non-empty set):  
$$S(n, k) = \frac{1}{k!} \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} j^n$$
- Pick' s Theorem :  $A = i + b/2 - 1$   
其面積  $A$  和內部格點數目  $i$ 、邊上格點數目  $b$  的關係
- Catalan number :  $C_n = \binom{2n}{n} / (n + 1)$   
$$C_{n+m}^{n+m} - C_{n+1}^{n+m} = (m + n)! \frac{n-m+1}{n+1} \quad \text{for } n \geq m$$
  
$$C_n = \frac{1}{n+1} \binom{2n}{n} = \frac{(2n)!}{(n+1)!n!}$$
  
$$C_0 = 1 \quad \text{and} \quad C_{n+1} = 2 \binom{2n+1}{n+2} C_n$$
  
$$C_0 = 1 \quad \text{and} \quad C_{n+1} = \sum_{i=0}^n C_i C_{n-i} \quad \text{for } n \geq 0$$
- Euler Characteristic:  
planar graph:  $V - E + F - C = 1$   
convex polyhedron:  $V - E + F = 2$   
 $V, E, F, C$ : number of vertices, edges, faces(regions), and components
- Kirchhoff's theorem :  
 $A_{ii} = \deg(i), A_{ij} = (i, j) \in E ? -1 : 0$ , Deleting any one row, one column, and cal the  $\det(A)$
- Polya' theorem ( $c$  為方法數,  $m$  為總數):  
$$\left(\sum_{i=1}^m c^{\gcd(i, m)}\right) / m$$
- Burnside lemma:  
$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$$

- 錯排公式: ( $n$  個人中, 每個人皆不再原來位置的組合數):  
$$dp[0] = 1; dp[1] = 0;$$
  
$$dp[i] = (i - 1) * (dp[i - 1] + dp[i - 2]);$$
- Bell 數 (有  $n$  個人, 把他們拆組的方法總數) :  
$$B_0 = 1$$
  
$$B_n = \sum_{k=0}^n s(n, k) \quad (\text{second - stirling})$$
  
$$B_{n+1} = \sum_{k=0}^n \binom{n}{k} B_k$$
- Wilson's theorem :  
$$(p - 1)! \equiv -1 \pmod{p}$$
- Fermat's little theorem :  
$$a^p \equiv a \pmod{p}$$
- Euler's totient function:  
$$A^{B^C} \bmod p = \text{pow}(A, \text{pow}(B, C, p - 1)) \bmod p$$
- 歐拉函數降冪公式:  
$$A^B \bmod C = A^{B \bmod \phi(C) + \phi(C)} \bmod C$$
- 6 的倍數:  
$$(a - 1)^3 + (a + 1)^3 + (-a)^3 + (-a)^3 = 6a$$

## 4 Geometry

### 4.1 definition

```

const ld EPS = 1e-8;
const ld PI = acos(-1);
int dcmp(ld x){ // float x (<, ==, >) y -> (-1, 0, 1)
    if(abs(x) < EPS) return 0;
    else return x < 0 ? -1 : 1;
}

struct Pt{
    ld x, y; // 改三維記得其他函式都要改
    Pt(ld _x = 0, ld _y = 0): x(_x), y(_y){}
    Pt operator+(const Pt &a) const{
        return Pt(x + a.x, y + a.y);
    }
    Pt operator-(const Pt &a) const{
        return Pt(x - a.x, y - a.y);
    }
    Pt operator*(const ld &a) const{
        return Pt(x * a, y * a);
    }
    Pt operator/(const ld &a) const{
        return Pt(x / a, y / a);
    }
    ld operator*(const Pt &a) const{ // dot product
        return x * a.x + y * a.y;
    }
    ld operator^(const Pt &a) const{ // cross product
        return x * a.y - y * a.x;
    }
    bool operator<(const Pt &a) const{
        return x < a.x || (x == a.x && y < a.y);
    }
    // return dcmp(x-a.x) < 0 ||
    // (dcmp(x-a.x) == 0 && dcmp(y-a.y) < 0);
    bool operator==(const Pt &a) const{
        return dcmp(x - a.x) == 0 && dcmp(y - a.y) == 0;
    }
    int qua() { // 在哪個象限(軸上點歸類到逆時針的象限)
        if(x > 0 && y >= 0) return 1;
        if(x <= 0 && y > 0) return 2;
        if(x < 0 && y <= 0) return 3;
        if(x >= 0 && y < 0) return 4;
    }
    ld angle() const{ // -pi ~ pi
        if(dcmp(x) == 0 && dcmp(y) == 0) return 0;
        return atan2(y, x);
    }
};

ld norm2(const Pt &a){
    return a * a;
}

ld norm(const Pt &a){ // norm(a - b) = dis of a, b
    return sqrt(norm2(a));
}

Pt perp(const Pt &a){ // 垂直向量(順時針旋轉90度)
    return Pt(-a.y, a.x);
}

Pt rotate(const Pt &a, ld ang){
    return Pt(a.x * cos(ang) - a.y * sin(ang),
        a.x * sin(ang) + a.y * cos(ang));
}

struct Line{
    Pt s, e, v; // start, end, end - start
    ld ang; // angle of v
    Line(Pt _s = Pt(0, 0), Pt _e = Pt(0, 0)):
        s(_s), e(_e) { v = e - s; ang = atan2(v.y, v.x); }
    bool operator<(const Line &L) const{ // sort by angle
        return ang < L.ang;
    }
};

struct Circle{
    Pt o; ld r;
    Circle(Pt _o = Pt(0, 0), ld _r = 0): o(_o), r(_r){}
    bool inside(const Pt &a) const {
        return norm2(a - o) <= r * r;
    }
};

```

## 4.2 halfPlaneIntersection \*

```

#define N 100010
#define EPS 1e-8
#define SIDE 100000000
struct PO{ double x , y ; } p[ N ] , o ;
struct LI{
    PO a , b ;
    double angle ;
    void in( double x1 , double y1 , double x2 , double
        y2 ){
        a.x = x1 ; a.y = y1 ; b.x = x2 ; b.y = y2 ;
    }
} li[ N ] , deq[ N ] ;
int n , m , cnt ;
inline int dc( double x ){
    if ( x > EPS ) return 1 ;
    else if ( x < -EPS ) return -1 ;
    return 0 ;
}
inline PO operator-( PO a , PO b ){
    PO c ;
    c.x = a.x - b.x ; c.y = a.y - b.y ;
    return c ;
}
inline double cross( PO a , PO b , PO c ){
    return ( b.x - a.x ) * ( c.y - a.y ) - ( b.y - a.y )
        * ( c.x - a.x ) ;
}
inline bool cmp( const LI &a , const LI &b ){
    if( dc( a.angle - b.angle ) == 0 ) return dc( cross(
        a.a , a.b , b.a ) ) < 0 ;
    return a.angle > b.angle ;
}
inline PO getpoint( LI &a , LI &b ){
    double k1 = cross( a.a , b.b , b.a ) ;
    double k2 = cross( a.b , b.a , b.b ) ;
    PO tmp = a.b - a.a , ans ;
    ans.x = a.a.x + tmp.x * k1 / ( k1 + k2 ) ;
    ans.y = a.a.y + tmp.y * k1 / ( k1 + k2 ) ;
    return ans ;
}
inline void getcut(){
    sort( li + 1 , li + 1 + n , cmp ) ; m = 1 ;
    for( int i = 2 ; i <= n ; i ++ )
        if( dc( li[ i ].angle - li[ m ].angle ) != 0 )
            li[ ++ m ] = li[ i ] ;
    deq[ 1 ] = li[ 1 ] ; deq[ 2 ] = li[ 2 ] ;
    int bot = 1 , top = 2 ;
    for( int i = 3 ; i <= m ; i ++ ){
        while( bot < top && dc( cross( li[ i ].a , li[ i ].
            b , getpoint( deq[ top ] , deq[ top - 1 ] ) ) )
            < 0 ) top -- ;
        while( bot < top && dc( cross( li[ i ].a , li[ i ].
            b , getpoint( deq[ bot ] , deq[ bot + 1 ] ) ) )
            < 0 ) bot ++ ;
        deq[ ++ top ] = li[ i ] ;
    }
    while( bot < top && dc( cross( deq[ bot ].a , deq[
        bot ].b , getpoint( deq[ top ] , deq[ top - 1 ] )
        ) ) < 0 ) top -- ;
    while( bot < top && dc( cross( deq[ top ].a , deq[
        top ].b , getpoint( deq[ bot ] , deq[ bot + 1 ] )
        ) ) < 0 ) bot ++ ;
    cnt = 0 ;
    if( bot == top ) return ;
    for( int i = bot ; i < top ; i ++ ) p[ ++ cnt ] =
        getpoint( deq[ i ] , deq[ i + 1 ] ) ;
    if( top - 1 > bot ) p[ ++ cnt ] = getpoint( deq[ bot
        ] , deq[ top ] ) ;
}
double px[ N ] , py[ N ] ;
void read( int rm ){
    for( int i = 1 ; i <= n ; i ++ ) px[ i + n ] = px[ i
        ] , py[ i + n ] = py[ i ] ;
    for( int i = 1 ; i <= n ; i ++ ){
        // half-plane from li[ i ].a -> li[ i ].b
        li[ i ].a.x = px[ i + rm + 1 ] ; li[ i ].a.y = py[ i
            + rm + 1 ] ;
        li[ i ].b.x = px[ i ] ; li[ i ].b.y = py[ i ] ;
        li[ i ].angle = atan2( li[ i ].b.y - li[ i ].a.y ,
            li[ i ].b.x - li[ i ].a.x ) ;
    }
}

```

```

}
}
inline double getarea( int rm ){
    read( rm ) ; getcut() ;
    double res = 0.0 ;
    p[ cnt + 1 ] = p[ 1 ] ;
    for( int i = 1 ; i <= cnt ; i ++ ) res += cross( o ,
        p[ i ] , p[ i + 1 ] ) ;
    if( res < 0.0 ) res *= -1.0 ;
    return res ;
}

```

## 4.3 Convex Hull \*

```

double cross(Pt o , Pt a , Pt b){
    return (a-o) ^ (b-o);
}
vector<Pt> convex_hull(vector<Pt> pt){
    sort(pt.begin(),pt.end());
    int top=0;
    vector<Pt> stk(2*pt.size());
    for (int i=0; i<(int)pt.size(); i++){
        while (top >= 2 && cross(stk[top-2],stk[top-1],pt[i
            ]) <= 0)
            top--;
        stk[top++] = pt[i];
    }
    for (int i=pt.size()-2, t=top+1; i>=0; i--){
        while (top >= t && cross(stk[top-2],stk[top-1],pt[i
            ]) <= 0)
            top--;
        stk[top++] = pt[i];
    }
    stk.resize(top-1);
    return stk;
}

```

## 4.4 Convex Hull trick \*

```

/* Given a convexhull, answer queries in O(\lg N)
CH should not contain identical points, the area should
be > 0, min pair(x, y) should be listed first */
double det( const Pt& p1 , const Pt& p2 )
{ return p1.X * p2.Y - p1.Y * p2.X; }
struct Conv{
    int n;
    vector<Pt> a;
    vector<Pt> upper, lower;
    Conv(vector<Pt> _a) : a(_a){
        n = a.size();
        int ptr = 0;
        for(int i=1; i<n; ++i) if (a[ptr] < a[i]) ptr = i;
        for(int i=0; i<=ptr; ++i) lower.push_back(a[i]);
        for(int i=ptr; i<n; ++i) upper.push_back(a[i]);
        upper.push_back(a[0]);
    }
    int sign( LL x ){ // fixed when changed to double
        return x < 0 ? -1 : x > 0 ;
    }
    pair<LL,int> get_tang(vector<Pt> &conv, Pt vec){
        int l = 0 , r = (int)conv.size() - 2 ;
        for( ; l + 1 < r ; ){
            int mid = (l + r) / 2 ;
            if(sign(det(conv[mid+1]-conv[mid],vec))>0)r=mid;
            else l = mid;
        }
        return max(make_pair(det(vec, conv[r]), r),
            make_pair(det(vec, conv[0]), 0));
    }
    void upd_tang(const Pt &p, int id, int &i0, int &i1){
        if(det(a[i0] - p, a[id] - p) > 0) i0 = id;
        if(det(a[i1] - p, a[id] - p) < 0) i1 = id;
    }
    void bi_search(int l, int r, Pt p, int &i0, int &i1){
        if(l == r) return;
        upd_tang(p, l % n, i0, i1);
        int sl=sign(det(a[l % n] - p, a[(l + 1) % n] - p));
        for( ; l + 1 < r ; ){
            int mid = (l + r) / 2 ;
            int smid=sign(det(a[mid%n]-p, a[(mid+1)%n]-p));
            if (smid == sl) l = mid;
            else r = mid;
        }
    }
}

```



```

    upd_tang(p, r % n, i0, i1);
}
int bi_search(Pt u, Pt v, int l, int r) {
    int sl = sign(det(v - u, a[l % n] - u));
    for( ; l + 1 < r; ) {
        int mid = (l + r) / 2;
        int smid = sign(det(v - u, a[mid % n] - u));
        if (smid == sl) l = mid;
        else r = mid;
    }
    return l % n;
}
// 1. whether a given point is inside the CH
bool contain(Pt p) {
    if (p.X < lower[0].X || p.X > lower.back().X)
        return 0;
    int id = lower_bound(lower.begin(), lower.end(), Pt
        (p.X, -INF)) - lower.begin();
    if (lower[id].X == p.X) {
        if (lower[id].Y > p.Y) return 0;
    } else if (det(lower[id-1]-p, lower[id]-p) < 0) return 0;
    id = lower_bound(upper.begin(), upper.end(), Pt(p.X
        , INF), greater<Pt>()) - upper.begin();
    if (upper[id].X == p.X) {
        if (upper[id].Y < p.Y) return 0;
    } else if (det(upper[id-1]-p, upper[id]-p) < 0) return 0;
    return 1;
}
// 2. Find 2 tang pts on CH of a given outside point
// return true with i0, i1 as index of tangent points
// return false if inside CH
bool get_tang(Pt p, int &i0, int &i1) {
    if (contain(p)) return false;
    i0 = i1 = 0;
    int id = lower_bound(lower.begin(), lower.end(), p)
        - lower.begin();
    bi_search(0, id, p, i0, i1);
    bi_search(id, (int)lower.size(), p, i0, i1);
    id = lower_bound(upper.begin(), upper.end(), p,
        greater<Pt>()) - upper.begin();
    bi_search((int)lower.size() - 1, (int)lower.size()
        - 1 + id, p, i0, i1);
    bi_search((int)lower.size() - 1 + id, (int)lower.
        size() - 1 + (int)upper.size(), p, i0, i1);
    return true;
}
// 3. Find tangent points of a given vector
// ret the idx of vertex has max cross value with vec
int get_tang(Pt vec) {
    pair<LL, int> ret = get_tang(upper, vec);
    ret.second = (ret.second + (int)lower.size() - 1) % n;
    ret = max(ret, get_tang(lower, vec));
    return ret.second;
}
// 4. Find intersection point of a given line
// return 1 and intersection is on edge (i, next(i))
// return 0 if no strictly intersection
bool get_intersection(Pt u, Pt v, int &i0, int &i1) {
    int p0 = get_tang(u - v), p1 = get_tang(v - u);
    if (sign(det(v-u, a[p0]-u)) * sign(det(v-u, a[p1]-u)) < 0) {
        if (p0 > p1) swap(p0, p1);
        i0 = bi_search(u, v, p0, p1);
        i1 = bi_search(u, v, p1, p0 + n);
        return 1;
    }
    return 0;
}
}
};

```

#### 4.5 掃描的線

```

ScanLine sl;
sl.add(兩點座標);
sl.run()

```

```

template <typename T>
struct SegmentTree {
    struct Node {
        T len = 0, tag = 0;
        int nl, nr;
        Node *l, *r;
    } *root;
    vector<T> vec;

```

```

    int n;
    SegmentTree() {}
    void init(vector<T> _vec) {
        vec = _vec;
        n = vec.size() - 1;
        root = build(0, n - 1);
    }
    Node* build(int l, int r) {
        Node *res = new Node();
        res->nl = l, res->nr = r;
        if (l == r) {
            res->l = res->r = nullptr;
            return res;
        }
        int mid = (l + r) >> 1;
        res->l = build(l, mid);
        res->r = build(mid + 1, r);
        return res;
    }
    void push(Node *cur) {
        int l = cur->nl, r = cur->nr;
        if (cur->tag) cur->len = vec[r + 1] - vec[l];
        else cur->len = l == r ? 0 : cur->l->len + cur->r->
            len;
    }
    void update(Node *cur, int ql, int qr, int x) {
        int l = cur->nl, r = cur->nr;
        if (vec[r + 1] <= ql || qr <= vec[l]) return;
        if (ql <= vec[l] && vec[r + 1] <= qr) {
            cur->tag += x;
            push(cur);
            return;
        }
        update(cur->l, ql, qr, x);
        update(cur->r, ql, qr, x);
        push(cur);
    }
    void update(int l, int r, int x) {
        update(root, l, r, x);
    }
};
template <typename T>
struct ScanLine {
    struct Line {
        T l, r, h, flag;
        bool operator<(const Line &rhs) {
            return h < rhs.h;
        }
    };
    vector<T> vec; vector<Line> line; SegmentTree<T> seg;
    int n, cnt = 0;
    ScanLine(int _n): n(_n << 1) {
        line.resize(n), vec.resize(n);
    }
    void add(int x1, int y1, int x2, int y2) {
        line[cnt] = {x1, x2, y1, 1}, line[cnt + 1] = {x1,
            x2, y2, -1};
        vec[cnt] = x1, vec[cnt + 1] = x2;
        cnt += 2;
    }
    T run() {
        T res = 0;
        sort(line.begin(), line.end());
        sort(vec.begin(), vec.end());
        vec.erase(unique(vec.begin(), vec.end()), vec.end()
            );
        seg.init(vec);
        for (int i = 0; i < n - 1; ++i) {
            seg.update(line[i].l, line[i].r, line[i].flag);
            res += seg.root->len * (line[i + 1].h - line[i].h
                );
        }
        return res;
    }
};

```

#### 4.6 Polar sort

```

sort(pl.begin(), pl.end(), [&](Pt a, Pt b) {
    // a = a - o, b = b - o;
    if (a.qua() == b.qua()) return (a ^ b) > 0;
    return a.qua() < b.qua();
});

```

```
}); // degree 0 to 359
sort(pl.begin(), pl.end(), [&](Pt a, Pt b){
    return (a - pt[i]).angle() < (b - pt[i]).angle();
}); // degree -180 to 180, slower
```

#### 4.7 Li Chao Segment Tree \*

```
struct LiChao_min{
    struct line{
        ll m,c;
        line(ll _m=0,ll _c=0){ m=_m; c=_c; }
        ll eval(ll x){ return m*x+c; } // overflow
    };
    struct node{
        node *l,*r; line f;
        node(line v){ f=v; l=r=NULL; }
    };
    typedef node* pnode;
    pnode root; ll sz,ql,qr;
#define mid ((l+r)>>1)
    void insert(line v,ll l,ll r,pnode &nd){
        /* if(!(ql<=l&&r<=qr)){
            if(!nd) nd=new node(line(0,INF));
            if(ql<=mid) insert(v,l,mid,nd->l);
            if(qr>mid) insert(v,mid+1,r,nd->r);
            return;
        } used for adding segment */
        if(!nd){ nd=new node(v); return; }
        ll trl=nd->f.eval(l),trr=nd->f.eval(r);
        ll vl=v.eval(l),vr=v.eval(r);
        if(trl<=vl&&trr<=vr) return;
        if(trl>vl&&trr>vr) { nd->f=v; return; }
        if(trl>vl) swap(nd->f,v);
        if(nd->f.eval(mid)<v.eval(mid))
            insert(v,mid+1,r,nd->r);
        else swap(nd->f,v),insert(v,l,mid,nd->l);
    }
    ll query(ll x,ll l,ll r,pnode &nd){
        if(!nd) return INF;
        if(l==r) return nd->f.eval(x);
        if(mid>=x)
            return min(nd->f.eval(x),query(x,l,mid,nd->l));
        return min(nd->f.eval(x),query(x,mid+1,r,nd->r));
    }
    /* -sz<=ll query_x<=sz */
    void init(ll _sz){ sz=_sz+1; root=NULL; }
    void add_line(ll m,ll c,ll l=-INF,ll r=INF){
        line v(m,c); ql=l; qr=r; insert(v,-sz,sz,root);
    }
    ll query(ll x) { return query(x,-sz,sz,root); }
};
```

#### 4.8 KD Tree \*

```
struct KDTree{ // O(sqrtN + K)
    struct Nd{
        LL x[MXK],mn[MXK],mx[MXK];
        int id,f;
        Nd *l,*r;
    }tree[MXN],*root;
    int n,k;
    LL dis(LL a,LL b){return (a-b)*(a-b);}
    LL dis(LL a[MXK],LL b[MXK]){
        LL ret=0;
        for(int i=0;i<k;i++) ret+=dis(a[i],b[i]);
        return ret;
    }
    void init(vector<vector<LL>> &ip,int _n,int _k){
        n=_n,k=_k;
        for(int i=0;i<n;i++){
            tree[i].id=i;
            copy(ip[i].begin(),ip[i].end(),tree[i].x);
        }
        root=build(0,n-1,0);
    }
    Nd* build(int l,int r,int d){
        if(l>r) return NULL;
        if(d==k) d=0;
        int m=(l+r)>>1;
        nth_element(tree+l,tree+m,tree+r+1,[&](const Nd &a,
            const Nd &b){return a.x[d]<b.x[d];});
        tree[m].f=d;

```

```
        copy(tree[m].x,tree[m].x+k,tree[m].mn);
        copy(tree[m].x,tree[m].x+k,tree[m].mx);
        tree[m].l=build(l,m-1,d+1);
        if(tree[m].l){
            for(int i=0;i<k;i++){
                tree[m].mn[i]=min(tree[m].mn[i],tree[m].l->mn[i]);
                tree[m].mx[i]=max(tree[m].mx[i],tree[m].l->mx[i]);
            }
        }
        tree[m].r=build(m+1,r,d+1);
        if(tree[m].r){
            for(int i=0;i<k;i++){
                tree[m].mn[i]=min(tree[m].mn[i],tree[m].r->mn[i]);
                tree[m].mx[i]=max(tree[m].mx[i],tree[m].r->mx[i]);
            }
        }
        return tree+m;
    }
    LL pt[MXK],md;
    int mID;
    bool touch(Nd *r){
        LL d=0;
        for(int i=0;i<k;i++){
            if(pt[i]<=r->mn[i]) d+=dis(pt[i],r->mn[i]);
            else if(pt[i]>=r->mx[i]) d+=dis(pt[i],r->mx[i]);
        }
        return d<md;
    }
    void nearest(Nd *r){
        if(!r||!touch(r)) return;
        LL td=dis(r->x,pt);
        if(td<md) md=td,mID=r->id;
        nearest(pt[r->f]<r->x[r->f]?r->l:r->r);
        nearest(pt[r->f]>r->x[r->f]?r->r:r->l);
    }
    pair<LL,int> query(vector<LL> &_pt,LL _md=1LL<<57){
        mID=-1,md=_md;
        copy(_pt.begin(),_pt.end(),pt);
        nearest(root);
        return {md,mID};
    }
} tree;
```

#### 4.9 多邊形面積

```
ld polygonArea(vector<Point> &poly, int n) {
    ld res = 0;
    for(int i = 0, j = 0; i < n; ++i) {
        j = (i + 1) % n;
        res += poly[i].x * poly[j].y - poly[j].x * poly[i].y;
    }
    return abs(res) / 2;
}
```

#### 4.10 Min Enclosing Circle

```
const int MXN = 1e7;
int n; Pt p[MXN]; // input n, p[0] ~ p[n - 1]
const Circle circumcircle(Pt a,Pt b,Pt c){
    Circle cir;
    ld fa,fb,fc,fd,fe,ff,dx,dy,dd;
    if( iszero( ( b - a ) ^ ( c - a ) ) ){
        if( ( ( b - a ) * ( c - a ) ) <= 0 )
            return Circle((b+c)/2,norm(b-c)/2);
        if( ( ( c - b ) * ( a - b ) ) <= 0 )
            return Circle((c+a)/2,norm(c-a)/2);
        if( ( ( a - c ) * ( b - c ) ) <= 0 )
            return Circle((a+b)/2,norm(a-b)/2);
    }else{
        fa=2*(a.x-b.x);
        fb=2*(a.y-b.y);
        fc=norm2(a)-norm2(b);
        fd=2*(a.x-c.x);
        fe=2*(a.y-c.y);
        ff=norm2(a)-norm2(c);
        dx=fc*fe-ff*fb;
        dy=fa*ff-fd*fc;
        dd=fa*fe-fd*fb;
        cir.o=Pt(dx/dd,dy/dd);

```

```

    cir.r=norm(a-cir.o);
    return cir; } }
inline Circle mec(int fixed,int num){
    int i;
    Circle cir;
    if(fixed==3) return circumcircle(p[0],p[1],p[2]);
    cir=circumcircle(p[0],p[0],p[1]);
    for(i=fixed;i<num;i++) {
        if(cir.inside(p[i])) continue;
        swap(p[i],p[fixed]);
        cir=mec(fixed+1,i+1);
    }
    return cir;
}
inline ld min_radius() {
    if(n<=1) return 0.0;
    if(n==2) return norm(p[0]-p[1])/2;
    random_shuffle(p, p+n);
    return mec(0,n).r; }

```

#### 4.11 Min Enclosing Ball

```

// Pt : { x , y , z }
const int MXN = 202020;
int n, nouter; Pt pt[MXN], outer[4], res;
ld radius,tmp;
void ball() {
    Pt q[3]; ld m[3][3], sol[3], L[3], det;
    int i,j; res.x = res.y = res.z = radius = 0;
    switch (nouter) {
        case 1: res=outer[0]; break;
        case 2: res=(outer[0]+outer[1])/2;
            radius=norm2(res - outer[0]); break;
        case 3:
            for (i=0; i<2; ++i) q[i]=outer[i+1]-outer[0];
            for (i=0; i<2; ++i) for(j=0; j<2; ++j)
                m[i][j]=(q[i] * q[j])*2;
            for (i=0; i<2; ++i) sol[i]=(q[i] * q[i]);
            if(fabs(det=m[0][0]*m[1][1]-m[0][1]*m[1][0])<EPS)
                return;
            L[0]=(sol[0]*m[1][1]-sol[1]*m[0][1])/det;
            L[1]=(sol[1]*m[0][0]-sol[0]*m[1][0])/det;
            res=outer[0]+q[0]*L[0]+q[1]*L[1];
            radius=norm2(res - outer[0]);
            break;
        case 4:
            for (i=0; i<3; ++i)
                q[i]=outer[i+1]-outer[0], sol[i]=(q[i] * q[i]);
            for (i=0; i<3; ++i) for(j=0; j<3; ++j)
                m[i][j]=(q[i] * q[j])*2;
            det= m[0][0]*m[1][1]*m[2][2]
                + m[0][1]*m[1][2]*m[2][0]
                + m[0][2]*m[1][0]*m[2][1]
                - m[0][2]*m[1][1]*m[2][0]
                - m[0][1]*m[1][0]*m[2][2]
                - m[0][0]*m[1][2]*m[2][1];
            if (fabs(det)<EPS) return;
            for (j=0; j<3; ++j) {
                for (i=0; i<3; ++i) m[i][j]=sol[i];
                L[j]=(m[0][0]*m[1][1]*m[2][2]
                    + m[0][1]*m[1][2]*m[2][0]
                    + m[0][2]*m[1][0]*m[2][1]
                    - m[0][2]*m[1][1]*m[2][0]
                    - m[0][1]*m[1][0]*m[2][2]
                    - m[0][0]*m[1][2]*m[2][1]) / det;
                for (i=0; i<3; ++i) m[i][j]=(q[i] * q[j])*2;
            } res=outer[0];
            for (i=0; i<3; ++i) res = res + q[i] * L[i];
            radius=norm2(res - outer[0]);
    }
}
void minball(int n){ ball();
    if(nouter < 4) for(int i = 0 ; i < n ; i ++){
        if(norm2(res - pt[i]) - radius > EPS){
            outer[nouter ++] = pt[i]; minball(i); --nouter;
            if(i>0){ Pt Tt = pt[i];
                memmove(&pt[1], &pt[0], sizeof(Pt)*i);pt[0]=Tt;
            }
        }
    }
}
ld solve(){
    // n points in pt
    random_shuffle(pt, pt+n); radius=-1;
    for(int i=0;i<n;i++) if(norm2(res-pt[i])-radius>EPS)
        nouter=1, outer[0]=pt[i], minball(i);
}

```

```

    return sqrt(radius);
}

```

## 5 Tree

### 5.1 LCA

求樹上兩點的最低共同祖先

$lca.init(n) \Rightarrow 0$ -base

$lca.addEdge(u, v) \Rightarrow u \leftrightarrow v$

$lca.build(root, root) \Rightarrow O(n \lg n)$

$lca.qlca(u, v) \Rightarrow O(\lg n)$   $u, v$  的 LCA

$lca.qdis(u, v) \Rightarrow O(\lg n)$   $u, v$  的距離 (可用倍增法帶權)

$lca.anc[u][i] \Rightarrow u$  的第  $2^i$  個祖先

```

const int MXN = 5e5 + 5;
struct LCA{
    int n, lgn, ti = 0;
    int anc[MXN][24], in[MXN], out[MXN];
    ll ancw[MXN][24];
    vector<pll> g[MXN];
    void init(int _n) {
        n = _n, lgn = __lg(n) + 5;
        for(int i = 0; i < n; i++) g[i].clear();
    }
    void addEdge(int u, int v, ll w = 1){
        g[u].PB(w, v), g[v].PB(w, u);
    }
    void build(int u, int f, ll w = 0) {
        in[u] = ti++;
        int cur = f;
        ll curw = w;
        for(int i = 0; i < lgn; ++i) {
            ancw[u][i] = curw, curw += ancw[cur][i];
            anc[u][i] = cur, cur = anc[cur][i];
        }
        for(auto i : g[u]) if(i.Y != f) build(i.Y, u, i.X);
        out[u] = ti++;
    }
    bool isanc(int a, int u) {
        return in[a] <= in[u] && out[u] <= out[a];
    }
    int qlca(int u, int v) {
        if(isanc(u, v)) return u;
        if(isanc(v, u)) return v;
        for(int i = lgn - 1; i >= 0; --i)
            if(!isanc(anc[u][i], v)) u = anc[u][i];
        return anc[u][0];
    }
    ll qdis(int u, int v) {
        ll dis = 0;
        for(int i = lgn - 1; i >= 0; --i) {
            if(!isanc(anc[u][i], v)) {
                dis += ancw[u][i];
                u = anc[u][i];
            }
            if(!isanc(anc[v][i], u)) {
                dis += ancw[v][i];
                v = anc[v][i];
            }
        }
        if(!isanc(u, v)) dis += ancw[u][0];
        if(!isanc(v, u)) dis += ancw[v][0];
        return dis;
    }
};

```

## 6 Graph

### 6.1 HeavyLightDecomposition \*

```

const int MXN = 200005;
template <typename T>
struct HeavyDecompose{ // 1-base, Need "ulimit -s unlimited"
    SegmentTree<T> st;
    vector<T> vec, tmp; // If tree point has weight
    vector<int> e[MXN];
    int sz[MXN], dep[MXN], fa[MXN], h[MXN];
    int cnt = 0, r = 0, n = 0;
    int root[MXN], id[MXN];
    void addEdge(int a, int b){
        e[a].emplace_back(b);
        e[b].emplace_back(a);
    }
    HeavyDecompose(int n, int r): n(n), r(r){
        vec.resize(n + 1); tmp.resize(n + 1);
    }
    void build(){
        dfs1(r, 0, 0);
        dfs2(r, r);
        st.init(tmp); // SegmentTree Need Add Method
    }
    void dfs1(int x, int f, int d){
        dep[x] = d, fa[x] = f, sz[x] = 1, h[x] = 0;
    }
}

```

```

for(int i : e[x]){
    if(i == f) continue;
    dfs1(i, x, d + 1);
    sz[x] += sz[i];
    if(sz[i] > sz[h[x]]) h[x] = i;
}
}
void dfs2(int x, int f){
    id[x] = cnt++; root[x] = f, tmp[id[x]] = vec[x];
    if(!h[x]) return;
    dfs2(h[x], f);
    for(int i : e[x]){
        if(i == fa[x] || i == h[x]) continue;
        dfs2(i, i);
    }
}
void update(int x, int y, T v){
    while(root[x] != root[y]){
        if(dep[root[x]] < dep[root[y]]) swap(x, y);
        st.update(id[root[x]], id[x], v);
        x = fa[root[x]];
    }
    if(dep[x] > dep[y]) swap(x, y);
    st.update(id[x], id[y], v);
}
T query(int x, int y){
    T res = 0;
    while(root[x] != root[y]){
        if(dep[root[x]] < dep[root[y]]) swap(x, y);
        res = (st.query(id[root[x]], id[x]) + res) % MOD;
        x = fa[root[x]];
    }
    if(dep[x] > dep[y]) swap(x, y);
    res = (st.query(id[x], id[y]) + res) % MOD;
    return res;
}
void update(int x, T v){
    st.update(id[x], id[x] + sz[x] - 1, v);
}
T query(int x){
    return st.query(id[x], id[x] + sz[x] - 1);
}
int getLca(int x, int y){
    while(root[x] != root[y]){
        if(dep[root[x]] > dep[root[y]]) x = fa[root[x]];
        else y = fa[root[y]];
    }
    return dep[x] > dep[y] ? y : x;
}
};

```

## 6.2 Centroid Decomposition \*

```

struct CentroidDecomposition {
    int n;
    vector<vector<int>> G, out;
    vector<int> sz, v;
    CentroidDecomposition(int _n) : n(_n), G(_n), out(
        _n), sz(_n), v(_n) {}
    int dfs(int x, int par){
        sz[x] = 1;
        for (auto &i : G[x]) {
            if(i == par || v[i]) continue;
            sz[x] += dfs(i, x);
        }
        return sz[x];
    }
    int search_centroid(int x, int p, const int mid){
        for (auto &i : G[x]) {
            if(i == p || v[i]) continue;
            if(sz[i] > mid) return search_centroid(i, x, mid);
        }
        return x;
    }
    void add_edge(int l, int r){
        G[l].PB(r); G[r].PB(l);
    }
    int get(int x){
        int centroid = search_centroid(x, -1, dfs(x, -1)/2);
        v[centroid] = true;
    }
};

```

```

for (auto &i : G[centroid]) {
    if(!v[i]) out[centroid].PB(get(i));
}
v[centroid] = false;
return centroid;
}
};

```

## 6.3 DominatorTree \*

```

struct DominatorTree{ // O(N)
#define REP(i,s,e) for(int i=(s);i<=(e);i++)
#define REPD(i,s,e) for(int i=(s);i>=(e);i--)
    int n, m, s;
    vector<int> g[ MAXN ], pred[ MAXN ];
    vector<int> cov[ MAXN ];
    int dfn[ MAXN ], nfd[ MAXN ], ts;
    int par[ MAXN ]; //idom[u] s到u的最後一個必經點
    int sdom[ MAXN ], idom[ MAXN ];
    int mom[ MAXN ], mn[ MAXN ];
    inline bool cmp( int u, int v )
    { return dfn[ u ] < dfn[ v ]; }
    int eval( int u ){
        if( mom[ u ] == u ) return u;
        int res = eval( mom[ u ] );
        if(cmp( sdom[ mn[ mom[ u ] ] ], sdom[ mn[ u ] ] ))
            mn[ u ] = mn[ mom[ u ] ];
        return mom[ u ] = res;
    }
    void init( int _n, int _m, int _s ){
        ts = 0; n = _n; m = _m; s = _s;
        REP( i, 1, n ) g[ i ].clear(), pred[ i ].clear();
    }
    void addEdge( int u, int v ){
        g[ u ].push_back( v );
        pred[ v ].push_back( u );
    }
    void dfs( int u ){
        ts++;
        dfn[ u ] = ts;
        nfd[ ts ] = u;
        for( int v : g[ u ] ) if( dfn[ v ] == 0 ){
            par[ v ] = u;
            dfs( v );
        }
    }
    void build(){
        REP( i, 1, n ){
            dfn[ i ] = nfd[ i ] = 0;
            cov[ i ].clear();
            mom[ i ] = mn[ i ] = sdom[ i ] = i;
        }
        dfs( s );
        REPD( i, n, 2 ){
            int u = nfd[ i ];
            if( u == 0 ) continue;
            for( int v : pred[ u ] ) if( dfn[ v ] ){
                eval( v );
                if( cmp( sdom[ mn[ v ] ], sdom[ u ] ) )
                    sdom[ u ] = sdom[ mn[ v ] ];
            }
            cov[ sdom[ u ] ].push_back( u );
            mom[ u ] = par[ u ];
            for( int w : cov[ par[ u ] ] ){
                eval( w );
                if( cmp( sdom[ mn[ w ] ], par[ u ] ) )
                    idom[ w ] = mn[ w ];
                else idom[ w ] = par[ u ];
            }
            cov[ par[ u ] ].clear();
        }
        REP( i, 2, n ){
            int u = nfd[ i ];
            if( u == 0 ) continue;
            if( idom[ u ] != sdom[ u ] )
                idom[ u ] = idom[ idom[ u ] ];
        }
    }
};

```

## 6.4 MaximumClique 最大團 \*

```

#define N 111
struct MaxClique{ // 0-base
    typedef bitset<N> Int;
    Int linkto[N], v[N];
};

```

```

int n;
void init(int _n){
    n = _n;
    for(int i = 0 ; i < n ; i++){
        linkto[i].reset(); v[i].reset();
    }
}
void addEdge(int a , int b)
{ v[a][b] = v[b][a] = 1; }
int popcount(const Int& val)
{ return val.count(); }
int lowbit(const Int& val)
{ return val._Find_first(); }
int ans , stk[N];
int id[N] , di[N] , deg[N];
Int cans;
void maxclique(int elem_num, Int candi){
    if(elem_num > ans){
        ans = elem_num; cans.reset();
        for(int i = 0 ; i < elem_num ; i ++){
            cans[id[stk[i]]] = 1;
        }
        int potential = elem_num + popcount(candi);
        if(potential <= ans) return;
        int pivot = lowbit(candi);
        Int smaller_candi = candi & (~linkto[pivot]);
        while(smaller_candi.count() && potential > ans){
            int next = lowbit(smaller_candi);
            candi[next] = !candi[next];
            smaller_candi[next] = !smaller_candi[next];
            potential --;
            if(next == pivot || (smaller_candi & linkto[next]).count()){
                stk[elem_num] = next;
                maxclique(elem_num + 1, candi & linkto[next]);
            }
        }
    }
    int solve(){
        for(int i = 0 ; i < n ; i++){
            id[i] = i; deg[i] = v[i].count();
        }
        sort(id , id + n , [&](int id1, int id2){
            return deg[id1] > deg[id2]; });
        for(int i = 0 ; i < n ; i++) di[id[i]] = i;
        for(int i = 0 ; i < n ; i++){
            for(int j = 0 ; j < n ; j++){
                if(v[i][j]) linkto[di[i]][di[j]] = 1;
            }
        }
        Int cand; cand.reset();
        for(int i = 0 ; i < n ; i++) cand[i] = 1;
        ans = 1;
        cans.reset(); cans[0] = 1;
        maxclique(0, cand);
        return ans;
    }
} } solver;

```

## 6.5 MaximalClique 極大團 \*

```

#define N 80
struct MaxClique{ // 0-base
    typedef bitset<N> Int;
    Int lnk[N] , v[N];
    int n;
    void init(int _n){
        n = _n;
        for(int i = 0 ; i < n ; i++){
            lnk[i].reset(); v[i].reset();
        }
    }
    void addEdge(int a , int b)
    { v[a][b] = v[b][a] = 1; }
    int ans , stk[N], id[N] , di[N] , deg[N];
    Int cans;
    void dfs(int elem_num, Int candi, Int ex){
        if(candi.none() && ex.none()){
            cans.reset();
            for(int i = 0 ; i < elem_num ; i ++){
                cans[id[stk[i]]] = 1;
            }
            ans = elem_num; // cans is a maximal clique
            return;
        }
        int pivot = (candilex)._Find_first();
        Int smaller_candi = candi & (~lnk[pivot]);
        while(smaller_candi.count()){
            int nxt = smaller_candi._Find_first();
            candi[nxt] = smaller_candi[nxt] = 0;
        }
    }
}

```

```

ex[nxt] = 1;
stk[elem_num] = nxt;
dfs(elem_num+1, candi & lnk[nxt], ex & lnk[nxt]);
} }
int solve(){
    for(int i = 0 ; i < n ; i++){
        id[i] = i; deg[i] = v[i].count();
    }
    sort(id , id + n , [&](int id1, int id2){
        return deg[id1] > deg[id2]; });
    for(int i = 0 ; i < n ; i++) di[id[i]] = i;
    for(int i = 0 ; i < n ; i++){
        for(int j = 0 ; j < n ; j++){
            if(v[i][j]) lnk[di[i]][di[j]] = 1;
        }
    }
    ans = 1; cans.reset(); cans[0] = 1;
    dfs(0, Int(string(n, '1')), 0);
    return ans;
} } solver;

```

## 6.6 Minimum Steiner Tree

```

const int MXNN = 105;
const int MXNK = 10 + 1;
template<typename T>
struct SteinerTree{ // 有重要點的MST權重和, 1-base
    int n, k;
    T inf;
    vector<vector<T>> dp;
    vector<vector<pair<int, T>>> edge;
    priority_queue<pair<T, int>, vector<pair<T, int>>,
        greater<pair<T, int>>> pq;
    vector<int> vis;
    void init(int _n, int _k, T _inf){
        // n points, 1~k 是重要點, type T的INF
        n = _n, k = _k, inf = _inf;
        dp.assign(n + 1, vector<T>(1 << k, inf));
        edge.resize(n + 1);
    }
    void addEdge(int u, int v, T w){ // u <-(w)-> v
        edge[u].emplace_back(v, w);
        edge[v].emplace_back(u, w);
    }
    void dijkstra(int s, int cnt){
        vis.assign(n + 1, 0);
        while(!pq.empty()){
            auto [d, u] = pq.top(); pq.pop();
            if(vis[u]) continue;
            vis[u] = 1;
            for(auto &[v, w] : edge[u]){
                // if(cnt > 1 && v <= k) continue;
                if(dp[v][s] > dp[u][s] + w){
                    dp[v][s] = dp[u][s] + w;
                    pq.push({dp[v][s], v});
                }
            }
        }
    }
    T run(){ // return total cost 0(nk*2^k + n^2*2^k)
        for(int i = 1; i <= k; ++i) dp[i][1 << (i - 1)] = 0;
        for(int s = 1; s < (1 << k); ++s){
            int cnt = 0, tmp = s;
            while(tmp) cnt += (tmp & 1), tmp >>= 1;
            for(int i = k + 1; i <= n; ++i){
                for(int sb = s & (s-1); sb; sb = s & (sb-1))
                    dp[i][s] =
                        min(dp[i][s], dp[i][sb] + dp[i][s ^ sb]);
                for(int i = (cnt > 1 ? k + 1 : 1); i <= n; ++i)
                    if(dp[i][s] != inf) pq.push({dp[i][s], i});
                dijkstra(s, cnt);
            }
        }
        T res = inf;
        for(int i = 1; i <= n; ++i)
            res = min(res, dp[i][(1 << k) - 1]);
        return res;
    }
}

```

## 6.7 BCC based on vertex \*

```

struct BccVertex {
    int n, nScc, step, dfn[MXNN], low[MXNN];
    vector<int> E[MXNN], sccv[MXNN];
    int top, stk[MXNN];
    void init(int _n) {
        n = _n; nScc = step = 0;
        for (int i=0; i<n; i++) E[i].clear();
    }
    void addEdge(int u, int v)
    { E[u].PB(v); E[v].PB(u); }
    void DFS(int u, int f) {
        dfn[u] = low[u] = step++;
    }
}

```



```

stk[top++] = u;
for (auto v:E[u]) {
    if (v == f) continue;
    if (dfn[v] == -1) {
        DFS(v,u);
        low[u] = min(low[u], low[v]);
        if (low[v] >= dfn[u]) {
            int z;
            sccv[nScc].clear();
            do {
                z = stk[--top];
                sccv[nScc].PB(z);
            } while (z != v);
            sccv[nScc++].PB(u);
        }
    } else
        low[u] = min(low[u], dfn[v]);
}
}
vector<vector<int>> solve() {
    vector<vector<int>> res;
    for (int i=0; i<n; i++)
        dfn[i] = low[i] = -1;
    for (int i=0; i<n; i++)
        if (dfn[i] == -1) {
            top = 0;
            DFS(i,i);
        }
    REP(i,nScc) res.PB(sccv[i]);
    return res;
}
}graph;

```

## 6.8 Strongly Connected Component \*

```

struct Scc{
    int n, nScc, vst[MXN], bln[MXN];
    vector<int> E[MXN], rE[MXN], vec;
    void init(int _n){
        n = _n;
        for (int i=0; i<MXN; i++)
            E[i].clear(), rE[i].clear();
    }
    void addEdge(int u, int v){
        E[u].PB(v); rE[v].PB(u);
    }
    void DFS(int u){
        vst[u]=1;
        for (auto v : E[u]) if (!vst[v]) DFS(v);
        vec.PB(u);
    }
    void rDFS(int u){
        vst[u] = 1; bln[u] = nScc;
        for (auto v : rE[u]) if (!vst[v]) rDFS(v);
    }
    void solve(){
        nScc = 0;
        vec.clear();
        FZ(vst);
        for (int i=0; i<n; i++)
            if (!vst[i]) DFS(i);
        reverse(vec.begin(),vec.end());
        FZ(vst);
        for (auto v : vec)
            if (!vst[v]){
                rDFS(v); nScc++;
            }
    }
};

```

## 6.9 差分約束 \*

約束條件  $V_j - V_i \leq W$  建邊  $V_i \rightarrow V_j$  權重為  $W$  → bellman-ford or spfa

## 7 String

### 7.1 PalTree \*

```

// len[s]是對應的回文長度
// num[s]是有幾個回文後綴
// cnt[s]是這個回文子字串在整個字串中的出現次數
// fail[s]是他長度次長的回文後綴，aba的fail是a
const int MXN = 1000010;
struct PalT{

```

```

    int nxt[MXN][26], fail[MXN], len[MXN];
    int tot, lst, n, state[MXN], cnt[MXN], num[MXN];
    int diff[MXN], sfail[MXN], fac[MXN], dp[MXN];
    char s[MXN]={'-1'};
    int newNode(int l, int f){
        len[tot]=l, fail[tot]=f, cnt[tot]=num[tot]=0;
        memset(nxt[tot], 0, sizeof(nxt[tot]));
        diff[tot]=(l>0?l-len[f]:0);
        sfail[tot]=(l>0&&diff[tot]==diff[f]?sfail[f]:f);
        return tot++;
    }
    int getfail(int x){
        while(s[n-len[x]-1]!=s[n]) x=fail[x];
        return x;
    }
    int getmin(int v){
        dp[v]=fac[n-len[sfail[v]]-diff[v]];
        if(diff[v]==diff[fail[v]])
            dp[v]=min(dp[v], dp[fail[v]]);
        return dp[v]+1;
    }
    int push(){
        int c=s[n]-'a', np=getfail(lst);
        if(!(lst=nxt[np][c])){
            lst=newNode(len[np]+2, nxt[getfail(fail[np])][c]);
            nxt[np][c]=lst; num[lst]=num[fail[lst]]+1;
        }
        fac[n]=n;
        for(int v=lst; len[v]>0; v=sfail[v])
            fac[n]=min(fac[n], getmin(v));
        return ++cnt[lst], lst;
    }
    void init(const char *_s){
        tot=lst=n=0;
        newNode(0,1), newNode(-1,1);
        for(; s[n];) s[n+1]=s[n], ++n, state[n-1]=push();
        for(int i=tot-1; i>1; i--) cnt[fail[i]]+=cnt[i];
    }
}palt;

```

### 7.2 SuffixArray

```

const int MXN = 1e6;
// sa[i]: idx of ith rank, rk[i]: rank of idx
// he[i]: sa[i], sa[i - 1] 前he[i]個字元相同
int ct[MXN], he[MXN], rk[MXN];
int sa[MXN], tsa[MXN], tp[MXN][2];
void suffix_array(string ip){ // 0-base
    int len = ip.size();
    int alp = 256;
    MEM(ct, 0);
    for(int i = 0; i < len; i++) ct[ip[i] + 1]++;
    for(int i = 1; i < alp; i++) ct[i] += ct[i - 1];
    for(int i = 0; i < len; i++) rk[i] = ct[ip[i]];
    for(int i = 1; i < len; i *= 2){
        for(int j = 0; j < len; j++){
            if(j + i >= len) tp[j][1] = 0;
            else tp[j][1] = rk[j + i] + 1;
            tp[j][0] = rk[j];
        }
        memset(ct, 0, sizeof(ct));
        for(int j = 0; j < len; j++) ct[tp[j][1] + 1]++;
        for(int j = 1; j < len+2; j++) ct[j] += ct[j - 1];
        for(int j = 0; j < len; j++) tsa[ct[tp[j][1]]++] = j;
        memset(ct, 0, sizeof(ct));
        for(int j = 0; j < len; j++) ct[tp[j][0] + 1]++;
        for(int j = 1; j < len+1; j++) ct[j] += ct[j - 1];
        for(int j = 0; j < len; j++)
            sa[ct[tp[j][0]]++] = tsa[j];
        rk[sa[0]] = 0;
        for(int j = 1; j < len; j++){
            if( tp[sa[j]][0] == tp[sa[j - 1]][0] &&
               tp[sa[j]][1] == tp[sa[j - 1]][1] )
                rk[sa[j]] = rk[sa[j - 1]];
            else
                rk[sa[j]] = j; } }
    for(int i = 0, h = 0; i < len; i++){
        if(rk[i] == 0) h = 0;
        else{
            int j = sa[rk[i] - 1];
            h = max(0, h - 1);
            for(; ip[i + h] == ip[j + h]; h++); }
    }
}

```

```
he[rk[i]] = h; } }
```

### 7.3 MinRoation \*

```
//rotate(begin(s),begin(s)+minRotation(s),end(s))
int minRotation(string s) {
    int a = 0, N = s.size(); s += s;
    rep(b,0,N) rep(k,0,N) {
        if(a+k == b || s[a+k] < s[b+k])
            {b += max(0, k-1); break;}
        if(s[a+k] > s[b+k]) {a = b; break;}
    } return a;
}
```

### 7.4 RollingHash

```
struct RollingHash { // 0-base, need MOD
    const int p1 = 44129; // 65537, 40961, 90001, 971651
    int n; vector<ll> pre, ppow;
    void init(string s) { // 0(n)
        n = s.size();
        pre.resize(n + 1); ppow.resize(n + 1);
        pre[0] = 0, ppow[0] = 1;
        for (int i = 0; i < n; i++)
            pre[i + 1] = (pre[i] * p1 + s[i]) % MOD,
            ppow[i + 1] = ppow[i] * p1 % MOD;
    }
    ll query(int l, int r) { // [l, r], 0(1)
        ll ret = pre[r + 1] - pre[l] * ppow[r - l + 1];
        return (ret % MOD + MOD) % MOD; } }
```

### 7.5 KMP

在 k 結尾的情況下，這個子字串可以由開頭長度為  $(k + 1) - (fail[k] + 1)$  的部分重複出現來表達  
 $fail[k] + 1$  為次長相同前綴後綴長度  
 如果我們不只想求最多，那可能的長度由大到小會是  
 $fail[k]+1, fail[fail[k]+1], fail[fail[fail[k]]]+1...$   
 直到有值為 -1 為止

```
const int MXN = 2e7 + 5;
int fail[MXN]; vector<int> mi;
void kmp(string &t, string &p){ // 0(n), 0-base
    // pattern match in target, idx store in mi
    mi.clear();
    if (p.size() > t.size()) return;
    for (int i = 1, j = fail[0] = -1; i < p.size(); ++i){
        while (j >= 0 && p[j + 1] != p[i]) j = fail[j];
        if (p[j + 1] == p[i]) j++;
        fail[i] = j; }
    for (int i = 0, j = -1; i < t.size(); ++i){
        while (j >= 0 && p[j + 1] != t[i]) j = fail[j];
        if (p[j + 1] == t[i]) j++;
        if (j == p.size() - 1)
            j = fail[j], mi.pb(i - p.size() + 1); } }
```

### 7.6 LCS & LIS

LIS: 最長遞增子序列

LCS: 最長共同子字串 (利用 LIS), 但常數可能較大

```
int lis(vector<ll> &v){ // 0(nlgn)
    vector<ll> p;
    for(int i = 0; i < v.size(); ++i)
        if(p.empty() || p.back() < v[i]) p.pb(v[i]);
        else *lower_bound(p.begin(), p.end(), v[i]) = v[i];
    return p.size(); }
```

```
int lcs(string s, string t){ // 0(nlgn)
    map<char, vector<int>> > mp;
    for(int i = 0; i < s.size(); ++i) mp[s[i]].pb(i);
    vector<int> p;
    for(int i = 0; i < t.size(); ++i){
        auto &v = mp[t[i]];
        for(int j = v.size() - 1; j >= 0; --j)
            if(p.empty() || p.back() < v[j]) p.pb(v[j]);
            else *lower_bound(p.begin(), p.end(), v[j]) = v[j];
    } return p.size(); }
```

### 7.7 Aho-Corasick \*

```
struct ACautomata{
    struct Node{
        int cnt,i;
        Node *go[26], *fail, *dic;
```

```
Node(){
    cnt = 0; fail = 0; dic = 0; i = 0;
    memset(go,0,sizeof(go));
    }
}pool[1048576],*root;
int nMem,n_pattern;
Node* new_Node(){
    pool[nMem] = Node();
    return &pool[nMem++];
}
void init() {
    nMem=0;root=new_Node();n_pattern=0;
    add("");
}
void add(const string &str) { insert(root,str,0); }
void insert(Node *cur, const string &str, int pos){
    for(int i=pos;i<str.size();i++){
        if(!cur->go[str[i]-'a'])
            cur->go[str[i]-'a'] = new_Node();
        cur=cur->go[str[i]-'a'];
    }
    cur->cnt++; cur->i=n_pattern++;
}
void make_fail(){
    queue<Node*> que;
    que.push(root);
    while (!que.empty()){
        Node* fr=que.front(); que.pop();
        for (int i=0; i<26; i++){
            if (fr->go[i]){
                Node *ptr = fr->fail;
                while (ptr && !ptr->go[i]) ptr = ptr->fail;
                fr->go[i]->fail=ptr?(ptr->go[i]:root);
                fr->go[i]->dic=(ptr->cnt?ptr:ptr->dic);
                que.push(fr->go[i]);
            } } } }
void query(string s){
    Node *cur=root;
    for(int i=0;i<(int)s.size();i++){
        while(cur&&!cur->go[s[i]-'a']) cur=cur->fail;
        cur=(cur?cur->go[s[i]-'a']:root);
        if(cur->i>=0) ans[cur->i]++;
        for(Node *tmp=cur->dic;tmp;tmp=tmp->dic)
            ans[tmp->i]++;
    } } // ans[i] : number of occurrence of pattern i
}AC;
```

### 7.8 Z Value \*

```
int z[MAXN];
void Z_value(const string& s) { //z[i] = lcp(s[1...],s[
    i...])
    int i, j, left, right, len = s.size();
    left=right=0; z[0]=len;
    for(i=1;i<len;i++){
        j=max(min(z[i-left],right-i),0);
        for(;i+j<len&&s[i+j]==s[j];j++);
        z[i]=j;
        if(i+z[i]>right) {
            right=i+z[i];
            left=i;
        }
    } }
```

### 7.9 manacher

```
const int MXN = 1e7 + 5;
struct Manacher{ // 0-base 每個點為中心的最長回文長度
    string st; int p[MXN * 2];
    void init(string s){ // 0(n)
        MEM(p, 0); st.clear();
        st.push_back('$'); st.push_back('#');
        for(int i = 0; i < s.size(); ++i)
            st.push_back(s[i]), st.push_back('#');
        st.push_back('*');
        int mx = 0, id = 0;
        for(int i = 1; i < st.size(); ++i){
            p[i] = mx > i ? min(p[id << 1] - i, mx - i) : 1;
            while(st[i + p[i]] == st[i - p[i]]) p[i]++;
            if(i + p[i] > mx) mx = i + p[i], id = i; } }
    // bt=1: middle between mid, mid+1
    int query(int mid, bool bt = 0) {
        return p[mid * 2 + 2 + bt] - 1; } }
```

## 8 Data Structure

### 8.1 Treap

Treap \*th = 0  
 th = merge(th, new Treap(val))  $\Rightarrow$  新增元素到 th  
 th = merge(merge(tl, tm), tr)  $\Rightarrow$  合併 tl,tm,tr 到 th  
 split(th, k, tl, tr)  $\Rightarrow$  分割 th, tl 的元素  $\leq k$  (失去 BST 性質後不能用)  
 kth(th, k, tl, tr)  $\Rightarrow$  分割 th, gsz(tl)  $\leq k$  (< when gsz(th) < k)  
 gsz  $\Rightarrow$  get size | gsum  $\Rightarrow$  get sum | th->rev ^= 1  $\Rightarrow$  反轉 th  
 帶懶標版本, 並示範 sum/rev 如何 pull/push  
 注意 Treap 複雜度好但常數大, 動作能用其他方法就用, 並做 io 等優化

```
struct Treap{
    Treap *l, *r;
    int pri, sz, rev;
    ll val, sum;
    Treap(int _val): l(0), r(0),
        pri(rand()), sz(1), rev(0),
        val(_val), sum(_val){};

    ll gsz(Treap *x){ return x ? x->sz : 0; }
    ll gsum(Treap *x){ return x ? x->sum : 0; }

    Treap* pull(Treap *x){
        x->sz = gsz(x->l) + gsz(x->r) + 1;
        x->sum = x->val + gsum(x->l) + gsum(x->r);
        return x; }
    void push(Treap *x){
        if(x->rev){
            swap(x->l, x->r);
            if(x->l) x->l->rev ^= 1;
            if(x->r) x->r->rev ^= 1;
            x->rev = 0; } }

    Treap* merge(Treap* a, Treap* b){
        if(!a || !b) return a ? a : b;
        push(a), push(b);
        if(a->pri > b->pri){
            a->r = merge(a->r, b);
            return pull(a); }
        else{
            b->l = merge(a, b->l);
            return pull(b); } }

    void split(Treap *x, int k, Treap *&a, Treap *&b){
        if(!x) a = b = 0;
        else{
            push(x);
            if(x->val <= k) a = x, split(x->r, k, a->r, b);
            else b = x, split(x->l, k, a, b->l);
            pull(x); } }

    void kth(Treap *x, int k, Treap *&a, Treap *&b){
        if(!x) a = b = 0;
        else{
            push(x);
            if(gsz(x->l) < k)
                a = x, kth(x->r, k - gsz(x->l) - 1, a->r, b);
            else b = x, kth(x->l, k, a, b->l);
            pull(x); } }
```

### 8.2 BIT

bit.init(n)  $\Rightarrow$  1-base  
 bit.add(i, x)  $\Rightarrow$  add a[i] by x  
 bit.sum(i)  $\Rightarrow$  get sum of [1, i]  
 bit.kth(k)  $\Rightarrow$  get kth small number (by using bit.add(num, 1))  
 維護差分可以變成區間加值, 單點求值

```
const int MXN = 1e6+5;
struct BIT{
    ll n, a[MXN];
    void init(int _n){ n = _n; MEM(a, 0); }
    void add(int i, int x){
        for(; i <= n; i += i & -i) a[i] += x; }
    int sum(int i){
        int ret = 0;
        for(; i > 0; i -= i & -i) ret += a[i];
        return ret; }
    int kth(int k){
        int res = 0;
        for(int i = 1 << __lg(n); i > 0; i >>= 1)
            if(res + i <= n && a[res+i] < k) k -= a[res+=i];
        return res; } }
```

### 8.3 二維偏序 \*

```
struct Node {
    int x, y, id;
    bool operator < (const Node &b) const {
        if(x == b.x) return y < b.y;
        return x < b.x;};
    struct TDPO {
        vector<Node> p; vector<ll> ans;
        void init(vector<Node> _p) {
            p = _p; bit.init(MXN);
            ans.resize(p.size());
            sort(p.begin(), p.end());
        }
        void bulid() {
            int sz = p.size();
            for(int i = 0; i < sz; ++i) {
                ans[p[i].id] = bit.sum(p[i].y - 1);
                bit.add(p[i].y, 1);};
        }
```

### 8.4 持久化 \*

```
struct Seg {
    // Persistent Segment Tree, single point modify,
    // range query sum
    // 0-indexed, [l, r)
    static Seg mem[M], *pt;
    int l, r, m, val;
    Seg* ch[2];
    Seg () = default;
    Seg (int _l, int _r) : l(_l), r(_r), m(l + r >> 1),
        val(0) {
        if (r - l > 1) {
            ch[0] = new (pt++) Seg(l, m);
            ch[1] = new (pt++) Seg(m, r);
        }
    }
    void pull() {val = ch[0]->val + ch[1]->val;}
    Seg* modify(int p, int v) {
        Seg *now = new (pt++) Seg(*this);
        if (r - l == 1) {
            now->val = v;
        } else {
            now->ch[p >= m] = ch[p >= m]->modify(p, v);
            now->pull();
        }
        return now;
    }
    int query(int a, int b) {
        if (a <= l && r <= b) return val;
        int ans = 0;
        if (a < m) ans += ch[0]->query(a, b);
        if (m < b) ans += ch[1]->query(a, b);
        return ans;
    }
} Seg::mem[M], *Seg::pt = mem;
// Init Tree
Seg *root = new (Seg::pt++) Seg(0, n);
```

### 8.5 2D 線段樹

```
// 2D range add, range sum in log^2
struct seg {
    int l, r;
    ll sum, lz;
    seg *ch[2];
    seg(int _l, int _r) : l(_l), r(_r), sum(0), lz(0) {}
    void push() {
        if (lz) ch[0]->add(l, r, lz), ch[1]->modify(l, r,
            lz), lz = 0;
    }
    void pull() {sum = ch[0]->sum + ch[1]->sum;}
    void add(int _l, int _r, ll d) {
        if (_l <= l && r <= _r) {
            sum += d * (r - l);
            lz += d;
            return;
        }
        if (!ch[0]) ch[0] = new seg(l, l + r >> 1), ch[1] =
            new seg(l + r >> 1, r);
        push();
        if (_l < l + r >> 1) ch[0]->add(_l, _r, d);
        if (l + r >> 1 < _r) ch[1]->add(_l, _r, d);
```

```

    pull();
}
ll qsum(int _l, int _r) {
    if (_l <= l && r <= _r) return sum;
    if (!ch[0]) return lz * (min(r, _r) - max(l, _l));
    push();
    ll res = 0;
    if (_l < l + r >> 1) res += ch[0]->qsum(_l, _r);
    if (l + r >> 1 < _r) res += ch[1]->qsum(_l, _r);
    return res;
}
};
struct seg2 {
    int l, r;
    seg v, lz;
    seg2 *ch[2];
    seg2(int _l, int _r) : l(_l), r(_r), v(0, N), lz(0, N)
    {
        if (l < r - 1) ch[0] = new seg2(l, l + r >> 1), ch[1] = new seg2(l + r >> 1, r);
    }
    void add(int _l, int _r, int _l2, int _r2, ll d) {
        v.add(_l2, _r2, d * (min(r, _r) - max(l, _l)));
        if (_l <= l && r <= _r) {
            lz.add(_l2, _r2, d);
            return;
        }
        if (_l < l + r >> 1) ch[0]->add(_l, _r, _l2, _r2, d);
        if (l + r >> 1 < _r) ch[1]->add(_l, _r, _l2, _r2, d);
    }
    ll qsum(int _l, int _r, int _l2, int _r2) {
        ll res = v.qsum(_l2, _r2);
        if (_l <= l && r <= _r) return res;
        res += lz.qsum(_l2, _r2) * (min(r, _r) - max(l, _l));
        if (_l < l + r >> 1) res += ch[0]->query(_l, _r, _l2, _r2);
        if (l + r >> 1 < _r) res += ch[1]->query(_l, _r, _l2, _r2);
        return res;
    }
};

```

## 8.6 Disjoint Set

```

struct DisjointSet {
    int fa[MXN], h[MXN], top;
    struct Node {
        int x, y, fa, h;
        Node(int _x = 0, int _y = 0, int _fa = 0, int _h = 0) : x(_x), y(_y), fa(_fa), h(_h) {}
    } stk[MXN];
    void init(int n) {
        top = 0;
        for (int i = 1; i <= n; i++) fa[i] = i, h[i] = 0;
    }
    int find(int x) {return x == fa[x] ? x : find(fa[x]);}
    void merge(int u, int v) {
        int x = find(u), y = find(v);
        if (h[x] > h[y]) swap(x, y);
        stk[top++] = Node(x, y, fa[x], h[y]);
        if (h[x] == h[y]) h[y]++;
        fa[x] = y;
    }
    void undo(int k=1) { //undo k times
        for (int i = 0; i < k; i++) {
            Node &it = stk[--top];
            fa[it.x] = it.fa;
            h[it.y] = it.h;
        }
    }
};

```

## 8.7 Black Magic

```

#include <bits/extc++.h>
using namespace __gnu_pbds;
typedef tree<int, null_type, less<int>, rb_tree_tag,
tree_order_statistics_node_update> set_t;
#include <ext/pb_ds/assoc_container.hpp>
typedef cc_hash_table<int, int> umap_t;
typedef priority_queue<int> heap;
#include <ext/rope>
using namespace __gnu_cxx;
int main() {

```

```

// Insert some entries into s.
set_t s; s.insert(12); s.insert(505);
// The order of the keys should be: 12, 505.
assert(*s.find_by_order(0) == 12);
assert(*s.find_by_order(3) == 505);
// The order of the keys should be: 12, 505.
assert(s.order_of_key(12) == 0);
assert(s.order_of_key(505) == 1);
// Erase an entry.
s.erase(12);
// The order of the keys should be: 505.
assert(*s.find_by_order(0) == 505);
// The order of the keys should be: 505.
assert(s.order_of_key(505) == 0);

heap h1, h2; h1.join(h2);

rope<char> r[2];
r[1] = r[0]; // persistenet
string t = "abc";
r[1].insert(0, t.c_str());
r[1].erase(1, 1);
cout << r[1].substr(0, 2);
}

```

## 9 Others

### 9.1 SOS dp \*

```

for(int i = 0; i < (1<<N); ++i)
    F[i] = A[i];
for(int i = 0; i < N; ++i) for(int mask = 0; mask < (1<<N); ++mask) {
    if(mask & (1<<i))
        F[mask] += F[mask^(1<<i)];
}

```

### 9.2 MO's Algorithm \*

```

struct MoSolver {
    struct query {
        int l, r, id;
        bool operator < (const query &o) {
            if (l / C == o.l / C) return (l / C) & 1 ? r > o.r : r < o.r;
            return l / C < o.l / C;
        }
    };
    int cur_ans;
    vector<int> ans;
    void add(int x) {
        // do something
    }
    void sub(int x) {
        // do something
    }
    vector<query> Q;
    void add_query(int l, int r, int id) {
        // [l, r]
        Q.push_back({l, r, id});
        ans.push_back(0);
    }
    void run() {
        sort(Q.begin(), Q.end());
        int pl = 0, pr = 0;
        cur_ans = 0;
        for (query &i : Q) {
            while (pl > i.l) add(a[--pl]);
            while (pr < i.r) add(a[pr++]);
            while (pl < i.l) sub(a[pl++]);
            while (pr > i.r) sub(a[--pr]);
            ans[i.id] = cur;
        }
    }
};

```











